Wednesday, December 02, 2009
10:03 AM

Solution to Quiz 10

The wavefunction of a particle in infinite potential well, is given by

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} & 0 \le x \le L \\ 0 & x < 0 \ \& \ x > L \end{cases}$$

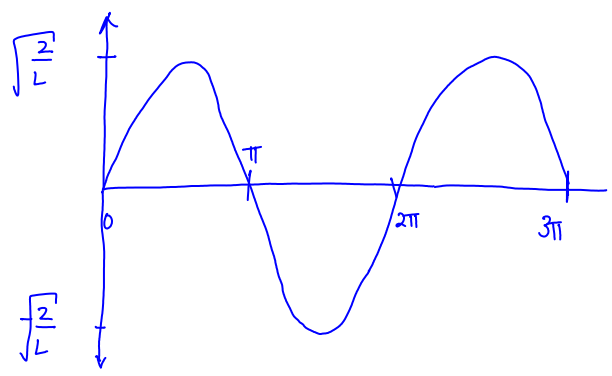
Here, $n = 1, 2, \ldots$ etc. For $n = 3$

$$\psi_3(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$$

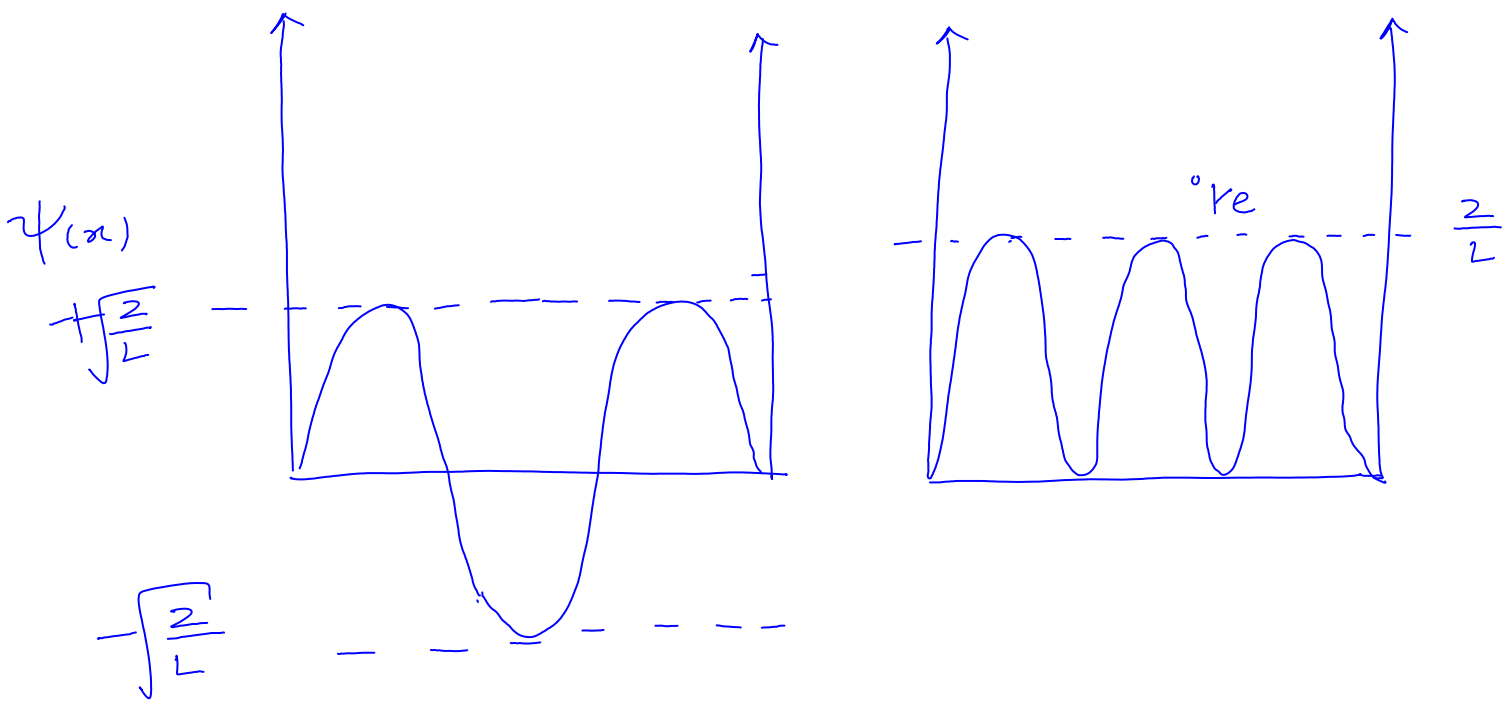
as x goes from 0 to L,

$\frac{3\pi x}{L}$ goes from 0 to 3π

Hence the wavefunction is simply a sine function, plotted from 0 to 3π, with amplitude $\sqrt{2/L}$



Hence $\psi(x)$ & $|\psi(x)|^2$ will look like



The particle is most likely to be found where $|\psi(x)|^2$ has a maxima, or whereever $\psi(x)$ has a maxima or minima. ie. extrema

maxima or minima, i.e. extrema

Since Sin function has an extrema at $(n+\frac{1}{2})\pi$,

the particle is most likely to be found at

$$\frac{3x\cancel{\pi}}{L} = (n+\tfrac{1}{2})\cancel{\pi} \qquad 0 \le x \le L$$

$$\Rightarrow \quad x = \frac{L}{3}(n+\tfrac{1}{2})$$

$$n=0, \quad x = \frac{L}{6} \; ; \quad n=1, \; x = \frac{L}{2} \; ; \; n=2, \; x = \frac{5L}{6}$$

For $n = 3,4,5\ldots$ etc we get $x > L$ where we know there is no wavefunction.

Hence the positions where the particle is most likely to be is

$$x = \frac{L}{6}, \quad \frac{3L}{6}\left(=\frac{L}{2}\right), \quad \frac{5L}{6}$$