Phys 273 – Formula Sheet #2

$$\ddot{y}_p + 2\omega_0^2 y_p - \omega_0^2(y_{p+1} + y_{p-1}) = 0,\ \omega_0^2 = T/ml, \rightarrow \vec{y}(t) = \sum_{n=1}^{N} c_n \vec{q}_n e^{i\omega_n t}$$

$$\vec{q}_n = \left(\sin\left(\frac{n\pi}{N+1}\right), \sin\left(\frac{2n\pi}{N+1}\right), \ldots, \sin\left(\frac{Nn\pi}{N+1}\right)\right),\ A_{pn} = \sin\left(\frac{pn\pi}{N+1}\right), c_n = a_n + ib_n\ , a_n = \frac{\vec{y}_0 \cdot \vec{q}_n}{|q_n|^2},\quad b_n = \frac{-\vec{v}_0 \cdot \vec{q}_n}{\omega_n |q_n|^2},$$

$$\omega_n = 2\omega_0 \sin\left(\frac{n\pi}{2(N+1)}\right)$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 y}{\partial t^2},\ v = \sqrt{T/\rho}$$

$$y(x,t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right) e^{i\omega_n t},\ \omega_n = \sqrt{\frac{T}{\rho}}\frac{n\pi}{L},\ c_n = a_n + ib_n,\ a_n = \frac{2}{L}\int_0^L y(x,t=0)\sin\left(\frac{n\pi x}{L}\right)dx,$$

$$b_n = \frac{-2}{\omega_n L}\int_0^L \dot{y}(x,t=0)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$y(x,t) = A\sin(kx - \omega t),\ k = \frac{2\pi}{\lambda},\ \omega = 2\pi f = \frac{2\pi}{T},\ v = \lambda f = \frac{\omega}{k}$$

$$y(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk A(k) e^{i(kx+\omega t)},\ A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx f(x) e^{-ikx}$$

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)dx = \frac{L}{2}\delta_{nm},\ \sum_{j=1}^{N}\sin\left(\frac{jn\pi}{N+1}\right)\sin\left(\frac{jm\pi}{N+1}\right) = \left(\frac{N+1}{2}\right)\delta_{nm}$$

$$Z = \frac{F}{\dot{y}} = \frac{T}{v_p} = \rho v_p$$

$$\frac{A_2}{A_1} = \frac{2Z_1}{Z_1 + Z_2},\quad \frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$P = \frac{1}{2}Z\omega^2 A^2$$

$$v = \frac{1}{\sqrt{L_0 C_0}},\ Z_0 = \sqrt{\frac{L_0}{C_0}}$$

$$\frac{V_-}{V_+} = \frac{Z_L - Z_0}{Z_L + Z_0},\quad \frac{V_L}{V_+} = \frac{2Z_L}{Z_L + Z_0},\quad \frac{I_-}{I_+} = \frac{Z_0 - Z_L}{Z_0 + Z_L},\ \frac{I_L}{I_+} = \frac{2Z_0}{Z_L + Z_0}$$

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$$y(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk A(k) e^{i(kx-\omega(k)t)}$$

$$v_p \equiv \frac{\omega(k)}{k},\ v_g = \frac{d\omega}{dk}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right],$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx,\ b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/L},\ c_n = \frac{1}{2L}\int_{-L}^{L} f(x) e^{-in\pi x/L} dx,$$

$$a_n = c_n + c_{-n},\ b_n = i(c_n - c_{-n}),\ a_0 = 2c_0$$

$$c_n = \begin{cases} \frac{1}{2}(a_{-n} + ib_{-n}), & n < 0 \\ \frac{1}{2}a_0, & n = 0 \\ \frac{1}{2}(a_n - ib_n), & n > 0 \end{cases}$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk A(k) e^{ikx},\ A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx f(x) e^{-ikx}$$

$$\frac{\partial^2 P'}{\partial x^2} = \frac{1}{v_s^2}\frac{\partial^2 P'}{\partial t^2},\ v_s = \sqrt{\frac{\gamma P_0}{\rho_0}}$$