Twin Paradox

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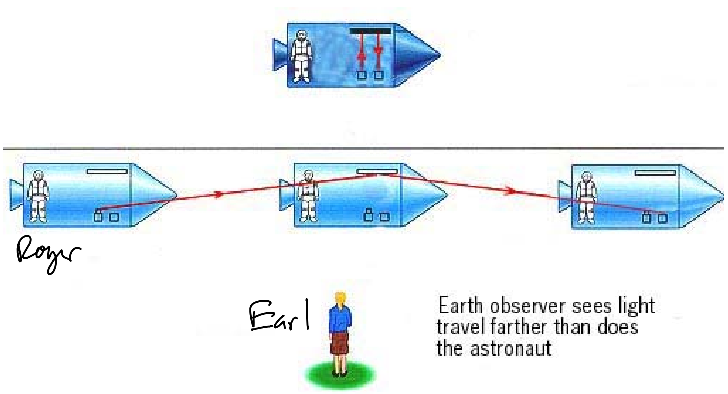
Rocket - Roger

Earth - Earl

Earl measures proper time. From Earl's perspective, there are Two events: rocket leaves + rocket returns. He measures Both Events with one clock. He is in an inertial reference frame.

Roger does not measure proper time. His rocket must decelerate + accelerate to turn around + go back to earth. He can not apply special relativity arguments since he is in a non inertial reference frame.

We must analyze entire problem in Earth's reference frame in order to correctly utilize special relativity:



Earl observes Roger's clock running slower

The time that passes for 1 tick on the Rocket's clock (Δt) is longer than 1 tick on the Earth's clock ($\Delta \tau$), so $\Delta t > \Delta \tau$:

$$\Delta t = \frac{\Delta \tau}{\sqrt{1-(v/c)^2}}$$

That is, Earl's clock runs faster than Roger's clock as observed by Earl:

(Rate of Earl's clock $= \frac{1}{\Delta\tau}$, Rate of Roger's clock $= \frac{1}{\Delta t}$)

$$\frac{1}{\Delta t} < \frac{1}{\Delta\tau}$$

Since Earl's heart rate is faster, he ages more quickly than Roger. Earl is older than Roger upon Roger's return.

The time that elapses between the two events (rocket leaves & rocket returns) as measured by Earl's Clock (Δt_E) is therefore more than that measured by Roger's clock (Δt_R) since the Earth clock is running faster:

$$\Delta t_E > \Delta t_R \quad \Rightarrow \quad \Delta t_E = \frac{\Delta t_R}{\sqrt{1-(v/c)^2}}$$

where Δt_E is time between take off & return as measured by Earl with Earth clock

Δt_R is time between take off & return using Roger's Clock (which is the same as the time measured by Roger between the two events)

that is; Let ΔT be time difference between the two events as measured by the Earth clock

$$\Delta t_E = \left[\frac{1}{\Delta\tau}\right]\Delta T \quad , \quad \Delta t_R = \left[\frac{1}{\Delta t}\right]\Delta T$$

Rate of Earl's clock

Rate of Roger's clock as observed by Earl

$$\therefore \frac{\Delta t_E}{\Delta t_R} = \frac{\Delta t}{\Delta \tau} = \gamma \Rightarrow \boxed{\Delta t_E = \frac{\Delta t_R}{\sqrt{1-(v/c)^2}}}$$

The elapsed time between take off + return as measured by Earl $= \Delta t_E$

The elapsed time between take off + return as measured by Roger $= \Delta t_R$

The Following solutions are wrong but obtain the correct answer. Proper time is not measured by the astronauts! The "boxed" formula above must be applied to obtain the correct answer!

51\. ||| A starship voyages to a distant planet 10 ly away. The explorers stay 1 yr, return at the same speed, and arrive back on earth 26 yr after they left. Assume that the time needed to accelerate and decelerate is negligible.

a. What is the speed of the starship?

b. How much time has elapsed on the astronauts' chronometers?

37.51. Model: The earth is frame S and the starship is frame S′. S′ moves relative to S with a speed v.

Solve: (a) The speed of the starship is

$$v = \frac{20\text{ ly}}{25\text{ y}} = \frac{(20\text{ y})c}{25\text{ y}} = 0.80c$$

Wrong

(b) The astronauts measure the proper time while they are traveling. This is

$$\Delta\tau = \sqrt{1-\frac{v^2}{c^2}}\Delta t = \sqrt{1-(0.8)^2}(25\text{ y}) = 15\text{ y}$$

Correct

Because the explorers stay on the planet for one year, the time elapsed on their chronometer is 16 years.

19\. || a. How fast must a rocket travel on a journey to and from a distant star so that the astronauts age 10 years while the Mission Control workers on earth age 120 years?

b. As measured by Mission Control, how far away is the distant star?

37.19. Model: Let S be the earth's reference frame and S′ the rocket's reference frame.

Solve: (a) The astronauts measure proper time $\Delta t' = \Delta\tau$. Thus

Wrong

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-(v/c)^2}} \Rightarrow 120\text{ y} = \frac{10\text{ y}}{\sqrt{1-(v/c)^2}} \Rightarrow v = 0.9965c$$

(b) In frame S, the distance of the distant star is

$$\Delta x = v\Delta t = (0.9965c)(60\text{ y}) = (0.9965\text{ ly/y})(60\text{ y}) = 59.8\text{ ly}$$

Correct

20\. || You fly 5000 km across the United States on an airliner at 250 m/s. You return two days later at the same speed.

a. Have you aged more or less than your friends at home?

b. By how much?

Hint: Use the binomial approximation.

37.20. Model: The earth's frame is S and the airliner's frame is S′. S′ moves relative to S with velocity v. Also, assume zero acceleration/ deceleration times.

The first event is when the airliner takes off and almost instantly attains a speed of v = 250 m/s. The second event is when the airliner returns to its original position after 2 days. It is clear that the two events occur at the *same position* in frame S′ and can be measured with just one clock. This is however not the case for an observer in frame S.

Solve: (a) You have aged less because your proper time is less than the time in the earth frame.

Wrong

(b) In the S frame (earth),

$$\Delta t = \frac{2\times 5\times 10^6\text{ m}}{250\text{ m/s}} = 4.0\times 10^4\text{ s}$$

In the S′ frame (airliner),

$$\Delta\tau = \Delta t\left(1-v^2/c^2\right)^{\frac{1}{2}} \approx \Delta t\left(1-\frac{v^2}{2c^2}\right)$$

$$\Rightarrow \Delta t - \Delta\tau \approx \Delta t\frac{v^2}{2c^2} = \left(4.0\times 10^4\text{ s}\right)\frac{1}{2}\left(\frac{250\text{ m/s}}{3.0\times 10^8\text{ m}}\right)^2 = 1.4\times 10^{-8}\text{ s} = 14\text{ ns}$$

You age 14 ns less than your stay-at-home friends.

Correct