UNIVERSITY OF MARYLAND, PHYSICS DEPARTMENT

PHYS 272, HW 6 Solutions

Jonathan Echevers

April 1, 2014

1 PROBLEM I

1.1 PART I

Consider the dot product inside the integral. The first vector field has the same orientation and is parallel to the parth at every point in space. Clearly, this dot product will yield a positive value, so the corresponding integral will be positive as well. The second field is also parallel to the path, but follows the opposite direction as the path C, therefore this dot product will yield a negative value, so the circulation integral is negative. The last vector field is orthogonal to the path C at every point, so the dot product will yield zero in this case, therefore the circulation is zero.

1.2 PART II

Consider the vector field $\mathbf{v} = \frac{y}{x^2+y^2+z^2}\hat{x} - \frac{x}{x^2+y^2+z^2}\hat{y}$ (see Fig. 1.1.) The vector field is exactly parallel to the path at every point in space! So the dot product can be computed directly without doing any calculations, because the dot product will simply yield one, divided by the normalization (this is easier to see if you rewrite $\mathbf{v} = \mathbf{u}/||\mathbf{u}||^2$, where $\mathbf{u} = y\hat{x} - x\hat{y}$), as follows:

$$\begin{aligned}\oint_C \mathbf{v}\cdot d\mathbf{l} &= -\oint_C \frac{dl}{\sqrt{R}} \\ &= -2\pi\sqrt{R} \end{aligned} \tag{1.1}$$

For parts (iii) through (iv) you can simply compute the formula given in the homework, which in turn is just equal to the following determinant (ignore if you haven't taken linear algebra, either way, you can just use the formula given in the homework):

$$\nabla \times \mathbf{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix} \tag{1.2}$$

1.3 Part iii

See Fig 1.2.

$$\nabla \times \mathbf{v} = 0\hat{x} + 0\hat{y} - 2\hat{z} \tag{1.3}$$

1.4 Part iv

See Fig. 1.3

$$\nabla \times \mathbf{v} = 0\hat{x} + 0\hat{y} + 2\hat{z} \tag{1.4}$$

1.5 Part v

See Fig. 1.4

$$\nabla \times \mathbf{v} = 0\hat{x} + 0\hat{y} + 0\hat{z} \tag{1.5}$$

1.6 Part vi

See Fig. 1.5

$$\nabla \times \mathbf{v} = 0\hat{x} + 0\hat{y} + 0\hat{z} \tag{1.6}$$

2 Problem II (graded)

In the limit that $L >> R$ we can approximate $L \approx \infty$, so we can find the electric field by applying Gauss' law:

$$\begin{aligned} \int E da &= \frac{Q}{\epsilon_0} \\ 2\pi r L E &= \frac{Q}{\epsilon_0} \\ E &= \frac{Q}{2\pi\epsilon_0 r L} \end{aligned} \tag{2.1}$$

Where r is the radius of the Gaussian surface and is $R_2 < r < R_1$ (everywhere else the electric field is zero) and Q is the charge enclosed, which then simply corresponds to the charge of the cylinder with radius R_2. Then:

$$\begin{aligned} V &= -\int_{R_2}^{R_1} drE(r) \\ &= -\frac{Q}{2\pi\epsilon_0 L}\int_{R_2}^{R_1}\frac{dr}{r} \\ &= -\frac{Q}{2\pi\epsilon_0 L} log(R_1/R_2) \end{aligned} \tag{2.2}$$

Then the capacitance is simply:

$$\begin{aligned} C &= Q/V \\ &= -\frac{2\pi\epsilon_0 L}{log(R_1/R_2)} \end{aligned} \tag{2.3}$$

3 Problem III

3.1 Part I

By Gauss' law:

$$\begin{aligned} \int Eda &= \frac{Q_{enc}}{\epsilon_0} \\ E(r) &= \frac{Q}{4\pi r^2 \epsilon_0} \end{aligned} \tag{3.1}$$

So:

$$\begin{aligned} V &= -\int_{R_2}^{R_1} drE(r) \\ &= -\frac{Q}{4\pi}\int_{R_2}^{R_1}\frac{dr}{r^2} \\ &= \frac{Q}{4\pi}[1/R_1 - 1/R_2] \\ &= \frac{Q}{4\pi}\frac{R_2 - R_1}{R_1 R_2} \end{aligned} \tag{3.2}$$

For $R_2 < r < R_1$. The electric field is zero everywhere else, and so is the potential. Then:

$$
\begin{aligned}
C &= \frac{Q}{V} \\
&= 4\pi \frac{R_1 R_2}{R_2 - R_1}
\end{aligned}
\tag{3.3}
$$

3.2 Part II

We know that the energy stored in a capacitor is simply $U = CV^2/2$, express $Q = 4\pi r^2 \epsilon_0 E = 4\pi\epsilon_0 R_2^2 E$.

$$
\begin{aligned}
U &= \frac{1}{2} C V^2 \\
&= 4\pi \frac{1}{2} \frac{R_1 R_2}{R_2 - R_1} \left[\frac{Q}{4\pi} \frac{R_2 - R_1}{R_1 R_2} \right]^2 \\
&= 4\pi \frac{1}{2} \frac{R_1 R_2}{R_2 - R_1} \left[\epsilon_0 R_2^2 E \right]^2
\end{aligned}
\tag{3.4}
$$

You can plug in the given values to find the final result.

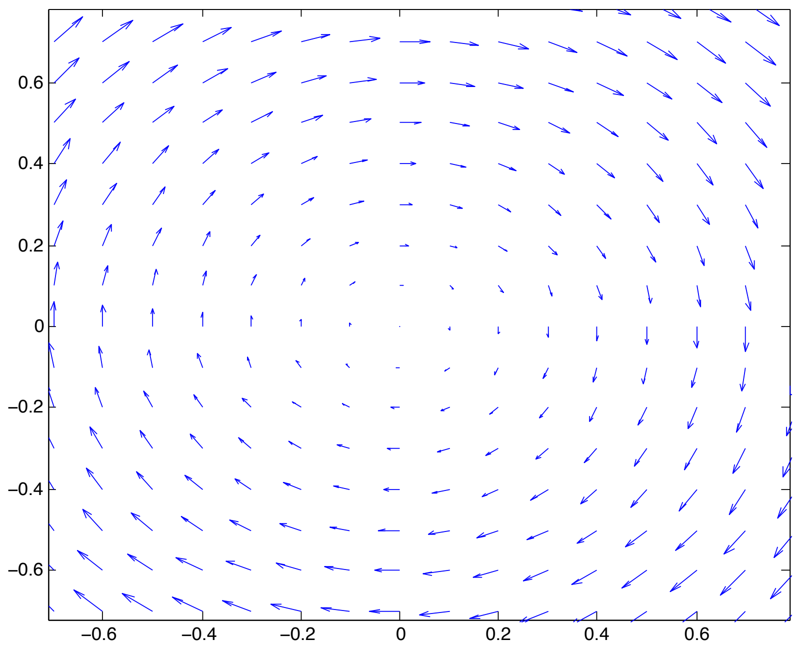

Figure 1.1: Vector field $\mathbf{v} = y\hat{x} - x\hat{y}$. The actual vector field $\mathbf{v} = \frac{y}{x^2+y^2+z^2}\hat{x} - \frac{x}{x^2+y^2+z^2}\hat{y}$ looks exactly the same, except that it is normalized twice, so every vector has exactly the same length as the others, which is less than unit length (this is not terribly enlightning when plot, that is the reason for plotting $\mathbf{v} = y\hat{x} - x\hat{y}$ instead. Other than vector length, the two fields look exactly the same).

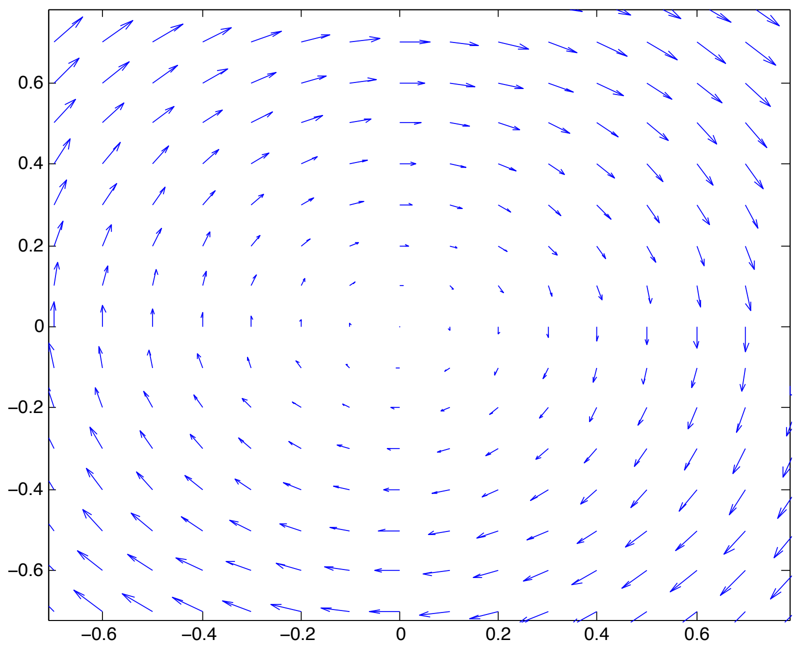

Figure 1.2: Vector field $\mathbf{v} = y\hat{x} - x\hat{y}$.

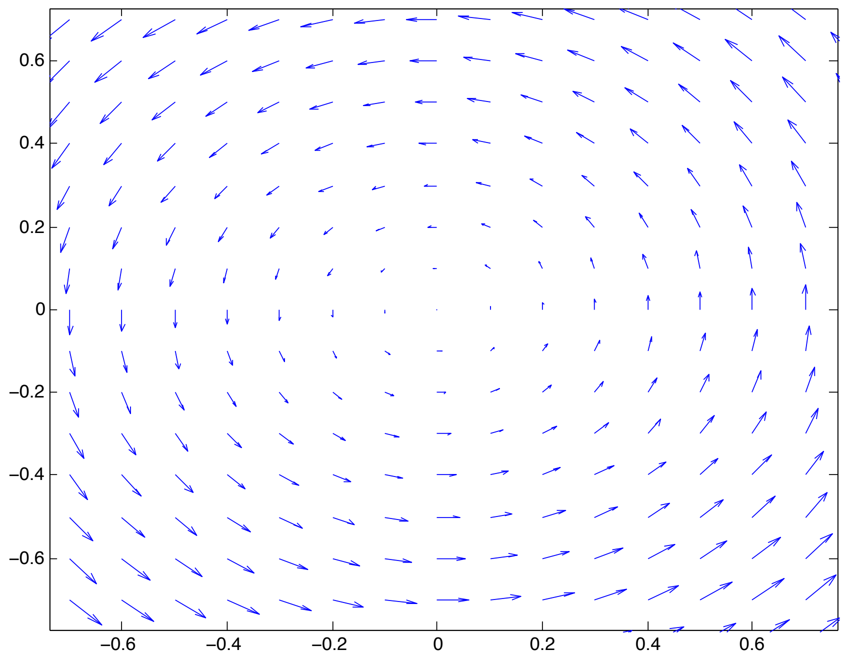

Figure 1.3: Vector field $\mathbf{v} = -y\hat{x} + x\hat{y}$.

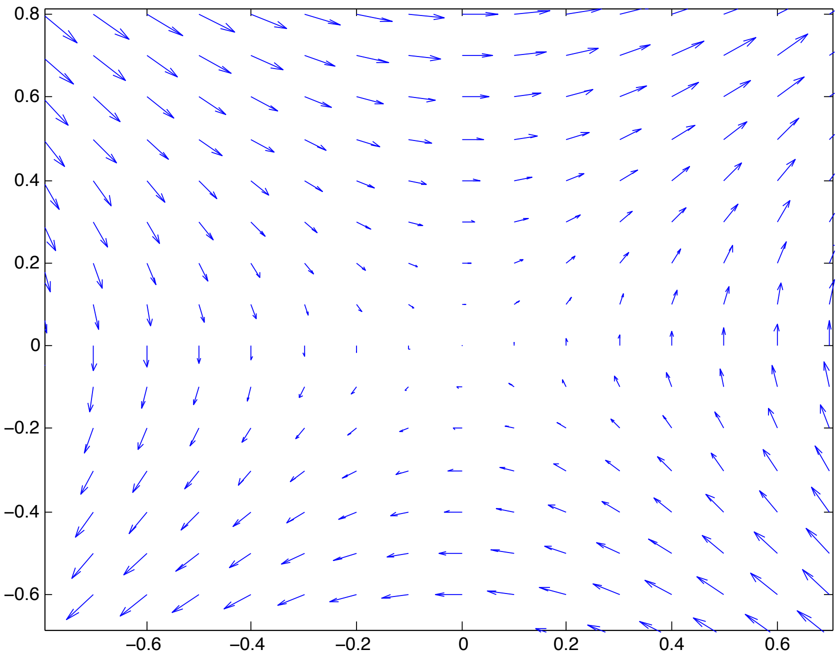

Figure 1.4: Vector field $\mathbf{v} = y\hat{x} + x\hat{y}$.

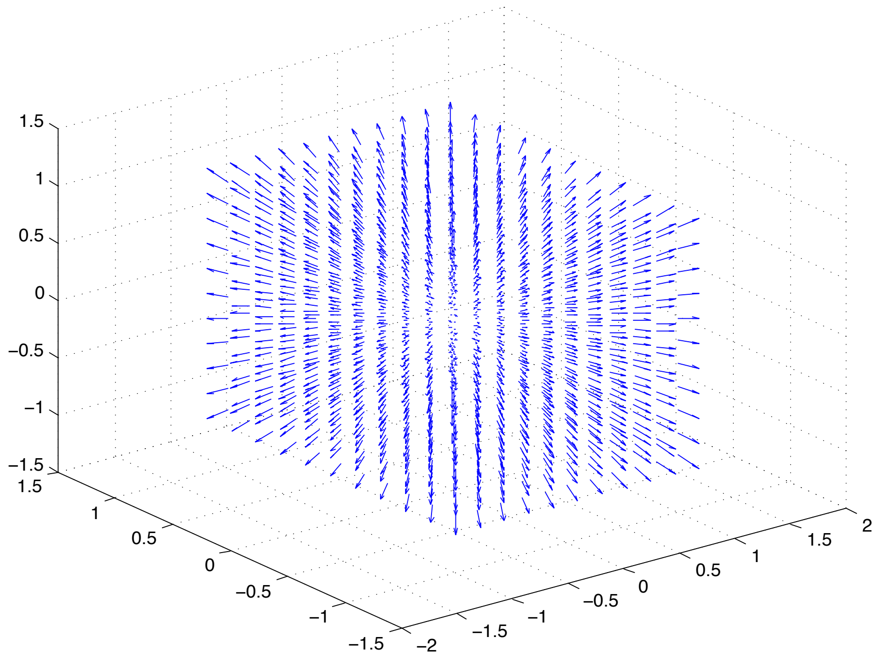

Figure 1.5: Vector field $\mathbf{v} = x\hat{x} + y\hat{y} + z\hat{z}$.