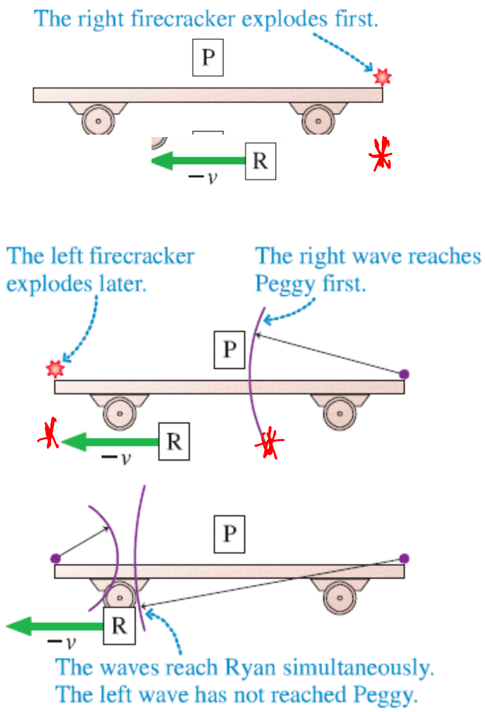
Length-contraction

Sunday, November 01, 2009
4:03 PM



What is the relationship between

① Length of the car (in the car's rest frame)

② Distance between scorch marks in Ryan's frame (Length of car as measured by Ryan)

③ Distance between scorch marks in Peggy's frame?

Length Contraction analysis:

I. Ryan measures simultaneously the front & back of the train & measures a length L

II. Ryan measures at 1 position the time the front of the train passes (event 1) & the time the back of the train passes (event 2), Δt. Then, $L = v\Delta t$, which gives the same result

Note: this Δt is the proper time $\Delta\tau$ since the two event were timed with a single clock

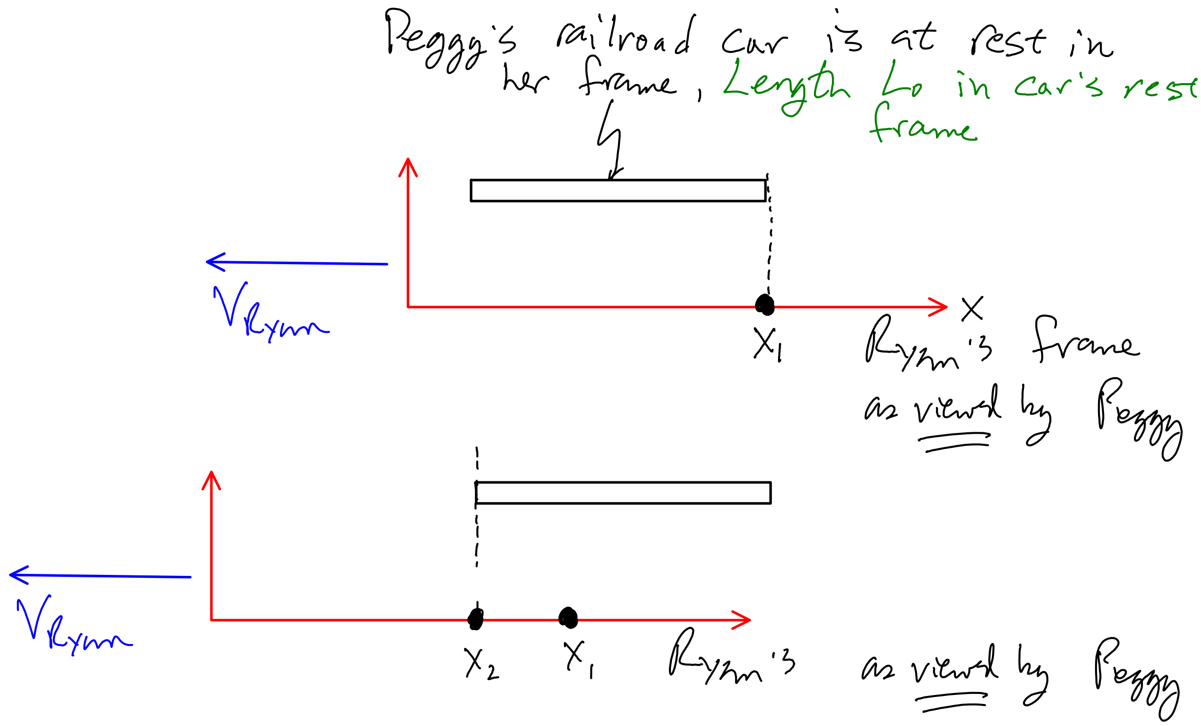
Case I: What Does Peggy See?

Recall that simultaneous events (firecrackers) at front & rear of train in Ryan's frame implies the front event

Recall that simultaneous events (firecrackers) at front & rear of train in Ryan's frame implies the front event occurred first in Peggy's Frame!

The Events are now Ryan taking a measurement.

∴ In Peggy's Frame, Ryan measures the Front of the train first and at a later time measures the Back of the train:



Peggy "see's" Ryan measure front of train then, at a later time, the back of the train.

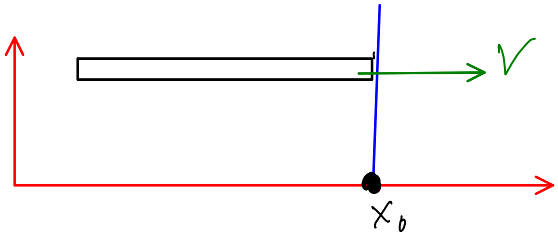
Peggy would conclude that Ryan screwed up the Measurement, & measured a shorter length

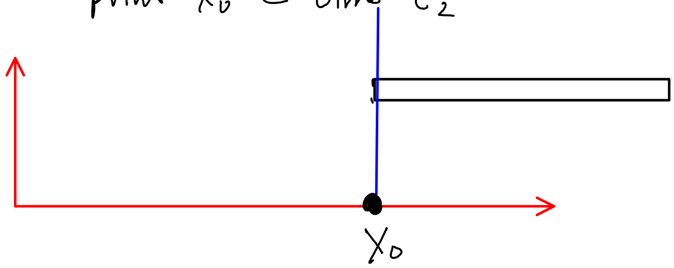
How do we relate the two lengths, the one measured by Ryan & the rest frame length L_0?

Case II: Ryan measures train length with a single clock: $\Delta\tau = \frac{L}{V}$

event 1: front of train crosses point x_0 @ time t_1

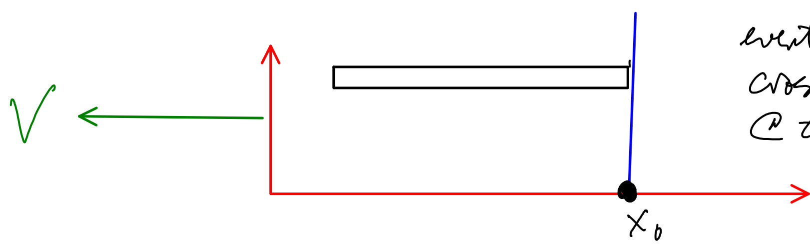
event 2: Back of train crosses point x_0 @ time t_2



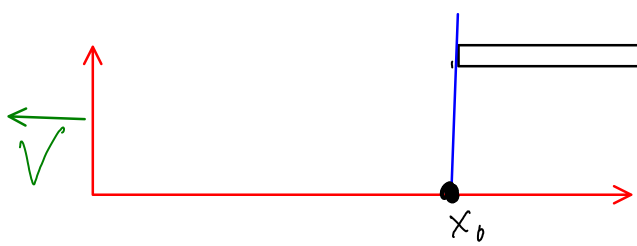


Ryan's frame: $\Delta\tau = t_2 - t_1 = \frac{L}{V}$

From Peggy's perspective, Ryan moves to left @ speed V



event 1: ryan's point x_0 crosses front of train @ time t_1



event 2: Ryan's point x_0 crosses rear of train @ time t_2

In Peggy's frame, train is at rest + the Length is L_0.

Δt in Peggy's frame $= \frac{L_0}{V}$

So we have: $V = \frac{L_0}{\Delta t} = \frac{L}{\Delta\tau}$

L_0 + Δt as measured in Peggy's frame

L + $\Delta\tau$ as measured in Ryan's frame

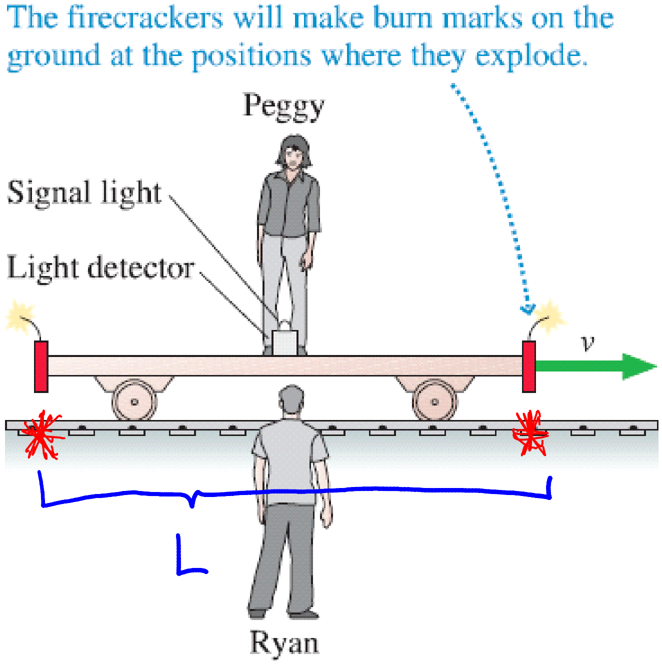
$$\therefore \quad L = L_0 \frac{\Delta\tau}{\Delta t} \quad , \quad \Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}}$$

$$\therefore \quad L = L_0 \sqrt{1-\beta^2}$$

Ryan measures that the train is shorter

Reconsider firecrackers + burn marks on tracks:

FIGURE 37.16 A railroad car traveling to the right with velocity v.

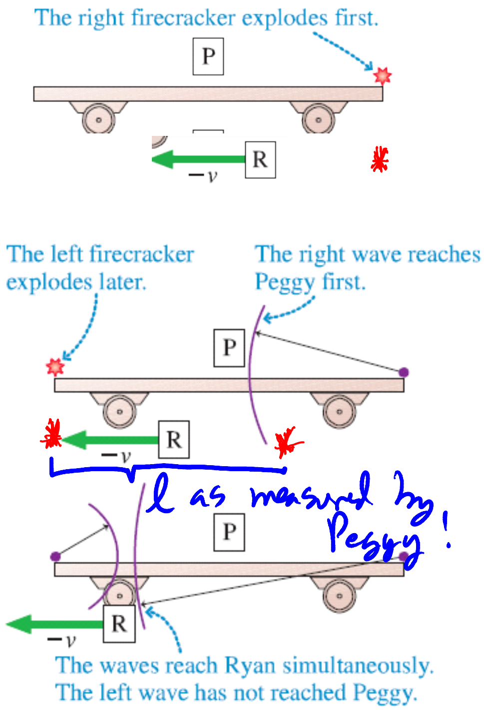


$L < L_0$.

The spacing on the tracks is smaller than the rest length L_0 of the car.

Definition: "proper length" is the length of an object as measured in the rest frame of the object

In Peggy's Frame:



Length of railroad car in rest frame L_0

Length of car as measured by Ryan
$$L = L_0\sqrt{1-\beta^2}$$
= "length" of scorch marks in Ryan's frame

What is the distance between scorch marks as measured by Peggy?
Call it "length of scorch marks", l

Consider Peggy making a second pass. The "proper" length is now L, the length of scorch marks in Ryan's frame.

Peggy will measure $l = L\sqrt{1-\beta^2}$ $\quad L = L_0\sqrt{1-\beta^2}$

or $l = L_0(1-\beta^2)$