Physics 273 - Homework #2

1) (3 points) Complex numbers are commonly expressed in two forms: cartesian ($z = x + iy$) and polar ($z = Ae^{i\vartheta}$). Convert $\frac{1}{1-i}$ to both of these forms, and state explicitly the values of x, y, A, and θ.

2) For any complex number, $\left|\frac{z^*}{z}\right| = 1$.

a) (3 points) Show that this is true using the cartesian form ($z = x + iy$).

b) (3 points) Show that this is true using the polar form ($z = Ae^{i\vartheta}$).

Which method is easier?

3) (3 points each) Prove the following identities for complex numbers. You may use either the cartesian form ($z = x + iy$) or the polar form ($z = Ae^{i\vartheta}$) .

a) Re(z) = (z +z*)/2
b) Im(z) = (z - z*)/2i
c) cos(θ) = [exp(iθ) + exp(-iθ)]/2
d) sin(θ) = [exp(iθ) - exp(-iθ)]/2i

4) (3 points each)

a) What is the amplitude and phase of the following complex function?:

$$E(t) = (1 - 2i)e^{i\omega t}$$

b) What is the amplitude and phase of the complex conjugate function?

Continued on next page.

Questions highlighted in yellow are optional.

5a) **(Optional)** The position of a simple harmonic oscillator as a function of time is described by

$$x(t) = Ae^{i(\omega_0 t+\delta)},$$

and by differentiating we can see that its velocity is described by:

$$\dot{x}(t) = i\omega_0 Ae^{i(\omega_0 t+\delta)}$$

Question: what is the significance of the factor of (i) which appears in front of the velocity function? In other words, does the (i) have any physical consequences? (You may answer in one or two sentences.)

5b) **(Optional)** **Initial conditions.** Suppose that the simple harmonic oscillator is set into motion with the following initial conditions: $x(t=0) = x_0$, and $\dot{x}(t=0) = 0$, where x_0 is a constant with units of meters. Solve for (A) and (δ) under these conditions.

5c) **(Optional)** **Initial conditions.** Now suppose that the simple harmonic oscillator is set into motion with the following initial conditions: $x(t=0) = 0$, and $\dot{x}(t=0) = v_0$, where v_0 is a constant with units of meters per second. Solve for (A) and (δ) under these conditions.

6) **Numerical solution of the simple harmonic oscillator.** In this problem we will extend your numerical solutions from Homework #1 to a more complicated physical system, the simple harmonic oscillator. Like before, the velocity and position at each moment in time will be given by

$$v_{i+1} = v_i + a_{i+1} \cdot \Delta t$$
$$x_{i+1} = x_i + v_{i+1} \cdot \Delta t$$

Since the force law for the simple harmonic oscillator is F(x) = -kx, the acceleration at each moment in time is given by

$$a = \frac{-kx}{m}.$$

In this case, the acceleration depends upon the position, which varies as a function of time. Numerically, we can say that the new acceleration at time step number (i+1) is given by

$$a_{i+1} = \frac{-kx_{i+1}}{m}.$$

However, we won't know the new position variable (x_{i+1}) until *after* we have calculated the acceleration and velocity so we can't use (x_{i+1}) to calculate (a_{i+1}). To avoid this difficulty, we instead use the position from the previous time step (x_i) to calculate (a_{i+1}):

$$a_{i+1} = \frac{-kx_i}{m}$$

This is fine as long as the time step is small enough that the position is not changing very much in one step (i.e., as long as (x_{i+1}) is not very different from (x_i)).

6a) Let the mass of the oscillator be 10 kg, the spring constant be 10,000 N/m, and the time step be $\Delta t = 0.001$ seconds. Calculate the numerical solution for the position as a function of time for 1,000 time steps (a total of one second). Let the initial conditions be $x(t=0) = 0$, and $v(t=0) = 3$ m/s. Make a plot of the resulting x(t) function.

6b) Measure the frequency of the oscillation that you observe in your numerical solution, and compare it to the expected frequency for this oscillator.

6c) Measure the amplitude and phase of the oscillation that you observe in your numerical solution, and compare them to the expected values based upon your answer to question (5c).

6d) Try changing the initial velocity from 3 m/s to a few other values, while keeping the initial position set to zero. Does the frequency of the oscillation change? Does the amplitude of the oscillation change? Does the phase of the oscillation change?