UNIVERSITY OF MARYLAND, PHYSICS DEPARTMENT

PHYS 272, HW 5 Solutions

Jonathan Echevers

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1 PROBLEM A

1.1 PART I

See Fig. 1.1

$$\mathbf{r}_1 = L\hat{x} + L\hat{y}$$
$$\mathbf{r}_2 = L\hat{x} + 0\hat{y}$$
$$\mathbf{r}_3 = 0\hat{x} + 0\hat{y}$$
$$\mathbf{r}_4 = 0\hat{x} + L\hat{y}$$

1.2 PART II

$$\begin{aligned} |\mathbf{r}_1 - \mathbf{r}_3| &= |(L-0)\hat{x} + (L-0)\hat{y}| \\ &= [L^2 + L^2]^{1/2} \\ &= L\sqrt{2} \end{aligned}$$

1.3 PART III

$$\begin{aligned} |\mathbf{r}_4 - \mathbf{r}_2| &= |(0-L)\hat{x} + (L-0)\hat{y}| \\ &= [L^2 + L^2]^{1/2} \\ &= L\sqrt{2} \end{aligned}$$

1.4 PART IV

Define the variable l to be the length of the diagonal.

$$l = |\mathbf{r}_1 - \mathbf{r}_3| = |\mathbf{r}_1| = L\sqrt{2}$$

1.5 PART V

Since the origin is center at $\mathbf{r}_3$, the distance will be:

$$d = |\mathbf{r} - \mathbf{r}_3| = |(x-0)\hat{x} + (y-0)\hat{y} + (z-0)\hat{z}| = [x^2 + y^2 + z^2]^{1/2}$$

1.6 PART VI

$$d = |\mathbf{r} - \mathbf{r}_1| = |(x-L)\hat{x} + (y-L)\hat{y} + (z-0)\hat{z}| = [(x-L)^2 + (y-L)^2 + z^2]^{1/2}$$

1.7 PART VII

Denote the angle between the vectors $\mathbf{r}_4$ and $\mathbf{r}_1 + \mathbf{r}_2$ as θ. Draw a picture, notice that the vector $\mathbf{r}_4$ is simply the projection of the vector $\mathbf{r}_1 + \mathbf{r}_2$ onto the y-axis. Then:

$$\begin{aligned} |\mathbf{r}_4| &= \cos\theta |\mathbf{r}_1 + \mathbf{r}_2| \\ \rightarrow \theta &= \cos^{-1} \frac{|\mathbf{r}_4|}{|\mathbf{r}_1 + \mathbf{r}_2|} \\ &= \cos^{-1} \frac{L}{L(4+1)} \\ &\approx 63^\circ \end{aligned}$$

1.8 PART VIII

See Fig. 1.2. Look at it and make sure you understand everything that's going on in the picture (and recall the pythagorean theorem.) Denote the radius of the largest circle that fits in the space between the circle and the square as r. Then:

$$\begin{aligned} r\sqrt{2} &= L\frac{\sqrt{2}}{2} - r \\ \rightarrow r &= L\frac{\sqrt{2}}{2(1+\sqrt{2})} \end{aligned}$$

2 PROBLEM B

2.1 PART I

See Part II for a detailed explanation (the same logic applies here).

$$\begin{aligned} V &= \frac{Q}{4\pi\epsilon_0} \sum_i \frac{1}{r_i} \\ &= \frac{Q}{4\pi\epsilon_0} [\frac{1}{(x^2+y^2+z^2)^{1/2}} + \frac{1}{((x-L)^2+y^2+z^2)^{1/2}} + \frac{1}{((x-L/2)^2+(y-L)^2+z^2)^{1/2}}] \end{aligned}$$

2.2 PART II

See Fig. 2.1 for field lines.

The electric field produced by a point charge is $\mathbf{E} = \frac{q\hat{\mathbf{r}}}{4\pi\epsilon_0 r^2}$, where $\mathbf{r}$ denotes the position vector that goes from the charge to the point in space at which we are measuring the field, and r denotes the magnitude of such vector (See Fig. 2.2). Let the field point be denoted by the arbitrary position vector $\mathbf{p} = x\hat{x} + y\hat{y} + z\hat{z}$, clearly this works for any position in space that we choose. Then, the total electric field produced by the three charges will simply be the sum of the field produced by individual charges:

$$\mathbf{E}_{total} = \sum_i \mathbf{E}_i = \frac{Q}{4\pi\epsilon_0} \sum_i \frac{\hat{\mathbf{r}}_i}{r_i^2}$$
$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{x\hat{x}+y\hat{y}+z\hat{z}}{(x^2+y^2+z^2)^{3/2}} + \frac{(x-L)\hat{x}+y\hat{y}+z\hat{z}}{((x-L)^2+y^2+z^2)^{3/2}} + \frac{(x-L/2)\hat{x}+(y-L)\hat{y}+z\hat{z}}{((x-L/2)^2+(y-L)^2+z^2)^{3/2}}\right]$$

2.3 Part III

At a point very far away, almost all the potential energy will be transformed into kinetic energy (by energy conservation), so the total kinetic energy will approximately correspond to the initial potential energy. Denote the total kinetic energy by T. The initial potential energy will simply correspond to the Coulomb potential:

$$T \approx V$$

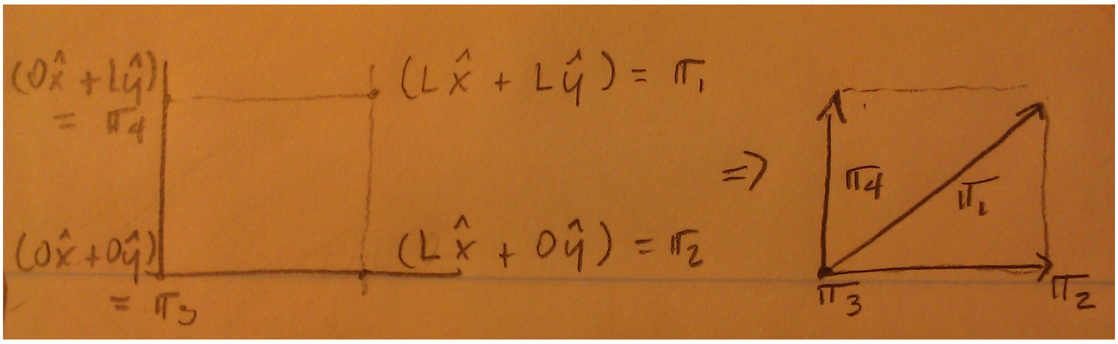

Figure 1.1: Problem A

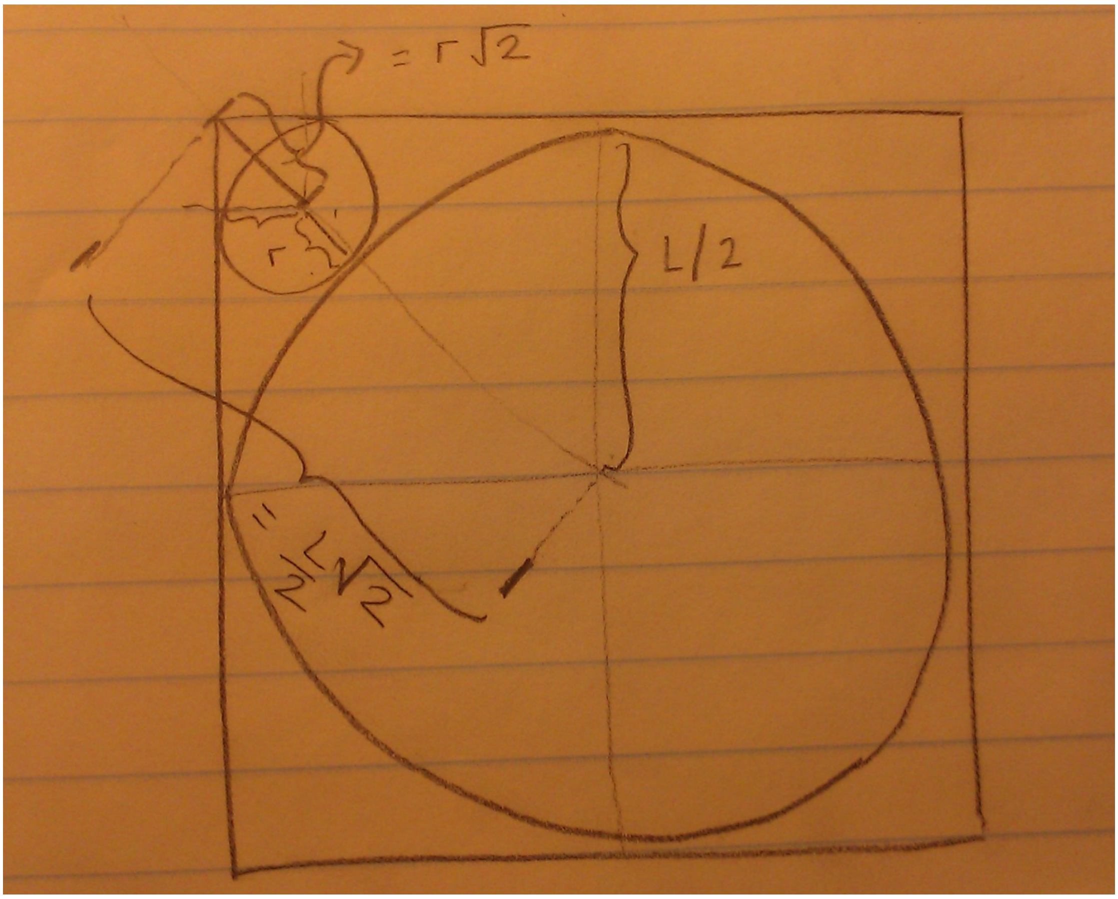

Figure 1.2: Problem A(viii)

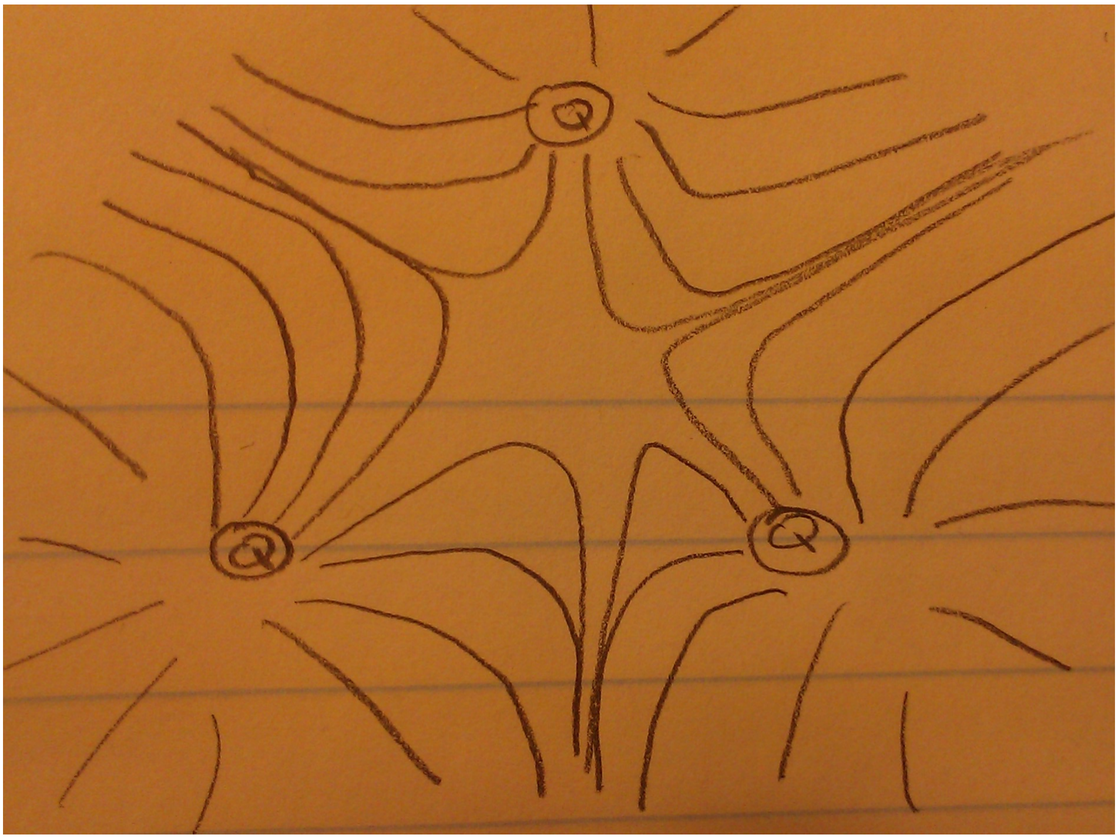

Figure 2.1: Problem B

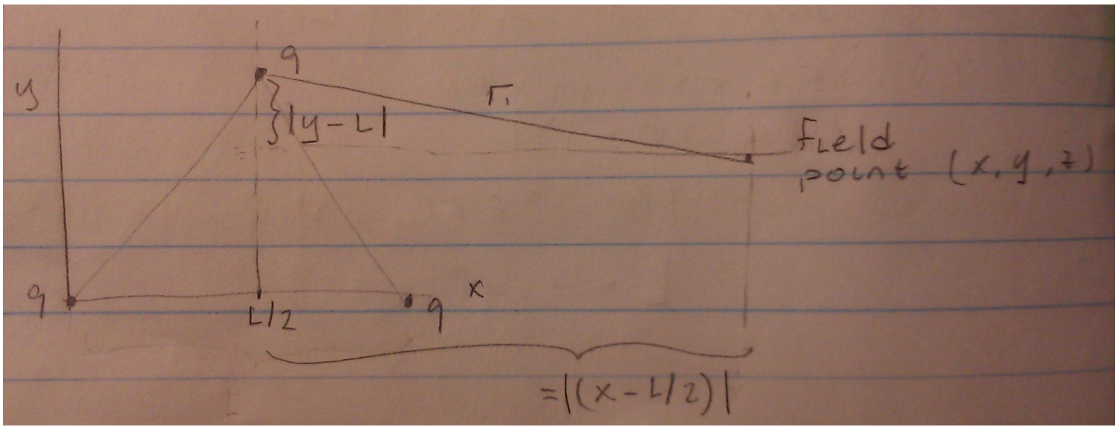

Figure 2.2: Problem B