Sunday, November 08, 2009
4:41 PM

Solution to Quiz 7-d
Section 0103

(a) The two events are

1: the astronaut starting from earth

2: the astronaut returning to earth

The proper time will be measured in a frame which is at rest with respect to the moving rocket; hence it has to be the astronaut.

In other words, the two events, 1 & 2 will be measured by the same clock in the astronaut's inertial frame

(b) $\Delta\tau = \sqrt{1-\beta^2}\,\Delta t$

$\Delta\tau = \frac{10}{2}$ years (half of time taken; to go from Earth to the star)

$\Delta t = \frac{20}{2} = 10$ years

$\Rightarrow 5 = 10\sqrt{1-\beta^2} \Rightarrow \sqrt{1-\beta^2} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow 1-\beta^2 = \frac{1}{4} \Rightarrow \beta^2 = 1-\frac{1}{4} = \frac{3}{4} \Rightarrow \beta = \sqrt{3}/2$

$\Rightarrow v/c = \beta = \sqrt{3}/2 = 0.866 \Rightarrow v = 0.866c$

(c) The distance would be

$\Delta x = v \cdot \Delta t$

Note that every term in the above eqn is w.r.t. the Earth frame

Δx: Dist. b/w Earth & star, as measured in Earth frame

v: velocity of rocket, measured in Earth frame

Δt: time elapsed in Earth frame

$\Rightarrow \quad \Delta x = 0.866\,c \times \frac{20}{2}$ years

$\Rightarrow \quad \Delta x = 8.66\,c \times$ years

$= 8.66$ light-years

[20 years were taking for the round journey which traversed Earth to Star twice]