Phys 273 - Homework #8

Questions highlighted in yellow are optional.

1) Consider the function

$$y(x) = \begin{cases} 0 & -L < x < -1/2 \\ d & -1/2 < x < 1/2 \\ 0 & 1/2 < x < L \end{cases}$$, and repeating with period 2L.

In this expression, (d) is a constant, and L > 1.

a) Sketch this function for the case L = 1, assuming d = 1.

b) Sketch this function for the case L =4, assuming d = 1.

c) Consider L = 1 and d = 1 again. Calculate the $\{c_n\}$ expansion coefficients for the complex Fourier Series for this function. Recall that the complex Fourier Series is given by

$$f(x) = \sum_{n=-\infty}^{n=\infty} c_n e^{in\pi x/L}$$

where $$c_n = \frac{1}{2L}\int_{-L}^{L} f(x) e^{-in\pi x/L} dx$$

d) Let q = nπ. Re-write the expansion coefficients from part (c) as a function of (q), and make a plot of the expansion coefficients as a function of (q).

e) Now we let L go to infinity, so that the function is no longer periodic. Sketch the function again and calculate its Fourier Transform, A(k). Recall that A(k) is given by:

$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

f) Plot the Fourier Transform A(k) as a function of k.

g) Compare the result from part (d) to the result from part (f). What are the similarities, and what are the differences?

2) A transverse wave on an infinitely long string is described by

$$y(x,t) = 0.5 \sin\left(\frac{\pi x}{2} - 50\pi t\right)$$

a) What are the amplitude, wavelength, and wave number of the wave?

b) What are the frequency (f), period, and velocity of the wave?

c) What is the maximum transverse speed of any particle in the string.

3) A wave on a string with a frequency of 20 Hz travels with a velocity of 80 m/s.

a) If the mass density of the string is 0.1 kg/m, what is the tension of the string?

b) What is the distance between two points on the wave which have a phase difference of 30 degrees?

4) Recall problem #2 of homework 7, where we calculated the Fourier Series for this square wave:

$$f(x) = \begin{cases} -1, -L < x < 0 \\ 1, 0 < x < L \end{cases}, \text{ periodic with period 2L.}$$

The result was

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{n\pi} \sin\left(\frac{n\pi x}{L}\right), \text{ odd (n) only}$$

Let's re-calculate this Fourier Series again, but this time let's use the complex form of the series:

$$f(x) = \sum_{n=-\infty}^{n=\infty} c_n e^{in\pi x/L}$$

Fourier's trick tells us that the coefficients $\{c_n\}$ can be calculated according to:

$$c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-in\pi x/L} dx$$

a) Calculate (c_0), for this square.
b) Calculate the rest of the Fourier coefficients $\{c_n\}$ for this square wave.
c) Since this f(x) is purely real, the coefficients $\{c_n\}$ should have the property that

$$c_n = c_{-n}^*$$

Check to see if this is true using your result from part (b).

d) The real coefficients $\{a_n\}$ and $\{b_n\}$ that you calculated on homework #7 should be related to the complex coefficients from parts (a) and (b) according to:

$$a_n = c_n + c_{(-n)}$$
$$b_n = i(c_n - c_{(-n)})$$
$$a_0 = 2c_0$$

Check to see if this is true.