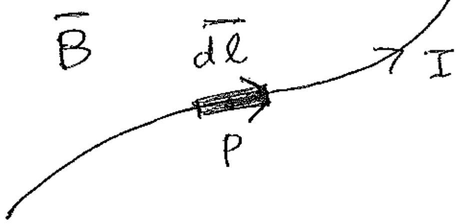
$\boxed{\text{Phys 272}}$ Dr. Agashe

Derivation of magnetic force (due to non-uniform magnetic field) on (thin) current carrying wire (of arbitrary shape) $\boxed{\text{from}}$ magnetic force on moving (point) charge: $\bar{F}_{mag} = q\,\bar{v} \times \bar{B}$

- Consider a thin wire carrying current I (same value at all points along wire) kept in a (in general) non-uniform magnetic field ($\bar{B}$)

$\boxed{\text{Step 1}}$ (and conquer!)

- (Divide) the wire into small elements of (infinitesimal) length $\boxed{dl}$ (located) at point $\boxed{P}$:



- $\boxed{\text{Step 2}}$ small charge in this element, $dq = \lambda\, dl$ ...(i); it is moving with $\bar{v}_{drift}$ in direction of current at P (i.e., along tangent to wire at P) ⇒

charge per unit length (of carriers of I) [λ]; $\boxed{\text{at P}}$ [$\bar{v}_{drift}$]

: treat like point charge

$d\bar{F}_{mag}$ (small force on this element) $= dq\ \bar{v}_{drift} \times \bar{B}$

$= \left[\text{using Eq. (i)}\right] (\lambda\, dl)\, \bar{v}_{drift} \times \bar{B}$ ...(ii)

- $\boxed{\text{Step 3}}$ But v_{drift} is related to I: in time dt, (any) charge travels $(v_{drift}\, dt)$ so that charge contained in (small) length $(v_{drift}\, dt)$ passes through point P in time dt, i.e., a charge of $(v_{drift}\, dt) \times \lambda$ ⇒

$\boxed{I}$ (charge passing P per unit time) $= (v_{drift}\, dt) \times \lambda / dt$

$= v_{drift}\, \lambda$ ...(iii)

- $\boxed{\text{Step 4}}$ Combining Eqs (ii) & (iii) gives

$d\bar{F}_{mag} = I\, \bar{dl} \times \bar{B}$, where direction of dl is taken to be tangent at P (i.e., direction of $\bar{v}_{drift}$) ...(iv)

- $\boxed{\text{step 5}}$ As usual, integrate to get total force on wire:

$$\bar{F}_{mag} = \int \underbrace{I}_{\substack{\text{same in all} \\ d\ell\text{'s} \Rightarrow \text{comes out} \\ \text{of } \int}} \overline{d\ell} \times \bar{B}$$

(in general) different for various $d\ell$'s (stays inside $\int$)

i.e., $\boxed{\bar{F}_{mag} = I \int \overline{d\ell} \times \bar{B}}$

again, direction along tangent to wire at P

evaluate at $d\ell$ (or P)