PHYS 272 (Spring 2019): Introductory Physics: Fields
Summary of topics/formulae for final exam

Chapter 21 of Giancoli

1. Basics of Coulomb's law (force between $Q_{1,2}$): $F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{12}}$ (along line joining $Q_{1,2}$)
2. Basics of Electric Field (force on "test" charge q due to source charge Q): $\mathbf{F} = q\mathbf{E}$, where E (due to charge Q at distance r from it) $= \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ (directed away/toward Q)
3. Principle of superposition for force, electric field and potential

Chapter 22 of Giancoli

1. Flux, $\Phi_E = \int \mathbf{E}.d\mathbf{A}$
2. Gauss's law: $\oint \mathbf{E}.d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$
3. Conductors: only surface charge; $\mathbf{E} = 0$ *in*side; $\mathbf{E} = \frac{\sigma}{\epsilon_0}$ at surface (normal to surface) and entire conductor has *same* potential

Chapter 23 of Giancoli

1. Potential energy of charge q in a potential V (due to other charges), $U = qV$
2. Basics of potential and field: $V = -\int^b a\mathbf{E}.d\mathbf{l}$; $\mathbf{E}_x = -\frac{\partial V}{\partial x}$
3. V due to point charge Q at a distance r from it $= \frac{Q}{4\pi\epsilon_0}\frac{1}{r}$
4. Potential energy of point charges: $U = \frac{1}{4\pi\epsilon_0}\sum_{i\neq j}\frac{Q_i Q_j}{r_{ij}}$

Chapter 24 of Giancoli

1. Capacitance: $Q = CV$; Parallel plate capacitor: $C = \frac{\epsilon A}{d}$, with $\epsilon = \epsilon_0 K$ (K is dielectric constant)
2. Equivalent capacitance of combination of two capacitors: $C_1 + C_2$ (in series: charges same, voltages add up to that of equivalent); $\left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$ (in parallel: voltages same, charges add up to that of equivalent)
3. Energy stored in capacitor: $U = \frac{1}{2}CV^2$
4. Energy stored in electric field: $u = \frac{1}{2}\epsilon E^2$

Chapter 25 of Griffiths

1. Ohm's law: $V = IR$; $I = \int \mathbf{j}.d\mathbf{A}$; $\mathbf{j} = nq\mathbf{v}_d = \sigma\mathbf{E}$ (where σ is conductivity and $\mathbf{v}_d$, q and n are drift velocity, charge and number density, respectively, of charge carriers)
2. For wire with uniform cross-section, $R = \rho\frac{l}{A}$ (where resistivity, $\rho = \frac{1}{\sigma}$)
3. Power: $P = VI$ (in general) $= I^2 R$ (in resistor)

Chapter 26 of Giancoli

1. Equivalent resistance of combinations of resistors: $R_{\text{eq}}^{\text{series}} = R_1 + R_2$ and $R_{\text{eq}}^{\text{parallel}} = (1/R_1 + 1/R_2)^{-1}$

2. Kirchhoff's junction rule: sum of all current entering any junction is equal to sum of all currents leaving that junction and Kirchhoff's loop rule: sum of potential differences across all the components along any closed path of the circuit (loop) must be zero.

3. Combinations of resistor and capacitor (RC circuit): voltage across capacitor while it is being charged by a battery via a resistance, $V_C = V_0 \left(1 - e^{-t/\tau}\right)$, where V_0 is the steady-state voltage (i.e., that of the battery) and $\tau = RC$ is the time constant. Whereas, during discharging of capacitor, $V_C = V_0 e^{-t/\tau}$.

Chapter 27 of Giancoli

1. Force due to magnetic field: $\mathbf{F}_{\text{mag}} = q(\mathbf{v} \times \mathbf{B})$ (point charge, giving uniform circular motion for $v \perp \mathbf{B}$); $\mathbf{F}_{\text{mag}} = I \int d\mathbf{l} \times \mathbf{B}$ (line current): in particular, $\mathbf{F}_{\text{mag}} = I\mathbf{l} \times \mathbf{B}$ for a straight wire of length l in a uniform magnetic field

2. Lorentz force equation: $\mathbf{F} = q\Big[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\Big]$

3. Torque on loop carrying current I: $\tau = \mu \times \mathbf{B}$, where magnetic dipole moment of current loop, $\mu = NI\mathbf{A}$ (with $\mathbf{A}$ being area of loop, directed along normal to surface, using right-hand rule)

Chapter 28 of Giancoli

1. Bio-Savart law for magnetic field due to a wire carrying current I: $\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$, where $\mathbf{r}$ is the vector from the infinitesimal element $d\mathbf{l}$ to the field point

2. Magnetic field due to an infinitely long straight wire carrying current I at a perpendicular distance r: $B = \frac{\mu_0 I}{2\pi r}$ (circumferential)

3. Magnetic field due to circular loop of radius R carrying current I at a distance x along axis: $B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$ (directed along axis)

4. Magnetic force (per unit length) between two long straight wires carrying currents $I_{1,\,2}$ and distance d apart: $F_{\text{mag}} = \frac{\mu_0 I_1 I_2}{2\pi d}$

5. Ampere's law: $\oint \mathbf{B}.d\mathbf{l} = \mu_0 I_{\text{enclosed}} = \int \mathbf{j}_{\text{enclosed}}.d\mathbf{A}$, where j is the current density (i.e., current per unit cross-sectional area) and use right-hand rule to determine direction of positive current

6. Magnetic field due to a solenoid with n turns per unit length carrying current I: $B = \mu_0 n I$ (inside, directed along axis) and *vanishing* outside

Chapter 29 of Giancoli

1. Faraday's law for emf induced in a circuit (including motional): $\mathcal{E} = -\frac{d\Phi_B}{dt}$, where magnetic flux, $\Phi_B = \int \mathbf{B}.d\mathbf{A}$ (including factor for number of loops in coil)

2. Lenz's law: direction of induced current is such that the flux it creates opposes the change of flux that induced it in the first place

3. A related version of Faraday's law (changing magnetic field/flux produces an electric field): $\oint \mathbf{E}.d\mathbf{l} = -\frac{d\Phi_B}{dt}$

Chapter 30 of Giancoli

1. Mutual inductance between coil 1, which carries current I_1 creating magnetic flux Φ_{21} through coil 2 with N_2 turns: $M = \frac{N_2 \Phi_{21}}{I_1}$ so that induced emf in coil 2 due to current in coil 1: $\mathcal{E}_2 = -M \frac{dI_1}{dt}$

2. Self-inductance of a single coil with N turns, which carries current I creating magnetic flux Φ_B through it: $L = N \frac{\Phi_B}{I}$ so that induced ("back/opposing") emf: $\mathcal{E} = -L \frac{dI}{dt}$

3. Energy stored in inductance L carrying current I: $U = \frac{1}{2} L I^2$, which can be thought of as being stored in the magnetic field, with energy density $u = \frac{1}{2} \frac{B^2}{\mu_0}$

4. Combination of inductance and resistance (LR circuit): when connected to a battery of emf V_0, the current rises as $I = I_0 \left(1 - e^{-t/\tau}\right)$, where $I_0 = V_0/R$ is the steady-state current and $\tau = L/R$ is the time constant. Whereas, if the battery is removed, then the current drops as $I = I_0 e^{-t/\tau}$

5. Combination of inductance and capacitance (LC circuit): capacitor discharging through an inductor exhibits oscillation (of charge on capacitor, current though inductor and energies stored in the two) with angular frequency $\omega = 1/\sqrt{LC}$, i.e., $Q = Q_0 \cos \omega t$ (where $Q_)$ is initial charge on capacitor) etc.

Chapter 31 of Giancoli

1. Changing electric field produces magnetic field (Maxwell's modification of Ampere's law): $\oint \mathbf{B}.d\mathbf{l} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$, where Φ_E is the electric flux ($\epsilon_0 \frac{d\Phi_E}{dt}$ being called the displacement current)

2. Maxwell's equations: $\oint \mathbf{E}.d\mathbf{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$ (Gauss's law); $\oint \mathbf{B}.d\mathbf{A} = 0$ (Gauss's law for magnetism); $\oint \mathbf{E}.d\mathbf{l} = -\frac{d\Phi_B}{dt}$ (Faraday's law) and $\oint \mathbf{B}.d\mathbf{l} = \mu_0 I_{\text{encl}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ (Ampere-Maxwell's law)

3. Plane sinusoidal electromagnetic (EM) wave traveling in x-direction: $E_y = E_0 \sin(kx - \omega t)$ and $B_z = B_0 \sin(kx - \omega t)$, where k (wave-number) $= 2\pi/\lambda$ in terms of wavelength (λ); $v = \lambda f$ is speed of the wave and f (ω) is (angular) frequency: Maxwell's equations give $\frac{E_0}{B_0} = v$, with $v = 1/\sqrt{\mu_0 \epsilon_0}$, which is (numerically) equal to the speed of light in vacuum, $c = 3 \times 10^8$ m/s

4. Energy carried by EM wave across unit area in unit time (Poynting vector):
$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$