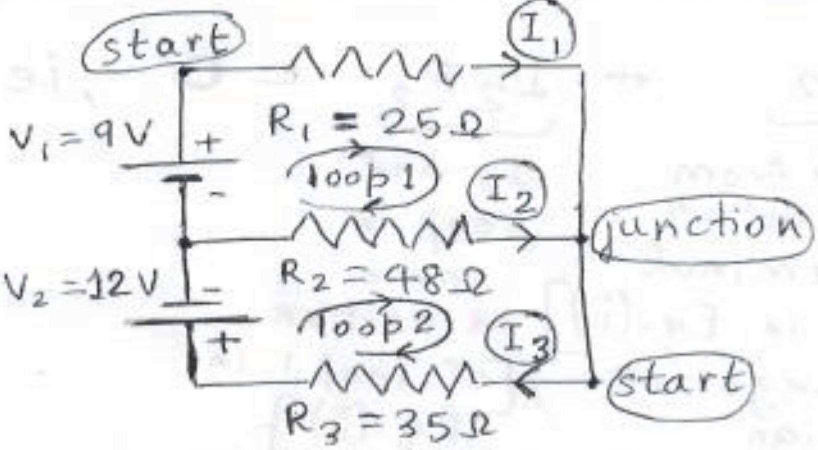
Kirchhoff's rules problem

Phys 272 | Dr. Agashe



Follow the recipe/strategy given in HW 7 and with #28 of Wed. session:

step (1): label current in each branch (i.e., part between 2 junctions) ($I_{1,2,3}$ above), choosing direction for each "randomly" (e.g. $I_{1,2}$ are "to right" vs. I_3 is "to left")

step (2): apply junction rule (at point marked):

$$I_1 + I_2 \text{ (sum of incoming)} = I_3 \text{ (outgoing)} \quad \ldots \text{(i)}$$

step (3): apply voltage rule to loops shown [we choose both directions to be clockwise just for simplicity: in general, directions could be different for each loop and loop direction is chosen independently of the currents in step (1)]:

loop 1 ("starting" at top left):

$$\underbrace{-I_1 R_1}_{\substack{I_1 \text{ to right} \\ \text{loop going to right} \Rightarrow \\ \text{same} \\ \text{negative sign}}} + \underbrace{I_2 R_2}_{\substack{I_2 \text{ to right, but loop going to left} \\ \Rightarrow \text{positive sign} \\ \text{(opposite directions)}}} + \underbrace{V_1}_{\substack{\text{loop going from "}-\text{" to "}+\text{" terminal} \Rightarrow \\ \text{positive sign}}} = 0, \text{ i.e.,}$$

$$-I_1 \, 25\Omega + I_2 \, 48\Omega + 9V = 0 \quad \ldots \text{(ii)}$$

$\underset{\text{any choice is ok}}{}$

loop 2 ("starting" at bottom right) :

$$\underbrace{-I_3 R_3}_{\substack{I_3 \text{ \& loop} \\ \text{same direction}}} \quad \underbrace{-V_2}_{\substack{\text{loop from} \\ \text{"+" to "−"} \\ \text{terminal} \\ \text{[cf. in Eq.(ii)]} \\ \Rightarrow \text{negative} \\ \text{sign}}} \quad \underbrace{-I_2 R_2}_{\substack{I_2 \text{ and} \\ \text{loop in} \\ \text{same} \\ \text{direction} \\ \text{[cf. loop 1 in} \\ \text{Eq.(ii)]}}} = 0 \quad \text{, i.e.,}$$

$$-I_3\, 35\,\Omega - 12\text{V} - I_2\, 48\,\Omega = 0 \quad \ldots \text{(iii)}$$

$\boxed{\text{step (4)}}$: We do have as many equations as unknowns here, i.e., Eqs (i)–(iii) for $I_{1,2,3}$ so that we do not need to apply rules further.

$\boxed{\text{step (5)}}$ Solve Eqs (i)–(iii), e.g., Eq. (ii) gives

$$I_1 = (9\text{V} + 48\, I_2)/(25\,\Omega) \quad \ldots \text{(iv)}$$

Plugging Eq. (iv) and I_3 from Eq. (i) into Eq. (iii) gives

$$-48\, I_2 - 12\,\text{V} - 35\,\Omega \underbrace{\left[I_2 + \overbrace{\left(\frac{9\,\text{V} + 48\, I_2}{25\,\Omega} \right)}^{I_1} \right]}_{I_3} = 0$$

(after bit of algebra!)

or $\boxed{I_2 = -\frac{123}{751}}$ A $= \boxed{0.164\text{A}}$, but to left (due to negative sign)

Plugging above value of I_2 in Eq. (iv) gives

$I_1 = 0.0455$ A (i.e., to right) and plugging values of $I_{1,2}$ in Eq. (i)

gives $I_3 = -0.118$ A (i.e., to right)

[Check with overall loop: $-I_1 R_1 - R_3 I_3 - V_2 + V_1 \overset{?}{=} 0$

(in V) $-0.0455 \times 25 + 35 \times 0.118 - 12 + 9 \overset{\checkmark}{=} 0$]