(1) Force between two point charges: (21.14)

(a) If the force is repulsive, both charges must be positive since the total charge is positive. Call the total charge \( Q \).

\[
Q_1 + Q_2 = Q, \quad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ(Q - Q)}{d^2} \rightarrow Q_1^2 - Q_1 + \frac{Fd^2}{k} = 0
\]

\[
Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}
\]

\[
= \frac{1}{2} \left[ (90.0 \times 10^{-6} \text{C}) \pm \sqrt{(90.0 \times 10^{-6} \text{C})^2 - 4 \frac{(12.0 \text{N})(1.16 \text{m})^2}{8.988 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2}} \right]
\]

\[
= 60.1 \times 10^{-6} \text{C}, \ 29.9 \times 10^{-6} \text{C}
\]

(b) If the force is attractive, then the charges are of opposite sign. The value used for \( F \) must then be negative. Other than that, the solution method is the same as for part (a).

\[
Q_1 + Q_2 = Q, \quad F = \frac{kQ_1Q_2}{d^2} = \frac{kQ(Q - Q)}{d^2} \rightarrow Q_1^2 - Q_1 + \frac{Fd^2}{k} = 0
\]

\[
Q_1 = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2} = \frac{Q \pm \sqrt{Q^2 - 4\frac{Fd^2}{k}}}{2}
\]

\[
= \frac{1}{2} \left[ (90.0 \times 10^{-6} \text{C}) \pm \sqrt{(90.0 \times 10^{-6} \text{C})^2 - 4 \frac{(-12.0 \text{N})(1.16 \text{m})^2}{8.988 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2}} \right]
\]

\[
= 106.8 \times 10^{-6} \text{C}, \ -16.8 \times 10^{-6} \text{C}
\]
(2) Force between more than two point charges: (21.16)

Determine the force on the upper right charge, and then use the symmetry of the configuration to determine the force on the other charges.

The force at the upper right corner of the square is the vector sum of the forces due to the other three charges. Let the variable $d$ represent the 0.100 m length of a side of the square, and let the variable $Q$ represent the 4.15 mC charge at each corner.

$$F_{4x} = k \frac{Q^2}{d^2} \quad \rightarrow \quad F_{4x} = -k \frac{Q^2}{d^2} \quad , \quad F_{4y} = 0$$  

$$F_{4x} = k \frac{Q^2}{2d^2} \quad \rightarrow \quad F_{4x} = k \frac{Q^2}{2d^2} \cos 45^\circ \quad = k \frac{\sqrt{2}Q^2}{4d^2} \quad , \quad F_{4y} = k \frac{-\sqrt{2}Q^2}{4d^2}$$

$$F_{4x} = k \frac{Q^2}{d^2} \quad \rightarrow \quad F_{4x} = 0 \quad , \quad F_{4y} = -k \frac{Q^2}{d^2}$$

Add the $x$ and $y$ components together to find the total force, noting that $F_{4x} = F_{4y}$.

$$F_{4} = F_{4x} + F_{4x} + F_{4y} = -k \frac{Q^2}{d^2} + k \frac{\sqrt{2}Q^2}{4d^2} + 0 = k \left( -1 + \frac{\sqrt{2}}{4} \right) = -0.64645k \frac{Q^2}{d^2} = F_{4y}$$

$$F_4 = \sqrt{F_{4x}^2 + F_{4y}^2} = k \frac{Q^2}{d^2} \left( 0.64645 \right) \sqrt{2} = k \frac{Q^2}{d^2} \left( 0.9142 \right)$$

$$= \left( 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( 4.15 \times 10^{-3} \text{ C} \right)^2 \left( 0.9142 \right) = 1.42 \times 10^7 \text{ N}$$

$$\theta = \tan^{-1} \left( \frac{F_{4y}}{F_{4x}} \right) = 225^\circ$$ from the x-direction, or exactly towards the center of the square.

For each charge, there are two forces that point towards the adjacent corners, and one force that points away from the center of the square. Thus for each charge, the net force will be the magnitude of $1.42 \times 10^7 \text{ N}$ and will lie along the line from the charge inwards towards the center of the square.

(3) Electric field due to two point charges: (21.32)

The field due to the negative charge will point towards the negative charge, and the field due to the positive charge will point towards the negative charge. Thus the magnitudes of the two fields can be added together to find the charges.

$$E_{net} = 2E_0 = 2k \frac{Q}{(l/2)^2} = \frac{8kQ}{\ell^2} \quad \rightarrow \quad Q = \frac{EF^2}{8k} = \frac{(586 \text{ N/C})(0.160 \text{ m})^2}{8 \left( 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right)} = 2.09 \times 10^{-10} \text{ C}$$
(4) Electric field due to line of charge: (21.53)

Choose a differential element of the rod $dx'$ a distance $x'$ from the origin, as shown in the diagram. The charge on that differential element is $dq = \frac{Q}{\ell} dx'$. The variable $x'$ is treated as positive, so that the field due to this differential element is $dE = \frac{1}{4\pi\varepsilon_0} \frac{dq}{(x + x')^2} = \frac{Q}{4\pi\varepsilon_0 \ell} \frac{dx'}{(x + x')^2}$. Integrate along the rod to find the total field.

$$E = \int dE = \int_0^\ell \frac{Q}{4\pi\varepsilon_0 \ell} \frac{dx'}{(x + x')^2} = \frac{Q}{4\pi\varepsilon_0 \ell} \left( \frac{1}{x + x'} \right)_0^\ell = \frac{Q}{4\pi\varepsilon_0 \ell} \left( \frac{1}{x + \ell} - \frac{1}{x} \right)$$

(5) Electric field due to surface charge: (21.54)

As suggested, we divide the plane into long narrow strips of width $dy$ and length $\ell$. The charge on the strip is the area of the strip times the charge per unit area: $dq = \sigma \ell dy$. The charge per unit length on the strip is $\lambda = \frac{dq}{\ell} = \sigma dy$. From Example 21-11, the field due to that narrow strip is

$$dE = \frac{\lambda}{2\pi\varepsilon_0 \sqrt{y^2 + z^2}} = \frac{\sigma dy}{2\pi\varepsilon_0 \sqrt{y^2 + z^2}}.$$ From Figure 21-68 in the textbook, we see that this field does not point vertically. From the symmetry of the plate, there is another long narrow strip a distance $y$ on the other side of the origin, which would create the same magnitude electric field. The horizontal components of those two fields would cancel each other, and so we only need calculate the vertical component of the field. Then we integrate along the $y$ direction to find the total field.

$$E = E_y = \int_{-\infty}^{\infty} \frac{\sigma z dy}{2\pi\varepsilon_0 (y^2 + z^2)} = \frac{\sigma z}{2\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{dy}{y^2 + z^2} = \frac{\sigma z}{2\pi\varepsilon_0} \frac{1}{z} \left( \tan^{-1} \left( \frac{y}{z} \right) \right)_{-\infty}^{\infty} = \frac{\sigma}{2\pi\varepsilon_0} \left[ \tan^{-1} (\infty) - \tan^{-1} (-\infty) \right] = \frac{\pi}{2} \left( \frac{\pi}{2} \right) = \frac{\sigma}{2\varepsilon_0}$$
(6) Gauss's law for sphere of charge (no integration required): (Example 22-4)

**APPROACH** Since the charge is distributed symmetrically in the sphere, the electric field at all points must again be symmetric. $\mathbf{E}$ depends only on $r$ and is directed radially outward (or inward if $Q < 0$).

**SOLUTION**

(a) For our gaussian surface we choose a sphere of radius $r$ ($r > r_0$), labeled $A_1$ in Fig. 22–12. Since $E$ depends only on $r$, Gauss’s law gives, with $Q_{encl} = Q$,

$$\oint \mathbf{E} \cdot d\mathbf{A} = E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

or

$$E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2}.$$  

Again, the field outside a spherically symmetric distribution of charge is the same as that for a point charge of the same magnitude located at the center of the sphere.

(b) Inside the sphere, we choose for our gaussian surface a concentric sphere of radius $r$ ($r < r_0$), labeled $A_2$ in Fig. 22–12. From symmetry, the magnitude of $\mathbf{E}$ is the same at all points on $A_2$, and $\mathbf{E}$ is perpendicular to the surface, so

$$\oint \mathbf{E} \cdot d\mathbf{A} = E(4\pi r^2).$$

We must equate this to $Q_{encl}/\epsilon_0$ where $Q_{encl}$ is the charge enclosed by $A_2$. $Q_{encl}$ is not the total charge $Q$ but only a portion of it. We define the charge density, $\rho_E$, as the charge per unit volume ($\rho_E = dQ/dV$), and here we are given that $\rho_E = \text{constant}$. So the charge enclosed by the gaussian surface $A_2$, a sphere of radius $r$, is

$$Q_{encl} = \left(\frac{\frac{4}{3} \pi r^3 \rho_E}{\frac{4}{3} \pi r_0^3 \rho_E}\right) Q = \frac{r^3}{r_0^3} Q.$$

Hence, from Gauss’s law,

$$E(4\pi r^2) = \frac{Q_{encl}}{\epsilon_0} = \frac{r^3}{r_0^3} \frac{Q}{\epsilon_0}$$

or

$$E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r_0^3} r.$$  

[r < r_0]

Thus the field increases linearly with $r$, until $r = r_0$. It then decreases as $1/r^2$, as plotted in Fig. 22–13.

**FIGURE 22–13** Magnitude of the electric field as a function of the distance $r$ from the center of a uniformly charged solid sphere.
(7) Gauss's law for a non-uniformly charged solid cylinder (integration required):

From the charge density \( \rho = \alpha r \), we know that the accumulated charge is higher for a larger \( r \), but it's the same for different \( \phi \) and \( z \). (\( \phi \) is the azimuthal angle of the cylinder, and \( z \) is the coordinate along the length of the cylinder.) Therefore, we can use the symmetry to integrate the charge inside the Gauss surface. That is, we can divide the cylinder into many thin cylindrical shells of thickness \( dr \).

The volume of the thin shell is \( dV = 2\pi r \cdot dr \cdot l \). Then the charge inside this thin shell is

\[
\delta q = \rho dV = (\alpha r)(2\pi rl) dr = 2\pi \alpha l r^2 dr
\]

(Gauss's law: \( \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{within}}}{\varepsilon_0} \))

\[
\mathbf{E} = \mathbf{E}(r) \quad d\mathbf{A} = 2\pi rl \, \mathbf{n} \quad \Rightarrow \quad \mathbf{E} \cdot d\mathbf{A} = (\mathbf{E}(r) \cdot \mathbf{n}) 2\pi rl
\]

(a) For \( r > R \)

\[
\oint_{\text{cylinder}} \mathbf{E} \cdot d\mathbf{A} = \oint_{\text{whole cylinder}} 2\pi rl \, d\mathbf{A} = \frac{2}{3} \pi \alpha l R^3
\]

\( \Rightarrow \) Gauss's law: \( (\mathbf{E}(r) \cdot \mathbf{n}) 2\pi rl = \frac{2}{3} \pi \alpha l R^3 \)

\( \Rightarrow \) \( \mathbf{E}(r) = \frac{\alpha R^2}{3\varepsilon_0} \mathbf{n} \)

(b) For \( r < R \)

\[
\oint_{\text{cylinder}} \mathbf{E} \cdot d\mathbf{A} = \oint_{\text{part of cylinder}} 2\pi rl \, d\mathbf{A}' = \frac{2}{3} \pi \alpha l r^2
\]

\( \Rightarrow \) Gauss's law: \( (\mathbf{E}(r) \cdot \mathbf{n}) 2\pi rl = \frac{2}{3} \pi \alpha l r^2 \)

\( \Rightarrow \) \( \mathbf{E}(r) = \frac{\alpha R^2}{3\varepsilon_0} \mathbf{n} \)

(c) You can check \( \mathbf{E}(r=R) \) are the same for the results of (a) and (b). So they match at the boundary of the cylinder.
(8) Gauss’s law for a non-uniformly charged solid sphere (integration required): (Example 22-5)

**APPROACH** We divide the sphere up into concentric thin shells of thickness $dr$ as shown in Fig. 22–14, and integrate (a) setting $Q = \int \rho_E \, dV$ and (b) using Gauss’s law.

**SOLUTION** (a) A thin shell of radius $r$ and thickness $dr$ (Fig. 22–14) has volume $dV = 4\pi r^2 \, dr$. The total charge is given by

$$Q = \int \rho_E \, dV = \int_0^{r_0} (\alpha r^2)(4\pi r^2 \, dr) = 4\pi \alpha \int_0^{r_0} r^4 \, dr = \frac{4\pi \alpha}{5} r_0^5.$$

Thus $\alpha = \frac{5Q}{4\pi r_0^5}$.

(b) To find $E$ inside the sphere at distance $r$ from its center, we apply Gauss’s law to an imaginary sphere of radius $r$ which will enclose a charge

$$Q_{encl} = \int_0^r \rho_E \, dV = \int_0^r (\alpha r^2) 4\pi r^2 \, dr = \int_0^r \left( \frac{5Q}{4\pi r_0^5} r^2 \right) 4\pi r^2 \, dr = \frac{Q r_0^5}{r^5}.$$

By symmetry, $E$ will be the same at all points on the surface of a sphere of radius $r$, so Gauss’s law gives

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0},$$

$$(E)(4\pi r^2) = \frac{Q}{\varepsilon_0 r_0^5},$$

so

$$E = \frac{Q r^3}{4\pi \varepsilon_0 r_0^5}.$$
(9) Gauss’s law for solid cylinder and thick cylindrical shells: (22.38)

\[ \oint E \cdot dA = \frac{Q_{\text{enc}}}{\varepsilon_0} \rightarrow E = \frac{Q_{\text{enc}}}{2\pi \varepsilon_0 R l} \]

(a) For \(0 < R < R_1\), the enclosed charge is the volume of charge enclosed, times the charge density.

\[ E = \frac{Q_{\text{enc}}}{2\pi \varepsilon_0 R l} = \frac{\rho_v \pi R^2 l}{2\pi \varepsilon_0 R l} = \frac{\rho_v R}{2\varepsilon_0} \]

(b) For \(R_1 < R < R_2\), the enclosed charge is all of the charge on the inner cylinder.

\[ E = \frac{Q_{\text{enc}}}{2\pi \varepsilon_0 R l} = \frac{\rho_v \pi R_2^2 l}{2\pi \varepsilon_0 R l} = \frac{\rho_v R_2^2}{2\varepsilon_0} \]

(c) For \(R_2 < R < R_1\), the enclosed charge is all of the charge on the inner cylinder, and the part of the charge on the shell that is enclosed by the gaussian cylinder.

\[ E = \frac{Q_{\text{enc}}}{2\pi \varepsilon_0 R l} = \frac{\rho_v \pi R_1^2 l + \rho_v \left( \pi R_1^2 l - \pi R_2^2 l \right)}{2\pi \varepsilon_0 R l} = \frac{\rho_v \left(R_1^2 + R^2 - R_2^2\right)}{2\varepsilon_0 R} \]

(d) For \(R > R_3\), the enclosed charge is all of the charge on both the inner cylinder and the shell.

\[ E = \frac{Q_{\text{enc}}}{2\pi \varepsilon_0 R l} = \frac{\rho_v \pi R_1^2 l + \rho_v \left( \pi R_3^2 l - \pi R_2^2 l \right)}{2\pi \varepsilon_0 R l} = \frac{\rho_v \left(R_1^2 + R_3^2 - R_2^2\right)}{2\varepsilon_0 R} \]

(10) Potential difference from electric field: (23.11)
Since the field is uniform, we may apply Eq. 23-4b. Note that the electric field always points from high potential to low potential.

(a) \(V_{BA} = 0\). The distance between the two points is exactly perpendicular to the field lines.

(b) \(V_{CB} = V_C - V_B = (-4.20 \text{ N/C})(7.00 \text{ m}) = -29.4 \text{ V}\)

(c) \(V_{CA} = V_C - V_A = V_C - V_B + V_B - V_A = V_{CB} + V_{BA} = -29.4 \text{ V} + 0 = -29.4 \text{ V}\)
(11) Potential due to point charges: (23.32)

Use Eq. 23-2b and Eq. 23-5.

\[ V_{BA} = V_B - V_A = \left( \frac{1}{4\pi\varepsilon_0} \frac{q}{d-b} + \frac{1}{4\pi\varepsilon_0} \frac{(-q)}{b} \right) - \left( \frac{1}{4\pi\varepsilon_0} \frac{q}{b} + \frac{1}{4\pi\varepsilon_0} \frac{(-q)}{d-b} \right) \]

\[ = \frac{q}{4\pi\varepsilon_0} \left( \frac{1}{d-b} - \frac{1}{b} - \frac{1}{b} + \frac{1}{d-b} \right) = \frac{2}{4\pi\varepsilon_0} q \left( \frac{1}{d-b} - \frac{1}{b} \right) = \frac{2q(2b - d)}{4\pi\varepsilon_0 b(d-b)} \]

(12) Potential (and electric field) due to a charge distribution: (23.33)

(a) For every element \( dq \) as labeled in Fig. 1 on the top half of the ring, there will be a diametrically opposite element of charge \(-dq\). The potential due to these two infinitesimal elements will cancel each other, and so the potential due to the entire ring is 0.

(b) Because the upper and lower halves of the ring are oppositely charged, the parallel components of the field from diametrically opposite infinitesimal segments of the ring will cancel each other. i.e. \( E_x \) cancels between charge \( +dq \) at \( \phi \) and \(-dq \) at \( (\pi-\phi) \).

\[ \Rightarrow E_x = 0. \]

(c) \( E_x \) cancels between charge at \( \phi \) and at \(-\phi \), but \( E_y \) will add up. \( E_y \) also adds up between charge \( +dq \) at \( \phi \) and \(-dq \) at \( (\pi-\phi) \), when \( E_x \) cancels. We can also calculate the electric field explicitly.

(d) \[ dE_x = \frac{dq}{4\pi\varepsilon_0} \left[ \cos^2 \theta - \sin^2 \phi \cos^2 \theta \right] \]

\[ = \frac{(-1)^{\theta/\pi}}{4\pi\varepsilon_0} \left[ \cos^2 \theta - \sin^2 \phi \cos^2 \theta \right] \]

\[ \Rightarrow E_x = \frac{1}{2} \int_{-\pi}^{\pi} dE_x + \frac{1}{2} \int_{0}^{\pi} dE_x \]

\[ \Rightarrow E_x = \frac{\sqrt{2\pi}}{4\pi\varepsilon_0} \left[ \cos^2 \theta - \sin^2 \phi \cos^2 \theta \right] \]

\[ = \frac{\sqrt{2\pi}}{4\pi\varepsilon_0} \left[ \frac{1}{2} \left( \cos^2 \theta + \sin^2 \phi \cos^2 \theta \right) \right] \]

\[ \sin \theta = \frac{R}{\sqrt{x^2 + R^2}} \]

\[ = -\frac{\sqrt{2\pi}}{2\pi\varepsilon_0} E_0 \frac{R}{(x^2 + R^2)^{3/2}} \]
(13) Four equal point charges: (23.60)

(a) \[ U_4 = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_1Q_4}{r_{14}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_2Q_4}{r_{24}} + \frac{Q_3Q_4}{r_{34}} \right) \]
\[ = \frac{Q^2}{4\pi\varepsilon_0} \left( \frac{1}{b} + \frac{1}{\sqrt{2}b} + \frac{1}{b} + \frac{1}{\sqrt{2}b} + \frac{1}{b} \right) = \frac{Q^2}{4\pi\varepsilon_0 b} \left( 4 + \sqrt{2} \right) \]

(b) The potential energy of the fifth charge is due to the interaction between the fifth charge and each of the other four charges. Each of those interaction terms is of the same magnitude since the fifth charge is the same distance from each of the other four charges.

\[ U_{5th\ charge} = \frac{Q^2}{4\pi\varepsilon_0 b} \left( 4\sqrt{2} \right) \]

(14) Basics of capacitance: (24.5)

After the first capacitor is disconnected from the battery, the total charge must remain constant. The voltage across each capacitor must be the same when they are connected together, since each capacitor plate is connected to a corresponding plate on the other capacitor by a constant-potential connecting wire. Use the total charge and the final potential difference to find the value of the second capacitor.

\[ Q_{\text{total}} = C_1 V_{\text{initial}} \]
\[ Q_1 = C_1 V_{\text{final}} \]
\[ Q_2 = C_2 V_{\text{final}} \]
\[ Q_{\text{total}} = Q_1_{\text{final}} + Q_2_{\text{final}} = (C_1 + C_2) V_{\text{final}} \rightarrow C_1 \frac{V_1}{V_{\text{initial}}} = (C_1 + C_2) V_{\text{final}} \rightarrow \]
\[ C_2 = C_1 \left( \frac{V_{\text{initial}}}{V_{\text{final}}} - 1 \right) = \left( 7.7 \times 10^{-6} \, \text{F} \right) \left( \frac{125 \, \text{V}}{15 \, \text{V}} - 1 \right) = 5.6 \times 10^{-5} \, \text{F} = 56 \mu\text{F} \]
(15) Two capacitors in Series and Parallel:
\[
\begin{align*}
C_1 + C_2 &= 8 \mu F \\
\frac{1}{C_1} + \frac{1}{C_2} &= \frac{1}{1.5 \mu F} \\
\implies C_1 C_2 &= 1.5 \mu F
\end{align*}
\]
\[
\implies \frac{C_1 C_2}{8 \mu F} = 1.5 \mu F
\]
\[
C_1 (8 \mu F - C_1) = 12 (\mu F)^2
\]
\[
C_1^2 - (8 \mu F) C_1 + 12 (\mu F)^2 = 0
\]
\[
(C_1 - 2 \mu F) \cdot (C_1 - 6 \mu F) = 0
\]
\[
C_1 = 2 \mu F \text{ or } C_1 = 6 \mu F
\]
i.e. \((C_1, C_2) = (2 \mu F, 6 \mu F) \text{ or } (6 \mu F, 2 \mu F)\)

(16) More than two capacitors in Series and Parallel (I): (24.32)

(a) From the diagram, we see that \(C_1\) and \(C_2\) are in parallel, and \(C_3\) and \(C_4\) are in parallel. Those two combinations are then in series with each other. Use those combinations to find the equivalent capacitance. We use subscripts to indicate which capacitors have been combined.
\[
\begin{align*}
C_{12} &= C_1 + C_2 \\
C_{34} &= C_3 + C_4 \\
\frac{1}{C_{1234}} &= \frac{1}{C_{12}} + \frac{1}{C_{34}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3 + C_4} \implies \\
C_{1234} &= \frac{(C_1 + C_2)(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}
\end{align*}
\]

(b) The charge on the equivalent capacitor \(C_{1234}\) is given by \(Q_{1234} = C_{1234} V\). This is the charge on both of the series components of \(C_{1234}\). Note that \(V_{12} + V_{34} = V\).
\[
\begin{align*}
Q_{12} &= C_{1234} V = C_{12} V_{12} \implies V_{12} = \frac{C_{1234}}{C_{12}} V = \frac{(C_1 + C_2)(C_3 + C_4)}{C_1 + C_2} V = \frac{(C_1 + C_4)}{(C_1 + C_2 + C_3 + C_4)} V \\
Q_{34} &= C_{1234} V = C_{34} V_{34} \implies V_{34} = \frac{C_{1234}}{C_{34}} V = \frac{(C_1 + C_2)(C_3 + C_4)}{C_3 + C_4} V = \frac{(C_1 + C_3)}{(C_1 + C_2 + C_3 + C_4)} V
\end{align*}
\]
The voltage across the equivalent capacitor $C_{12}$ is the voltage across both of its parallel components, and the voltage across the equivalent $C_{34}$ is the voltage across both its parallel components.

$$V_{12} = \frac{V_1 + V_2}{(C_1 + C_2 + C_3 + C_4)} = \frac{(C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$C_1 V_1 = \frac{Q_1}{(C_1 + C_2 + C_3 + C_4)} = \frac{C_1 (C_3 + C_4)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$V_{34} = \frac{V_3 + V_2}{(C_1 + C_2 + C_3 + C_4)} = \frac{(C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$C_3 V_3 = \frac{Q_3}{(C_1 + C_2 + C_3 + C_4)} = \frac{C_3 (C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V$$

$$C_4 V_4 = \frac{Q_4}{(C_1 + C_2 + C_3 + C_4)} = \frac{C_4 (C_1 + C_2)}{(C_1 + C_2 + C_3 + C_4)}V$$

(17) More than two capacitors in Series and Parallel (II): (24.45)

(a) $C_1 = C_2 = C_3 = C = 22.6 \mu F$

$$C_{net} = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{2}{3} C = 33.9 \mu F$$

(b)

<table>
<thead>
<tr>
<th>$\Delta V$</th>
<th>$Q$</th>
<th>$E = \frac{1}{2} C \Delta V^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$V$</td>
<td>$CV$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$V/2$</td>
<td>$CV/2$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$V/2$</td>
<td>$CV/2$</td>
</tr>
</tbody>
</table>

$$E_{net} = \frac{3}{4} CV^2 = 1.7 \times 10^{-3} J$$
(18) Change in value of capacitance (and hence voltage across it) up on filling it with dielectric: (24.62)

Since the capacitors each have the same charge and the same voltage in the initial situation, each has the same capacitance of \( C = \frac{Q_0}{V_0} \). When the dielectric is inserted, the total charge of \( 2Q_0 \) will not change, but the charge will no longer be divided equally between the two capacitors. Some charge will move from the capacitor without the dielectric \( (C_1) \) to the capacitor with the dielectric \( (C_2) \). Since the capacitors are in parallel, their voltages will be the same.

\[
\begin{align*}
V_1 &= V_2 \\
\frac{Q_1}{C_1} &= \frac{Q_2}{C_2} \\
\frac{Q_1}{C} &= \frac{2Q_0 - Q}{KC} \\
Q_1 &= \frac{2}{(K + 1)}Q_0 \quad \Rightarrow \quad Q_2 = \frac{0.48Q_0}{2.42} \\
V_1 &= \frac{0.48Q_0}{V_0} \\
V_2 &= \frac{1.52Q_0}{3.2Q_0/V_0} \\
\end{align*}
\]

(19) Basics of resistance of a wire: (25.21)

The wires have the same resistance and the same resistivity.

\[
R_{long} = R_{short} \quad \Rightarrow \quad \frac{\rho l_{long}}{A_1} = \frac{\rho l_{short}}{A_2} \quad \Rightarrow \quad \frac{(4)2l_{short}}{\pi d_{long}^2} = \frac{4l_{short}}{\pi d_{short}^2} \quad \Rightarrow \quad \frac{d_{long}}{d_{short}} = \sqrt{2}
\]

(20) Basics of power and Ohm’s law: (25.43)

Each bulb will draw an amount of current found from Eq. 25-6.

\[
P = IV \quad \Rightarrow \quad I_{bulb} = \frac{P}{V}
\]

The number of bulbs to draw 15 A is the total current divided by the current per bulb.

\[
I_{total} = nI_{bulb} = n \frac{P}{V} \quad \Rightarrow \quad n = \frac{VI_{total}}{P} = \frac{(120 \text{ V})(15 \text{ A})}{75 \text{ W}} = 24 \text{ bulbs}
\]
(21) Microscopic understanding of current: (25.57)

\[ j = nev_d \rightarrow v_d = \frac{j}{ne} = \frac{I}{neA} = \frac{I}{N(1 \text{ mole})(m(1 \text{ mole})\rho_o)\pi(\frac{1}{2}d)^2} = \frac{4Im}{N\rho_oed^2} \]

\[ v_d = \frac{4(2.3\times10^{-6} \text{ A})(63.5\times10^{-3} \text{ kg})}{(6.02\times10^{23})(8.9\times10^3 \text{ kg/m}^3)(1.60\times10^{-10} \text{ C})\pi(0.65\times10^{-3} \text{ m})^2} = 5.1\times10^{-10} \text{ m/s} \]

\[ j = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4I}{\pi d^2} = \frac{4(2.3\times10^{-6} \text{ A})}{\pi(6.5\times10^{-4} \text{ m})^2} = 6.931 \text{ A/m}^2 \approx 6.9 \text{ A/m}^2 \]

\[ j = \frac{1}{\rho} E \rightarrow E = \rho j = (1.68\times10^{-8} \Omega \cdot \text{m})(6.931 \text{ A/m}^2) = 1.2\times10^{-7} \text{ V/m} \]

(22) Power delivered to resistances and their parallel combination: (26.17)

\[ P = \frac{V^2}{R} \rightarrow \frac{1}{R} = \frac{P}{V^2} \]

\[ R_{eq} = \left( \frac{1}{R_{75}} + \frac{1}{R_{40}} \right)^{-1} = \left( \frac{75 \text{ W}}{(110 \text{ V})^2} + \frac{25 \text{ W}}{(110 \text{ V})^2} \right)^{-1} = 121 \Omega \approx 120 \Omega \]

(23) Series and parallel combinations of resistances: (26.10)

The resistors can all be connected in series.

\[ R_{eq} = R + R + R = 3R \]

The resistors can all be connected in parallel.

\[ \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} \rightarrow R_{eq} = \left( \frac{3}{R} \right)^{-1} = \frac{R}{3} \]

Two resistors in series can be placed in parallel with the third.

\[ \frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R + R} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R} \rightarrow R_{eq} = \frac{2R}{3} \]

Two resistors in parallel can be placed in series with the third.

\[ R_{eq} = R + \left( \frac{1}{R} + \frac{1}{R} \right)^{-1} = R + \frac{R}{2} = \frac{3}{2}R \]
(24) Non-zero resistance for conducting wire: (26.13)
We model the resistance of the long leads as a single resistor $r$. Since the bulbs are in parallel, the total current is the sum of the current in each bulb, and so $I = 8I_r$. The voltage drop across the long leads is $V_{lead} = Ir = 8I_r r = 8(0.24 \text{ A})(1.4 \Omega) = 2.688 \text{ V}$. Thus the voltage across each of the parallel resistors is $V_r = V_{kt} - V_{lead} = 110 \text{ V} - 2.688 \text{ V} = 107.3 \text{ V}$. Since we have the current through each resistor, and the voltage across each resistor, we calculate the resistance using Ohm's law.

$$V_r = I_r R \quad \rightarrow \quad R = \frac{V_r}{I_r} = \frac{107.3 \text{ V}}{0.24 \text{ A}} = 447.1 \Omega = 450 \Omega$$

The total power delivered is $P = V_{kt} I$, and the “wasted” power is $I^2r$. The fraction wasted is the ratio of those powers.

$$\text{fraction wasted} = \frac{I^2 r}{V_{tot} I} = \frac{Ir}{V_{tot}} = \frac{8(0.24 \text{ A})(1.4 \Omega)}{110 \text{ V}} = 0.024$$

So about 2.5% of the power is wasted.

(25) Network of resistances: (26.16)

(a) The equivalent resistance is found by combining the 820 $\Omega$ and 680 $\Omega$ resistors in parallel, and then adding the 960 $\Omega$ resistor in series with that parallel combination.

$$R_{eq} = \left( \frac{1}{820 \Omega} + \frac{1}{680 \Omega} \right)^{-1} + 960 \Omega = 372 \Omega + 960 \Omega = 1332 \Omega \approx 1330 \Omega$$

(b) The current delivered by the battery is $I = \frac{V}{R_{eq}} = \frac{12.0 \text{ V}}{1332 \Omega} = 9.009 \times 10^{-3} \text{ A}$. This is the current in the 960 $\Omega$ resistor. The voltage across that resistor can be found by Ohm’s law.

$$V_{460} = IR = \left(9.009 \times 10^{-3} \text{ A}\right)(960 \Omega) = 8.649 \text{ V} \approx 8.6 \text{ V}$$

Thus the voltage across the parallel combination must be $12.0 \text{ V} - 8.6 \text{ V} = 3.4 \text{ V}$. This is the voltage across both the 820 $\Omega$ and 680 $\Omega$ resistors, since parallel resistors have the same voltage across them. Note that this voltage value could also be found as follows.

$$V_{\text{parallel}} = IR_{\text{parallel}} = \left(9.009 \times 10^{-3} \text{ A}\right)(372 \Omega) = 3.351 \text{ V} = 3.4 \text{ V}$$
Another network of resistances: (26.19)

The resistors have been numbered in the accompanying diagram to help in the analysis. \( R_1 \) and \( R_2 \) are in series with an equivalent resistance of \( R_{12} = R + R = 2R \). This combination is in parallel with \( R_3 \), with an equivalent resistance of

\[
R_{123} = \left( \frac{1}{R} + \frac{1}{2R} \right)^{-1} = \frac{2}{3} R
\]

This combination is in series with \( R_4 \), with an equivalent resistance of \( R_{1234} = \frac{2}{3} R + R = \frac{5}{3} R \). This combination is in parallel with \( R_5 \), with an equivalent resistance of

\[
R_{12345} = \left( \frac{1}{R} + \frac{3}{5R} \right)^{-1} = \frac{8}{5} R
\]

Finally, this combination is in series with \( R_6 \), and we calculate the final equivalent resistance.

\[
R_{\text{eq}} = \frac{2}{3} R + R = \frac{12}{5} R
\]

We reduce the circuit to a single loop by combining series and parallel combinations. We label a combined resistance with the subscripts of the resistors used in the combination. See the successive diagrams.

\( R_1 \) and \( R_2 \) are in series.

\[
R_{12} = R_1 + R_2 = R + R = 2R
\]

\( R_{12} \) and \( R_3 \) are in parallel.

\[
R_{123} = \left( \frac{1}{R_{12}} + \frac{1}{R_3} \right)^{-1} = \left( \frac{1}{2R} + \frac{1}{R} \right)^{-1} = \frac{2}{3} R
\]

\( R_{123} \) and \( R_4 \) are in series.

\[
R_{1234} = R_{123} + R_4 = \frac{2}{3} R + R = \frac{5}{3} R
\]

\( R_{1234} \) and \( R_5 \) are in parallel.

\[
R_{12345} = \left( \frac{1}{R_{1234}} + \frac{1}{R_5} \right)^{-1} = \left( \frac{1}{\frac{5}{3} R} + \frac{1}{R} \right)^{-1} = \frac{8}{5} R
\]

\( R_{12345} \) and \( R_6 \) are in series, producing the equivalent resistance.

\[
R_{\text{eq}} = R_{12345} + R_6 = \frac{8}{5} R + R = \frac{12}{5} R
\]

Now work “backwards” from the simplified circuit. Resistors in series have the same current as their equivalent resistance, and resistors in parallel have the same voltage as their equivalent resistance.
\[ I_{eq} = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{\frac{15}{8} R} = \frac{8 \mathcal{E}}{13 R} = I_6 = I_{12345} \]

\[ V_5 = V_{1234} = V_{12345} = I_{1234} R_{1234} = \left( \frac{8 \mathcal{E}}{13 R} \right) \left( \frac{8}{5} R \right) = \frac{8}{13} \mathcal{E} \ ; \ I_5 = \frac{V_5}{R_5} = \frac{\frac{8}{13} \mathcal{E}}{R} = \frac{\mathcal{E}}{13 R} = I_5 \]

\[ I_{1234} = \frac{V_{1234}}{R_{1234}} = \frac{\frac{8}{13} \mathcal{E}}{\frac{8}{5} R} = \frac{3 \mathcal{E}}{13 R} = I_4 = I_{123} \ ; \ V_{123} = I_{123} R_{123} = \left( \frac{3 \mathcal{E}}{13 R} \right) \left( \frac{8}{5} R \right) = \frac{24}{13} \mathcal{E} = V_{12} = V_3 \]

\[ I_3 = \frac{V_3}{R_3} = \frac{2 \mathcal{E}}{13 R} = I_3 \ ; \ I_{12} = \frac{V_{12}}{R_{12}} = \frac{\frac{2 \mathcal{E}}{13 R}}{2R} = \frac{\mathcal{E}}{13 R} = I_1 = I_2 \]

\[ V_{AB} = V_3 = \frac{2}{13} \mathcal{E} \ ; \ I_1 = I_2 = \frac{\mathcal{E}}{13 R} \ ; \ I_3 = \frac{2 \mathcal{E}}{13 R} \ ; \ I_4 = \frac{3 \mathcal{E}}{13 R} \ ; \ I_5 = \frac{5 \mathcal{E}}{13 R} \ ; \ I_6 = \frac{8 \mathcal{E}}{13 R} \]

(27) **Power delivered to series and parallel combinations of resistances:** (26.35)

We are to find the ratio of the power used when the resistors are in series, to the power used when the resistors are in parallel. The voltage is the same in both cases. Use Eq. 25-7b, along with the definitions of series and parallel equivalent resistance.

\[ R_{\text{series}} = R_1 + R_2 + \cdots + R_n = nR \ ; \ R_{\text{parallel}} = \left( \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \right)^{-1} = \left( \frac{n}{R} \right)^{-1} = \frac{R}{n} \]

\[ \frac{P_{\text{series}}}{P_{\text{parallel}}} = \frac{V^2 / R_{\text{series}}}{V^2 / R_{\text{parallel}}} = \frac{R_{\text{parallel}} / R_{\text{series}}}{nR} = \frac{1}{n^2} \]

========================================================================
(28) Kirchhoff’s rules for circuit with two loops: (26.32)

There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff’s junction rule applied to the junction of the three branches at the top center of the circuit.

\[ I_1 = I_2 + I_3 \]

Another equation comes from Kirchhoff’s loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

\[ 58 - I_1 (120\, \Omega) - I_1 (82\, \Omega) - I_2 (64\, \Omega) = 0 \quad \rightarrow \quad 58 = 202I_1 + 64I_2 \]

The final equation comes from Kirchhoff’s loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

\[ 3.0 - I_3 (25\, \Omega) + I_2 (64\, \Omega) - I_3 (110\, \Omega) = 0 \quad \rightarrow \quad 3 = -64I_2 + 135I_3 \]

Substitute \( I_1 = I_2 + I_3 \) into the left loop equation, so that there are two equations with two unknowns.

\[ 58 = 202(I_2 + I_3) + 64I_2 = 266I_2 + 202I_3 \]

Solve the right loop equation for \( I_2 \) and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

\[ 3 = -64I_2 + 135I_3 \quad \rightarrow \quad I_2 = \frac{135I_3 - 3}{64} \quad ; \quad 58 = 266I_2 + 202I_3 = 266\left( \frac{135I_3 - 3}{64} \right) + 202I_3 \quad \rightarrow \]

\[ I_3 = \frac{0.09235}{64} \quad ; \quad I_2 = \frac{135I_3 - 3}{64} = 0.1479 \] ; \( I_1 = I_2 + I_3 = 0.24025 \, \text{A} \)

The current in each resistor is as follows:

| 120Ω: 0.24 A | 82Ω: 0.24 A | 64Ω: 0.15 A | 25Ω: 0.092 A | 110Ω: 0.092 A |

(29) Kirchhoff’s rules for circuit with single loop/battery internal resistances: (26.28)

Apply Kirchhoff’s loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

\[ -I (2.0\, \Omega) + 18 \, \text{V} - I (6.6 \, \Omega) - 12 \, \text{V} - I (1.0\, \Omega) = 0 \quad \rightarrow \quad I = \frac{6 \, \text{V}}{9.6 \, \Omega} = 0.625 \, \text{A} \]

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

18 V battery: \( V_{\text{terminal}} = -I (2.0 \, \Omega) + 18 \, \text{V} = -(0.625 \, \text{A})(2.0 \, \Omega) + 18 \, \text{V} = 16.75 \, \text{V} \approx 17 \, \text{V} \)

12 V battery: \( V_{\text{terminal}} = I (1.0 \, \Omega) + 12 \, \text{V} = (0.625 \, \text{A})(1.0 \, \Omega) + 12 \, \text{V} = 12.625 \, \text{V} \approx 13 \, \text{V} \)
(30) Circuit with two resistors and two capacitors: (26.45)

(a) Resistors in series: \( R_{eq} = R_1 + R_2 = 2 \times 2.2\, k\Omega = 4.4\, k\Omega \)

(b) Capacitors in parallel: \( C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{3.8\, \mu F}{2} = 1.9\, \mu F \)

(c) Time constant: \( \tau_{eq} = R_{eq}C_{eq} = 4.4k\Omega \times 1.9\mu F \approx 8.4\, ms \)

(d) When capacitors are uncharged, entire battery voltage appears across resistors.
\( \Rightarrow \) \( I_0 = \frac{\mathcal{E}}{R_{eq}} = 12V/4.4k\Omega = 2.73m\, A \)

(e) \( I = I_0 e^{-t/\tau_{eq}} \) so that \( I = 1.5m\, A \) at
\( t = \tau_{eq} \ln(I_0/I) = 8.4ms \times \ln (2.73/1.5) \approx 5\, ms. \)

(f) Use energy stored \( = Q^2/(2C) \), with \( Q = \mathcal{E}C(1-e^{-t/\tau_{eq}}) \).
For reaching 75% of final/maximum energy, we need charge to be \( \sqrt{0.75} \) of final.
\( \Rightarrow (1 - e^{-t/\tau_{eq}}) = \sqrt{0.75}, \text{ i.e.} \)
\( t = \tau_{eq} \ln \left( \frac{1}{1-\sqrt{0.75}} \right) \approx 2\tau_{eq} \approx 17\, ms. \)
(31) Charged particle in magnetic field: (27.21)

(a) From Example 27-7, we have that \( r = \frac{mv}{qB} \), and so \( v = \frac{r q B}{m} \). The kinetic energy is given by
\[
K = \frac{1}{2} m v^2 = \frac{1}{2} m \left( \frac{r q B}{m} \right)^2 = \frac{r^2 q^2 B^2}{2m} \quad \text{and so we see that} \quad K \propto r^2.
\]

(b) The angular momentum of a particle moving in a circular path is given by \( L = mvr \). From Example 27-7, we have that \( r = \frac{mv}{qB} \), and so \( v = \frac{r q B}{m} \). Combining these relationships gives
\[
L = mvr = m \frac{r q B}{m} r = q Br^2.
\]

(32) Force on a bent current-carrying wire due to magnetic field:

\[
\begin{align*}
\vec{F}_1 &= \frac{l}{2} \vec{B} = \frac{l}{2} \left[ \frac{\beta + \gamma}{\sqrt{2}} \right] = \frac{l}{2} \left[ \frac{\beta + \gamma}{\sqrt{2}} \right] = \frac{l}{2} \vec{B} \\
\vec{F}_2 &= \frac{l}{2} \vec{B} = \frac{l}{2} \left[ \frac{\beta + \gamma}{\sqrt{2}} \right] = \frac{l}{2} \vec{B} \\
\vec{F} &= \vec{F}_1 + \vec{F}_2 = \frac{l}{2} \vec{B} + \frac{l}{2} \vec{B} = l \vec{B} \\
|\vec{F}| &= l \vec{B} \left[ \sqrt{\beta^2 + \gamma^2} \right] = l \vec{B} \\
\theta &= \tan^{-1} \left( \frac{\beta}{\gamma} \right) = 35.3^\circ
\end{align*}
\]

(33) Parallel current-carrying wires:

The left wire will cause a field on the \( x \) axis that points in the \( y \) direction, and the right wire will cause a field on the \( x \) axis that points in the negative \( y \) direction. The distance from the left wire to a point on the \( x \) axis is \( x \), and the distance from the right wire is \( (d-x) \).

(a) \( \vec{B}_L = \frac{\mu_0 I}{2\pi x} \hat{y} \)

(b) \( \vec{B}_R = \frac{\mu_0 I}{2\pi (d-x)} \hat{y} \)

(c) \( \vec{B}_{net} = \vec{B}_L + \vec{B}_R = \frac{\mu_0 I}{2\pi x} \hat{y} - \frac{\mu_0 I}{2\pi (d-x)} \hat{y} = \frac{\mu_0 I}{2\pi x} \left( \frac{1}{x} - \frac{1}{d-x} \right) \hat{y} = \frac{\mu_0 I}{2\pi x} \left( \frac{d-2x}{x(d-x)} \right) \hat{y} \)
(34) A small wire loop inside a larger wire loop:

\[ \mathbf{B} = \frac{\mu_0 I}{2R} \mathbf{\hat{z}} \]
\[ \mathbf{A} = I \mathbf{\hat{a}} = I \mathbf{\hat{r}} \times \mathbf{\hat{g}} \]

We assume that the inner loop is sufficiently small that the magnetic field from the larger loop can be considered to be constant across the surface of the smaller loop.

\[ \mathbf{T} = \mathbf{r} \times \mathbf{B} = (\pi r^2 \mathbf{\hat{z}}) \times (\frac{\mu_0 I}{2R} \mathbf{\hat{z}}) = \frac{\mu_0 \pi I^2 r^2}{2R} \mathbf{\hat{z}} \]

This torque would cause the inner loop to rotate into the same plane as the outer loop with the currents flowing in the same direction.

(35) Ampere's law:

(a) \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mathbf{B} \cdot (2\pi r \mathbf{\hat{\phi}}) = \mu_0 I \]

(b) \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mathbf{B} \cdot (2\pi r \mathbf{\hat{\phi}}) = \mu_0 (I - 2I) \]

(36) Faraday's law:

(a) \[ \mathbf{B} = \mu_0 n \mathbf{I} = \mu_0 n \mathbf{I}_0 \cos(\omega t) \]

(b) \[ \Phi = \oint \mathbf{B} \cdot d\mathbf{A} = \mathbf{B} \cdot \mathbf{A}_t = \mu_0 n \mathbf{I}_0 \mathbf{A}_t \cos(\omega t) \]

(c) \[ \mathcal{E} = -\frac{d\Phi}{dt} = -\mu_0 n \mathbf{I}_0 \mathbf{A}_t \frac{d\cos(\omega t)}{dt} = \mu_0 n \omega \mathbf{I}_0 \mathbf{A}_t \sin(\omega t) \]
(37) Lenz’s law:

(a) Induced

I increasing

(b) I decreasing

Induced

(c) No change of magnetic flux, so no induced current.

(d) I increasing

(38) More Lenz’s law:
As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced current must be to the right, so the induced current is counter-clockwise as viewed from the right end of the solenoid.
Two ways to compute emf (Faraday's law and motional emf):

\[ E = -\frac{d\Phi}{dt} = -B \frac{dA}{dt} = -B (\frac{dW}{dt}) = -Blv \]

Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by \( I = \frac{E}{R} \).

\( E = Blv = (0.650T)(0.350m)(8.40m/s) = 0.774V \)

\( I = \frac{E}{R} = \frac{0.774V}{0.28\Omega} = 2.76A \)

(\( \vec{F} = q \vec{v} \times \vec{B} = q(\vec{v} \times (-\vec{B})) = q\vec{v}B\hat{j} \)

\[ E = \int \vec{F} \cdot d\vec{l} = \int (q\vec{v}B\hat{j}) \cdot d\vec{l} = q\vec{v}B\hat{j} \cdot d\vec{l} = q\vec{v}B \cdot \vec{l} \]

(b) (i) \[ P = \frac{E^2}{R} = \frac{(Blv)^2}{R} \]

\[ P = F \cdot v \Rightarrow F = \frac{P}{v} = \frac{B^2lv^2}{R} = \frac{(0.650T)(0.350m)(8.40m/s)}{0.28\Omega} = 0.628N \]

(ii) \( \vec{F} = I \vec{I} \times \vec{B} = I (\vec{v} \times (-\vec{B})) = -IL\vec{B} \hat{i} \)

\[ = -\left(\frac{E}{R}\right)IB \hat{i} = -0.628\hat{i} \]
(40) E from changing B:

(a) To compensate the increasing magnetic flux in +z direction, there must be increasing induced magnetic flux in -z direction, which equivalently produces induced electric field in the circumferential (clockwise) direction when viewed from left side.

(b) Induced electric field must go in the -z direction to cancel the increasing magnetic flux in the circumferential direction. 

\[ \mathbf{E} = \text{axial to compensate circumferential } \mathbf{B} \]

(41) Mutual inductance:

(a) \[ B_1 = \mu_0 n_1 I \]

(b) \[ \Phi_2 = B_1 A_2 = (\mu_0 n_1 I) (\pi R^2) = \mu_0 n_1 I \pi R^2 \]

(c) \[ M = \frac{N_2 \Phi_2}{I} = \frac{(N_2 I) (\mu_0 n_1 I \pi R^2)}{I} \]

\[ \Rightarrow \frac{M}{I} = \mu_0 n_1 n_2 \pi R^2, \text{ independent of } I. \]

(42) Magnetic energy stored (integration required):

(a) Amperes law: \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \]

For \( R < R' \), \[ I_{\text{enc}} = I \left( \frac{\pi R'^2}{\pi R^2} \right) = (\frac{R'}{R})^2 I \]

|\( B_1 \) only depends on \( R \), but uniform circumferentially. |

\[ \Rightarrow \oint \mathbf{B} \cdot d\mathbf{l} = B(2\pi R) = \mu_0 \left( \frac{R}{R} \right)^2 I \]

\[ \Rightarrow B = \frac{\mu_0 I}{2\pi R} \]

(b) Magnetic energy density: \[ u_B = \frac{1}{2} B^2 = \frac{1}{2} \mu_0 \left( \frac{\mu_0 I}{2\pi R} \right)^2 \]

volume of the infinitesimal cylindrical shells: \( dV = 2\pi R dr dl \)

We assume that the volume of the cylindrical shells are so infinitesimal that the magnetic field is constant within each shell.

\[ \Rightarrow \frac{U}{I} = \frac{1}{2} u_B dV = \int_0^R \frac{1}{2} \mu_0 \left( \frac{\mu_0 I}{2\pi R} \right)^2 2\pi R dr = \frac{\mu_0 I^2}{4\pi R^2} \int_0^R 2\pi r dr = \frac{\mu_0 I^2}{16} \]
(43) LR circuit:

(a) We use Eq. 30-6 to determine the energy stored in the inductor, with the current given by Eq. Eq 30-9.

\[ U = \frac{1}{2} L I^2 = \frac{L V_0^2}{2 R^2} \left(1 - e^{-t/\tau}\right)^2 \]

(b) Set the energy from part (a) equal to 99.9% of its maximum value and solve for the time.

\[ U = 0.999 \frac{V_0^2}{2R^2} = \frac{V_0^2}{2R^2} \left(1 - e^{-t/\tau}\right)^2 \rightarrow t = \tau \ln \left(1 - \sqrt{0.999}\right) \approx 7.6\tau \]

(44) LC circuit:

(a) \[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{0.175H \cdot (425 \times 10^{-12}F)}} = 18.45kHz \]

(b) \[ Q_0 = CV = (425 \times 10^{-12}F) \cdot (135V) = 5.74 \times 10^{-8} \text{ (Coulombs)} \]

\[ I_{max} = \omega Q_0 = \frac{Q_0}{\sqrt{LC}} = \frac{(5.74 \times 10^{-8} C)}{\sqrt{0.175H \cdot (425 \times 10^{-12}F)}} = 6.66mA \]

(c) \[ U = \frac{Q_0^2}{2C} = \frac{(5.74 \times 10^{-8} C)^2}{2(425 \times 10^{-12} F)} = 3.88 \times 10^{-6}J \]

(45) Modification of Ampere’s law by Maxwell:

The electric field between the plates is given by \[ E = \frac{V}{d} \], where \( d \) is the distance between the plates.

The displacement current is shown in section 31-1 to be \[ I_d = \varepsilon_0 A \frac{dE}{dt} \].

\[ I_d = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{1}{d} \frac{dV}{dt} = \varepsilon_0 A \frac{dV}{dt} = C \frac{dV}{dt} \]
(46) **EM wave:**

(a) The energy emitted in each pulse is the power output of the laser times the time duration of the pulse.

\[ P = \frac{\Delta W}{\Delta t} \rightarrow \Delta W = P \Delta t = \left(1.8 \times 10^{11} \text{ W}\right) \left(1.0 \times 10^{-9} \text{ s}\right) = 180 \text{ J} \]

(b) We find the rms electric field from the intensity, which is the power per unit area. That is also the magnitude of the Poynting vector. Use Eq. 31-19a with rms values.

\[ S = \frac{P}{A} = c \varepsilon_0 E_{\text{rms}}^2 \rightarrow \]

\[ E_{\text{rms}} = \sqrt{\frac{P}{A c \varepsilon_0}} = \sqrt{\frac{\left(1.8 \times 10^{11} \text{ W}\right)}{\pi \left(2.2 \times 10^{-3} \text{ m}\right)^2 \left(3.00 \times 10^8 \text{ m/s}\right) \left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)}} \]

\[ = 2.1 \times 10^9 \text{ V/m} \]

(c) Eq. (31-19b): \[ S = \frac{P}{A} = \frac{E_{\text{rms}} B_{\text{rms}}}{\mu_0} \]

\[ \Rightarrow B_{\text{rms}} = \frac{\mu_0 P}{E_{\text{rms}} A} = \frac{\left(4 \pi \times 10^{-7} \text{T} \cdot \text{m}/\text{A}\right) \left(1.8 \times 10^{11} \text{ W}\right)}{\left(2.1 \times 10^9 \text{ V/m}\right) \pi \left(2.2 \times 10^{-3} \text{ m}\right)^2} = 7.083 \text{T} \]

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