Chapter 38 and Chapter 39

State of 19th and very early 20th century physics:

Light:

1. E&M **Maxwell's equations** --> waves; J. J. Thompson's **double slit experiment** with light

2. Does light need a medium? --> Aether and **Michelson-Moreley experiment**

   1 and 2 lead to **Relativity**, 1905

3. Detected in discrete lumps --> **photoelectric effect**

   concept of photons

4. **Black Body radiation spectrum** -- Planck's proposition that **energy is quantized**

5. Gas discharge tube produces **discrete line spectrum** which depends upon atoms vs. black body radiation (incandescence) continuous spectrum

6. Emission spectra of hydrogen fairly simple (**Balmer formula**)

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Matter:

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6. **Rutherford’s foil experiment**, fires doubly ionized He at gold foil, most go through by some bounce back --> requires positively charged, heavy centers in gold foil --> nucleus surrounded by 'orbiting' electrons; Deduces the rough diameter of nucleus ~ 10 fm (10^-15 m)

7. 1910, J. J. Thompson develops **mass spectrometer** --> same element has different masses; first evidence of 'something else' besides electrons and positively charged atoms, leading to the discovery of neutrons

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<u>Some major as yet unsolved problems:</u>

Rutherford nuclear model:

1. accelerating electrons should emit E&M radiation losing energy and spiral into nucleus
2. should emit and absorb continuous spectrum (not discrete)

Planck's black body radiation required discrete energies and could not be explained by classical physics, what was viewed as a mathematical trick at the time (later to be viewed as a physical phenomena)

Ejection of electrons from a material from impinging E&M radiation dependent upon frequency, not intensity.

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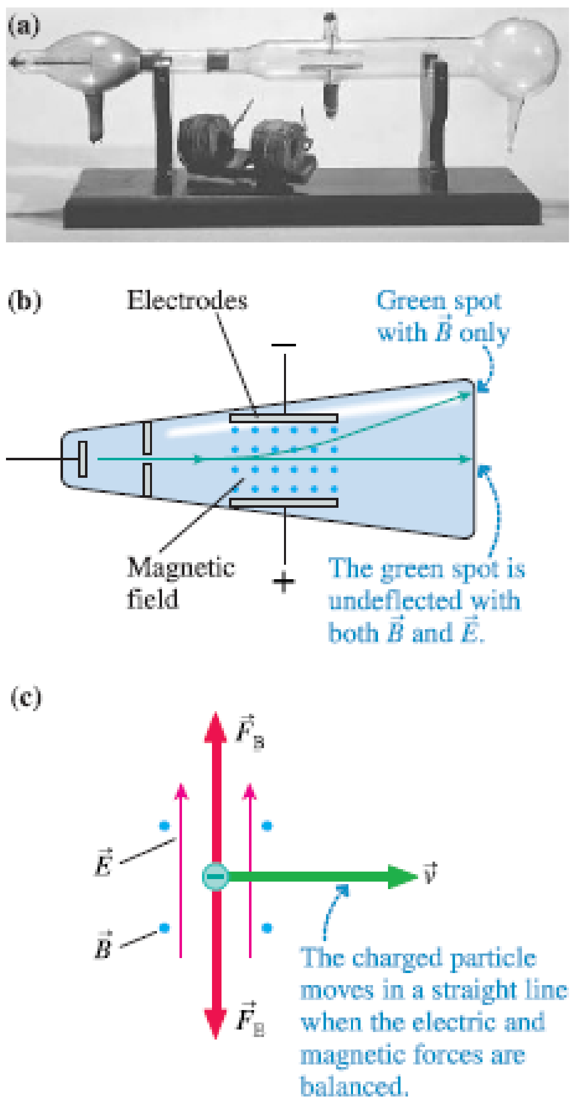
FIGURE 38.7 Thomson's crossed-field experiment to measure the velocity of cathode rays. The photograph shows his original tube and the coils he used to produce the magnetic field.



J. J. Thompson's measurement of e/m

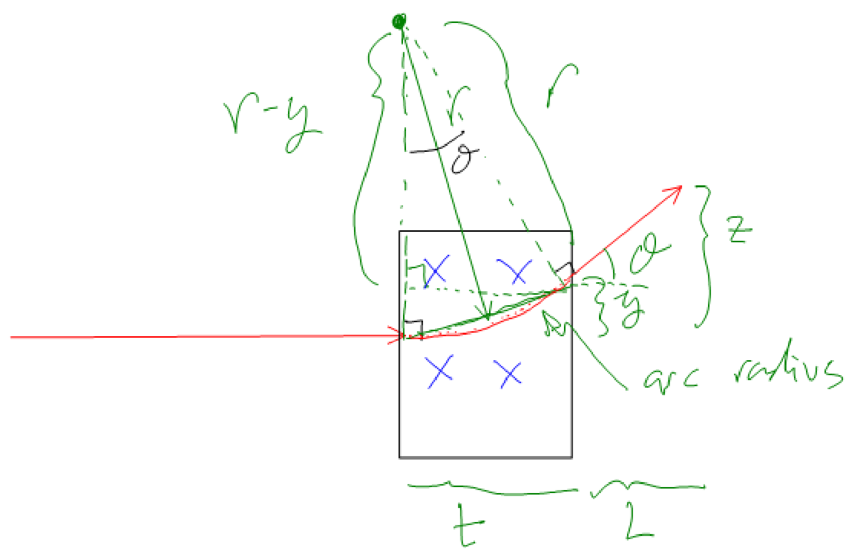
Case I: F_E Cancels F_B gives v

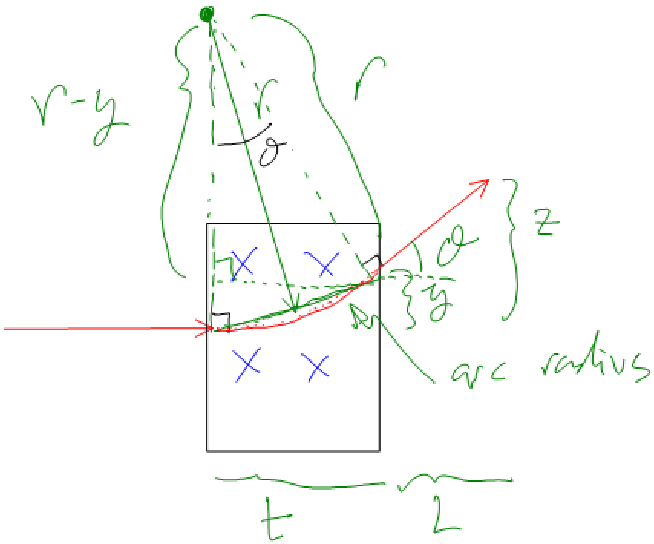
$$F_E = F_B \Rightarrow q\vec{E} = q\vec{v}\times\vec{B}$$

$$\Rightarrow qE = qvB$$

$$\Rightarrow v = \frac{E}{B}$$

Case II: E field off, B field on, measure radius of curvature of arc in B-field





Case II: E field off, B field on, measure radius of Curvature of arc in B-field

$$F = m\frac{v^2}{r} = qvB \quad , \quad v = \frac{E}{B} \text{ from Case I}$$

$$\Rightarrow \frac{q}{m} = \frac{v}{rB}$$

$$(r-y)^2 + t^2 = r^2 \quad , \quad \tan\vartheta = \frac{t}{(r-y)} = \frac{z-y}{L} \Rightarrow (r-y) = \frac{tL}{(z-y)}$$

$$\therefore r^2 = t^2 + \left(\frac{tL}{z-y}\right)^2$$

Measure, t, L, z to find r

for small deflection
$t \ll L$, $z \ll L$
$y \ll z$

$$\Rightarrow r^2 = t^2\left[1 + \left(\frac{L}{z}\right)^2\right]$$

$$\Rightarrow r^2 = \frac{t^2}{z^2}\left[z^2 + L^2\right]$$

$$\Rightarrow r \approx \frac{t}{z}L$$

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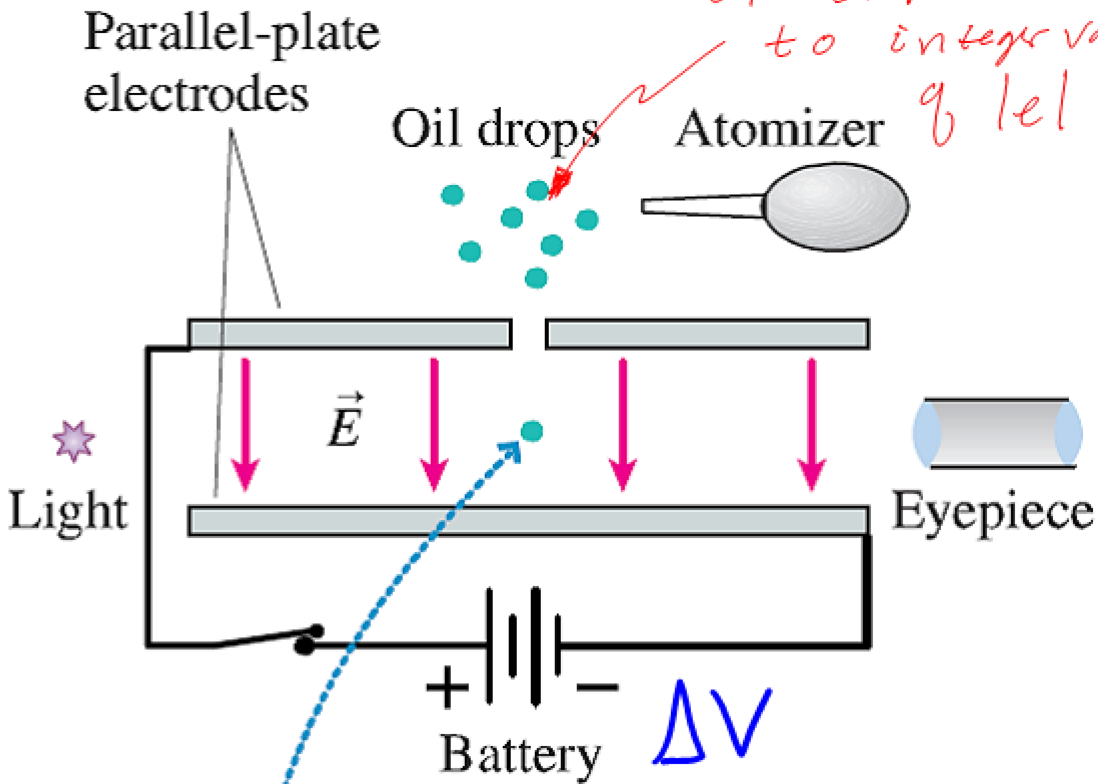
Millikan's Oil drop experiment: Measuring quantized q

oil drops charged to integer values q $|e|$



The upward electric force on a negatively charged droplet balances the downward gravitational force.

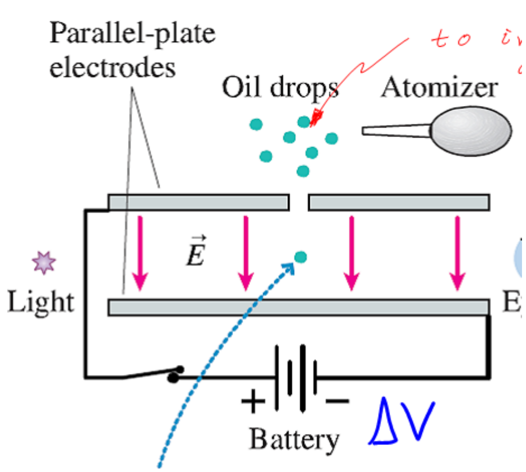
Three forces on oil

① Drag force when moving @ terminal velocity $= C\vec{v}^2$; C a constant

② Gravity, $m\vec{g}$

③ Electric force, $q\vec{E}$

Millikan's Oil drop experiment: Measuring quantized q



I. Battery off:

$\vec{F}_D = C\vec{v}_t^{\,2}$

$\vec{v}_t \downarrow$

$m\vec{g}$

at terminal velocity:

$F_D = mg \Rightarrow C = \dfrac{mg}{V_t^2}$

$\underline{\underline{V_t}}$ is measured

I. Battery off:

at terminal velocity:

$$F_D = mg \Rightarrow C = \frac{mg}{v_t^2}$$

II. Battery on:

at terminal velocity:

$$F_D' + mg = qE \,, \quad E = \Delta V/d \,, \; d = \text{plate separation}$$

$$\underbrace{C v'^2}_{= mg\frac{v'^2}{v^2}} + mg = q\,\Delta V/d$$

$$\Rightarrow mg\left(\frac{v_t'^2}{v_t^2} + 1\right) = q\,\frac{\Delta V}{d}$$

$$\Rightarrow q = \frac{mgd}{\Delta V}\left(\frac{v_t'^2}{v_t^2} + 1\right)$$

radius of oil droplet

$$m = \text{mass of oil droplet} = \rho\left(\frac{4}{3}\pi a^3\right)$$

density of oil

$$\Rightarrow q = \left(\frac{4}{3}\pi g\right)(\rho d)\,\underbrace{\frac{a^3}{\Delta V}\left(\frac{v_t'^2}{v_t^2} + 1\right)}$$

Constants

Measured before expt.,

ΔV, v_t' & v_t measured during experiment

"a" is difficult to measure

$$q = \left(\frac{4}{3}\pi g\right)(\rho\, d)\ \underbrace{\frac{a^3}{\Delta V}\left(\frac{v_t'^2}{v_t^2} + 1\right)}$$

Constants

Measured before expt.,

ΔV, v_t' + v_t measured during experiment

"a" is difficult to measure

$$F_D = \frac{C_d}{2}\, \rho_A\, v_t^2\, A_\perp \quad , A = \text{cross sectional area} = \pi\left(\frac{a}{2}\right)^2, a = \text{diameter of oil drop}$$

C_d = drag coefficient in air

ρ_A = density of air

For Case I:

$$F_D = mg \Rightarrow \frac{C_d}{8}\, \rho_A\, v_t^2\, \pi a^2 = \rho_{oil}\, \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 g$$

$$\Rightarrow a = \frac{3}{4}\, \frac{C_d}{g}\, \frac{\rho_A}{\rho_{oil}}\, v_t^2$$

$$\therefore q = \left(\frac{4}{3}\pi g\right)(\rho\, d)\ \frac{a^3}{\Delta V}\left(\frac{v_t'^2}{v_t^2} + 1\right)$$

Measured: $q = |ne|$ for all oil drops, n integer

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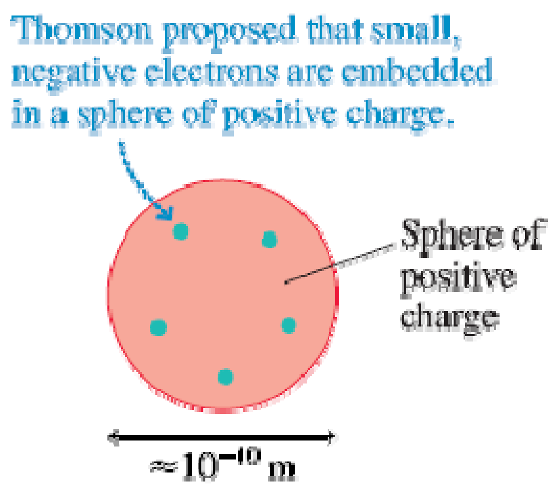
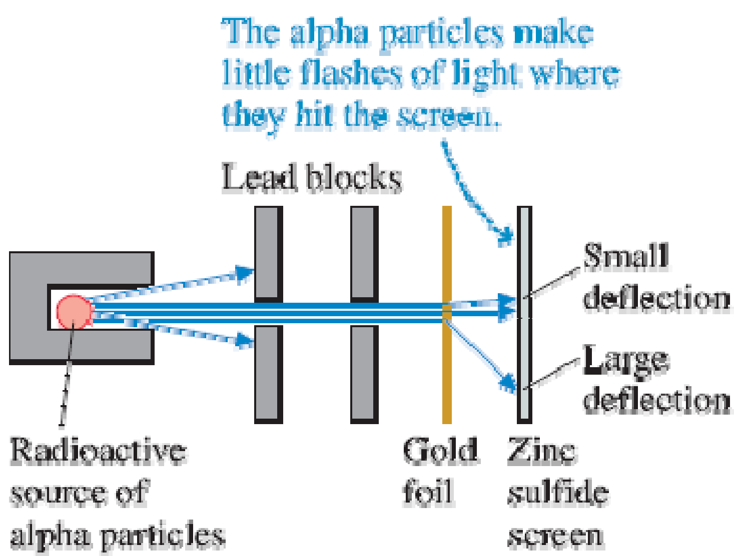
Rutherford Back scattering: discovery of nucleus



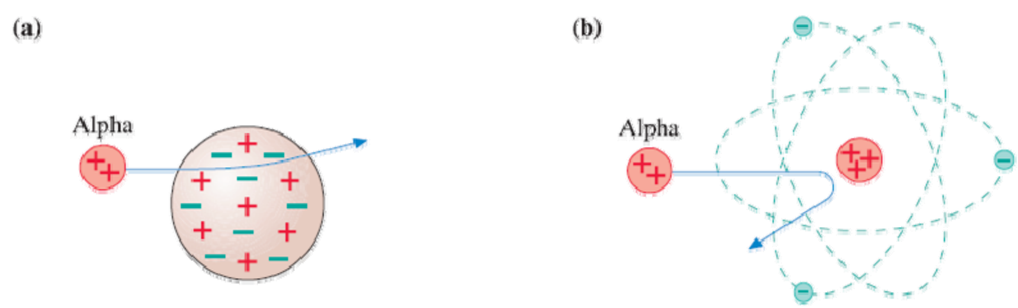


Uranium radioactive decay → beta rays (electrons) and alpha rays (doubly ionized Helium = 2 protons, 2 neutrons)

Use ballistics of particle rays to probe atomic structure: Fire alpha rays at a sheet of gold foil

Statistics of small number of back-scattering events suggested massive tiny cores of positive charge surrounded by negative charge

Deduced the size of the nucleus: 10 fm
(1 fm = 10^-15 m)



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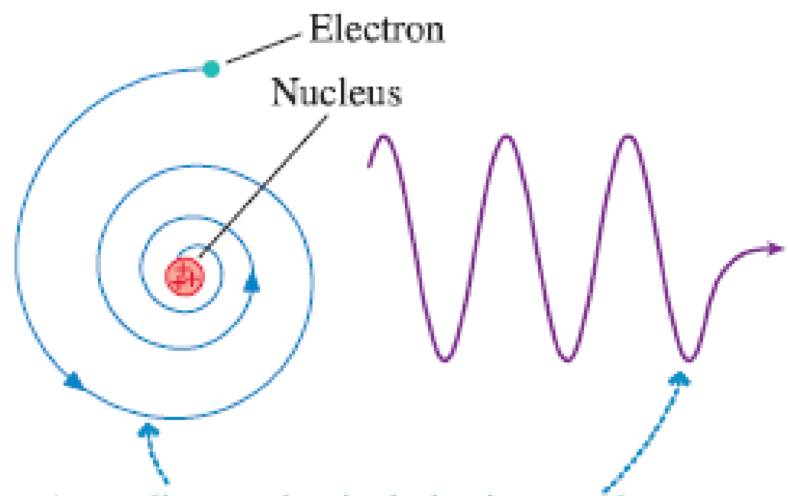

According to classical physics, an electron would spiral into the nucleus while radiating energy as an electromagnetic wave.

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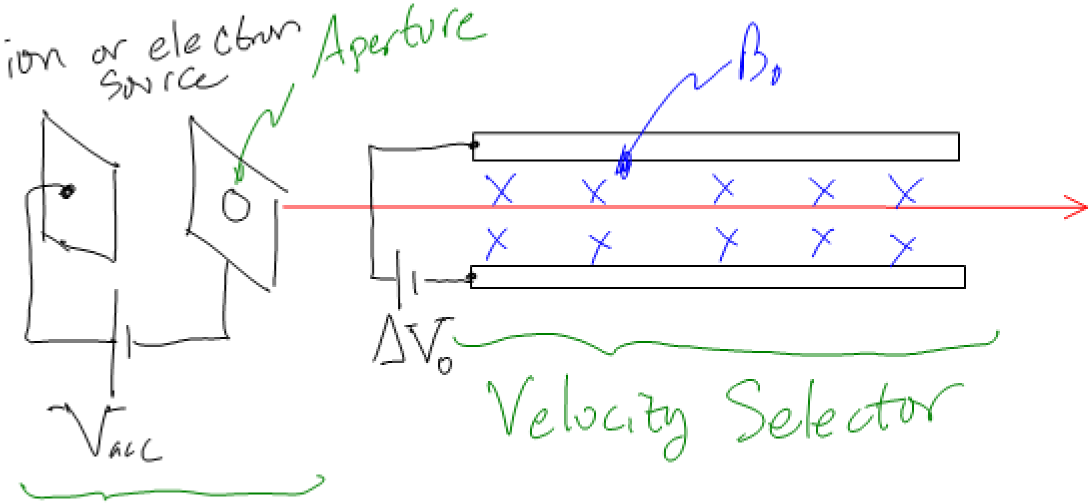
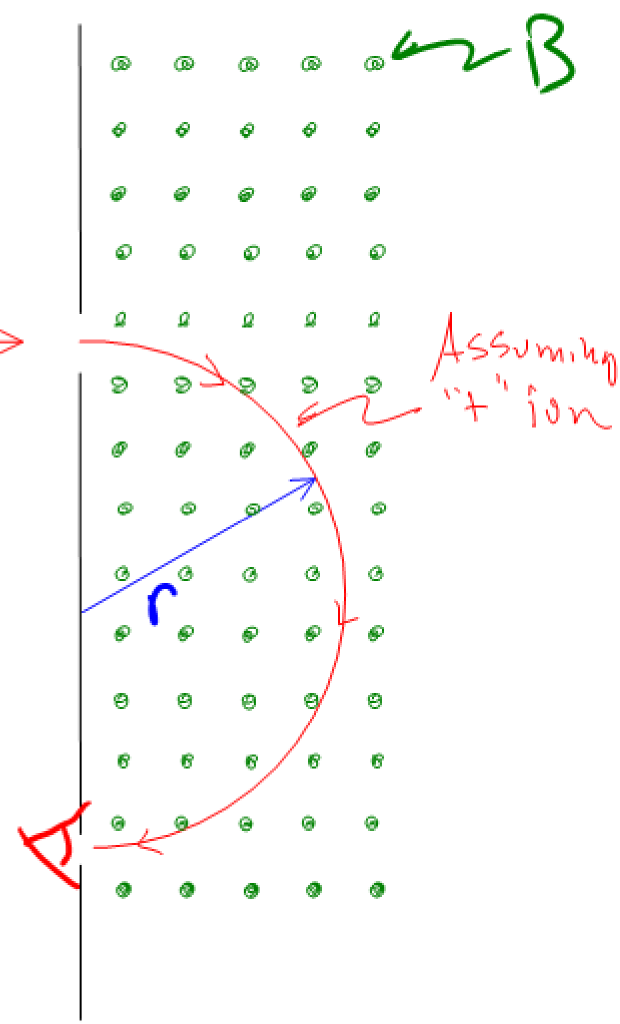
J. J. Thompson's mass spectrometer: First evidence of neutron





ion source:

$$\frac{1}{2}mv^2 \approx qV_{acc} \Rightarrow \frac{q}{m} = \frac{1}{2}\frac{v^2}{V_{acc}}$$

spread in v's

Velocity Selector:

$$F = qE_0 = qvB_0, \quad E = \frac{\Delta V_0}{d}$$

$$\Rightarrow \quad v = \frac{E_0}{B_0} = \frac{\Delta V_0}{B_0 d}$$

J. J. Thompson's mass spectrometer: First evidence of neutron

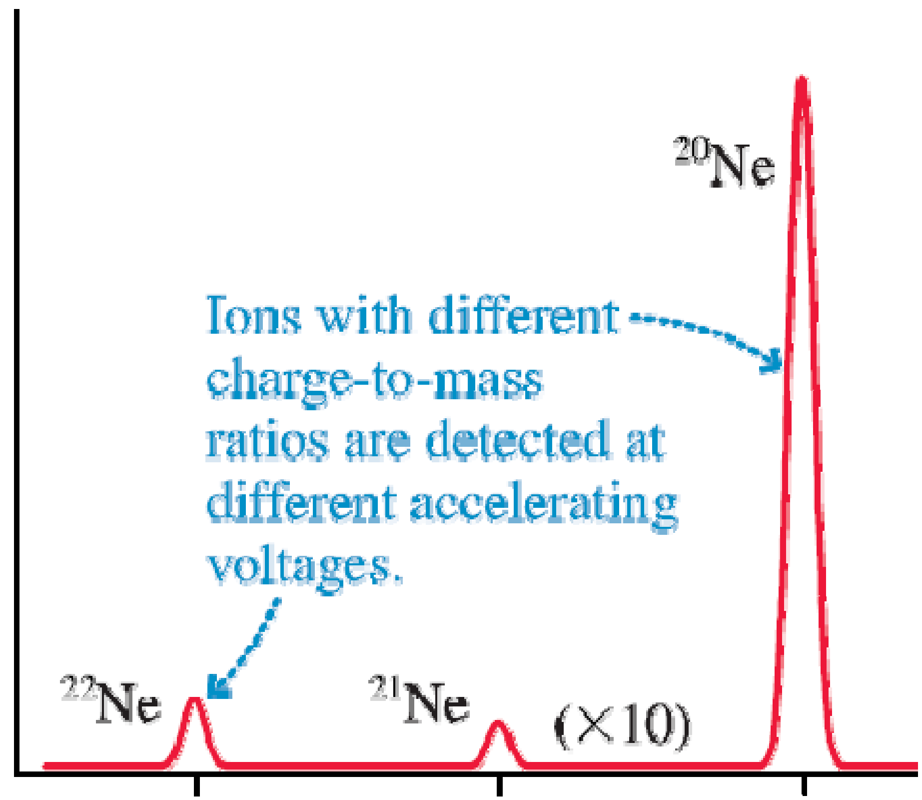
$$v = \frac{E_0}{B_0} = \frac{\Delta V_0}{B_0 d}$$

$$F = m\frac{v^2}{r} = qvB \Rightarrow m = \frac{rqB}{v} = q\left[\frac{rBB_0 d}{\Delta V_0}\right]$$

Know $q = |ne|$

So $m = n\left(\frac{|e| r B B_0 d}{\Delta V_0}\right)$

Discovery of isotopes



Some definitions

Definition of eV: energy required to move 1 electron across 1 V = 1.6 x 10^-19 J

Recall: $W = q\Delta V$ so $\text{Energy} = \overset{1.6\cdot 10^{-19}\,C}{e}\,(1\,V) = 1.6\cdot 10^{-19}\,J$

atomic number: number of electrons or protons, Z

Mass number: A=Z+N, N is the number of neutrons

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   concept of photons

4. **Black Body radiation spectrum** -- Planck's proposition that **energy is quantized**

5. Gas discharge tube produces **discrete line spectrum** which depends upon atoms vs. black body radiation (incandescence) continuous spectrum

6. Emission spectra of hydrogen fairly simple (**Balmer formula**)

Einstein's photon postulate

Relativity: $E^2 - (pc)^2 = (mc^2)^2$

Anything traveling @ speed of light $c \Rightarrow m = 0$

$$\Rightarrow \quad E = pc$$

Postulate: $E_{ph} = \hbar \omega \Rightarrow E_{ph} = hf$

$$\Rightarrow \quad p_{ph} = \frac{E_{ph}}{c} = h\frac{f}{c} \quad , \quad \& \ c = \lambda f$$

$$\Rightarrow \quad p_{ph} = \frac{h}{\lambda}$$

Note: Postulate includes that energy of E&M wave is quantized!

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Proof of photons: Photoelectric effect

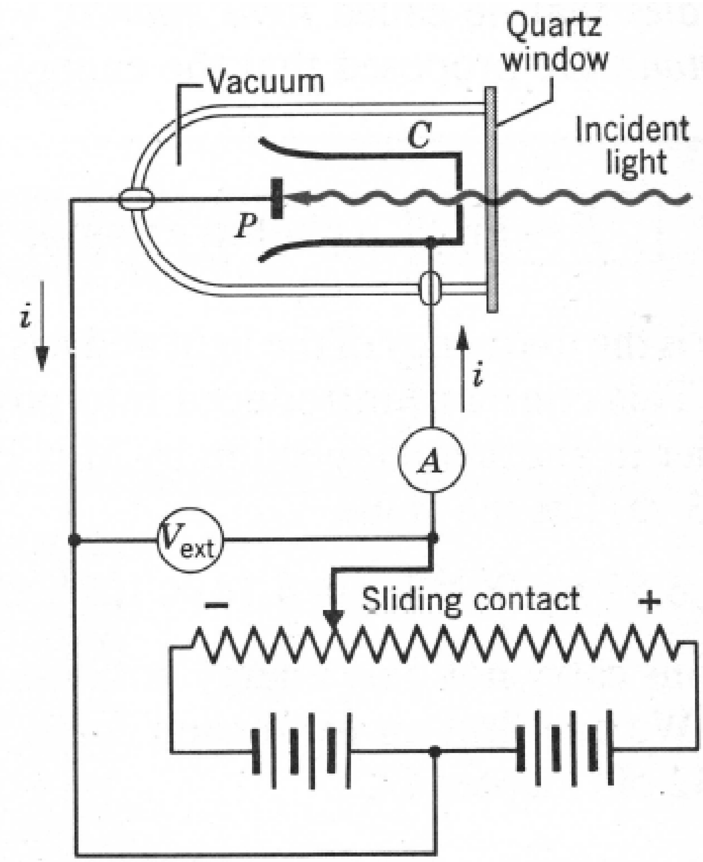
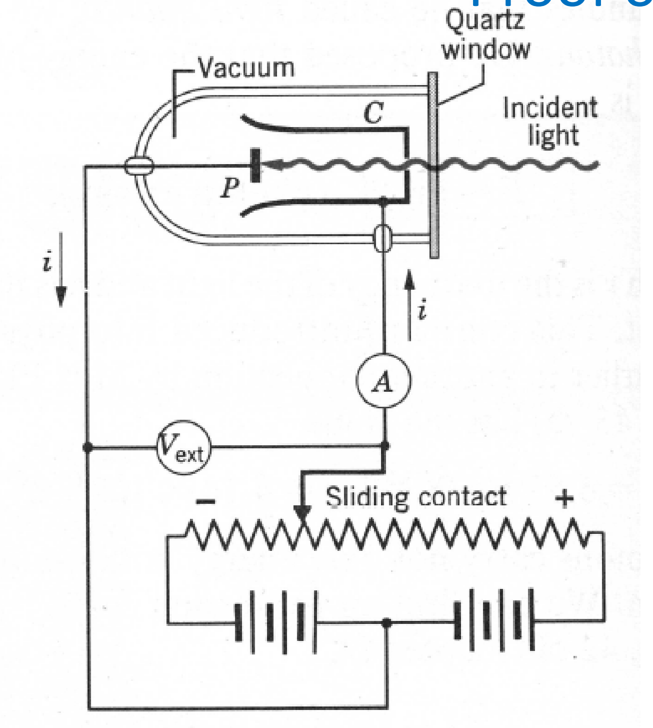
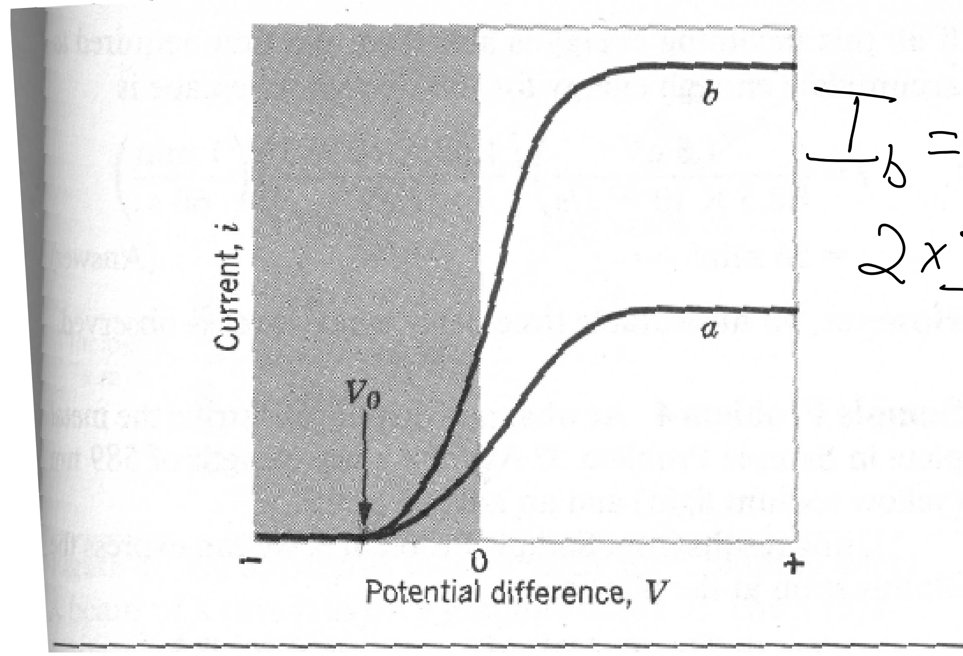


Figure 1 An apparatus used to study the photoelectric effect. The incident light falls on plate *P*, ejecting photoelectrons, which are collected by collector cup *C*. The photoelectrons move in the circuit in a direction opposite to that of the conventional current arrows.

Proof of photons: Photoelectric effect





$I_b = 2 \times I_a$

Stopping Potential

The K.E. of the most energetic electron is measured:

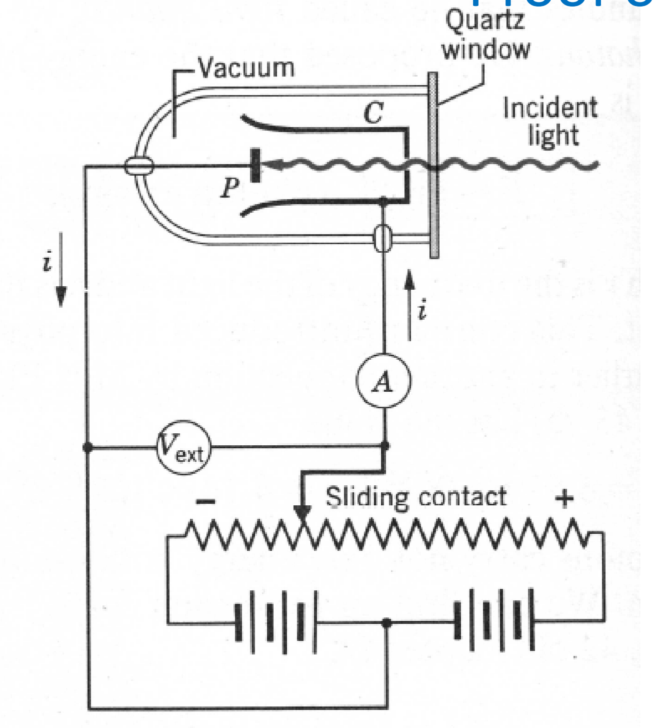
$KE_{max} = eV_0$, independent of intensity (curve a & b)

Classical Expectations:

(1) Energy of ejected electrons $\propto$ Intensity $\propto E^2$

$\Rightarrow$ Expect $KE_{max} \propto$ Intensity

Proof of photons: Photoelectric effect



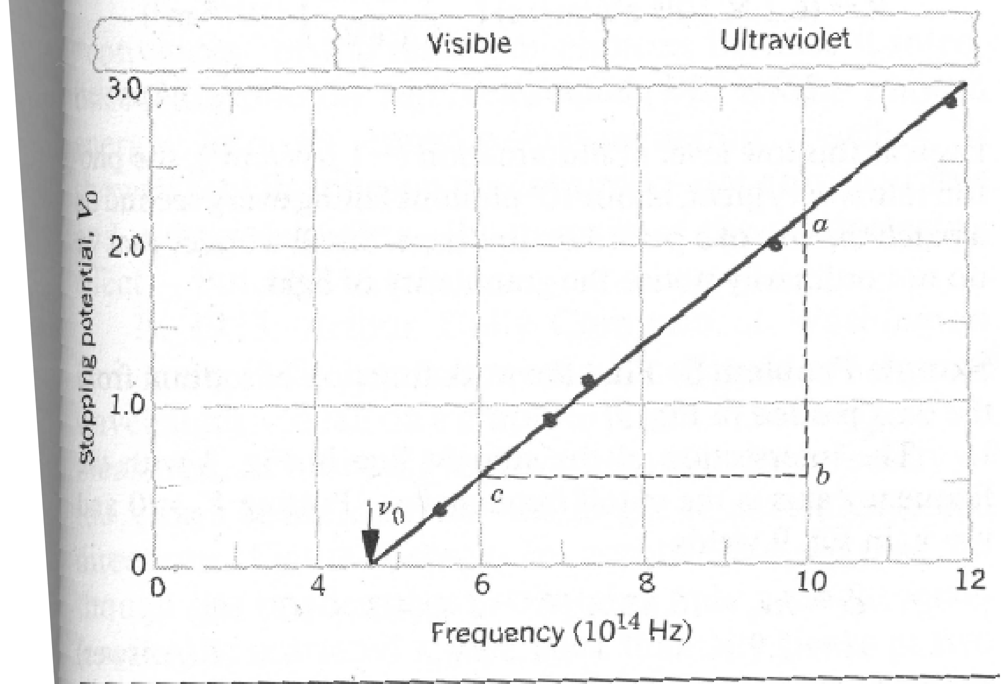


Figure 3 The stopping potential for sodium as a functi...

(2) There exists a cut-off frequency ν_0 below which there is no photoelectric effect

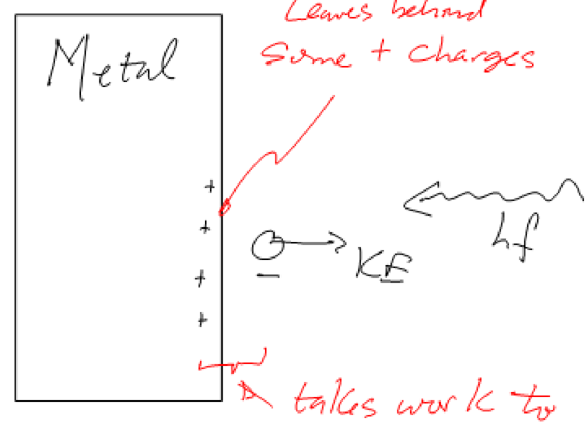
Classical Expectations:

(2) Photoelectric effect should occur at all frequencies provided

Using Einstein's photon model:

$$\underbrace{hf}_{\text{Photon Energy}} = \underbrace{\phi}_{\text{Work function}} + \underbrace{K_m}_{\text{KE of electron}}$$

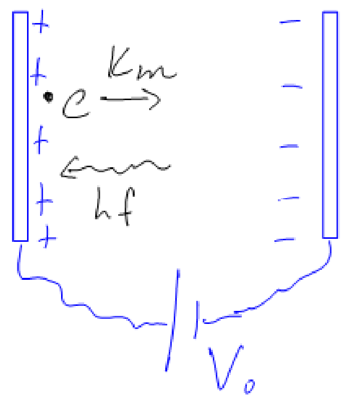
Leaves behind some + charges



takes work to pull e away from "+" charges, ϕ

or $K_m = hf - \phi$

From circuit analysis, $K_m = eV_0$



if $eV_0 > KE$ of e, it will not make it to the "Right" plate

if $eV_0 = KE$, it will barely make it to the "Right" plate

$\Rightarrow K_m = eV_0$

$$\Rightarrow eV_0 = hf - \phi$$

$$\text{or} \quad V_0 = \underbrace{\left(\frac{h}{e}\right)}_{③}f - \frac{\phi}{e}$$

③ Correct slope measured experimentally

① V_0 linear with f

② Minimum f required to observe photo generated current

Photo electric effect "time delay" issue:

Classically: The Energy of an ejected photo electron must be "soaked up" from the incident wave. The effective area in which the electron soaks up this energy ~ cross sectional area of atom. Therefore, if Intensity is feeble, there will exist a time delay as enough (time integrated) energy is absorbed by an electron

This delay is <u>not</u> <u>measured</u>

Photon Picture: photon energy is delivered to the ejected electron in a single collision event

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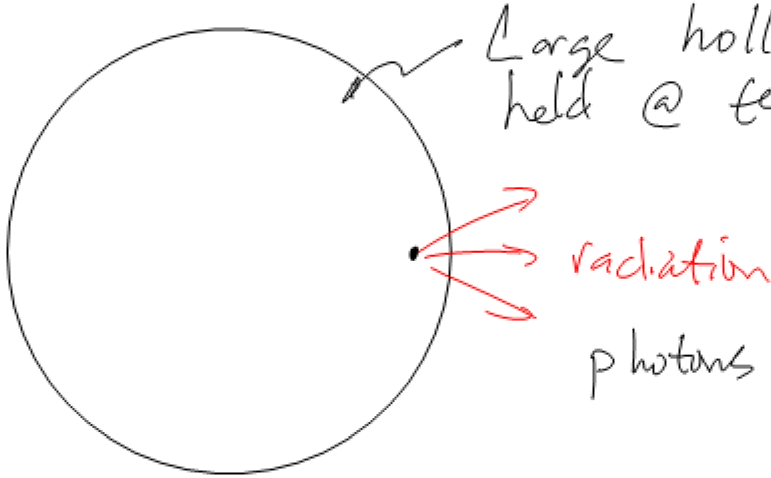
More proof of quantization of energy: Planck's Black-body radiation

Ideal radiator: a radiator whose emitted radiation depends <u>only</u> on temperature (independent of material, nature of surface)

Make an ideal radiator = black body radiator:

Large hollow closed surface held @ temperature T with small hole



photons inside are in equilibrium w/walls

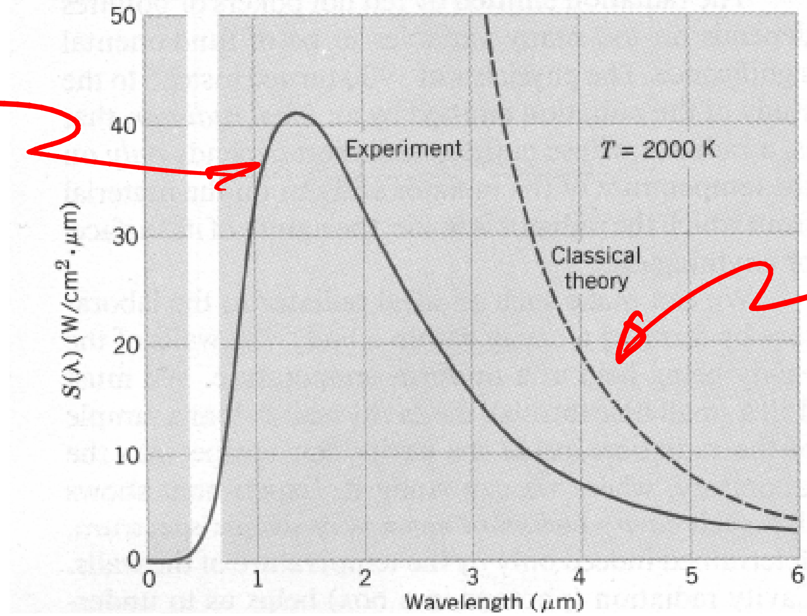
$S(\lambda)\,d\lambda$ = radiated Intensity in the interval of wavelength's λ & $\lambda + d\lambda$

$S(\lambda)$ called Spectral radiance

Classical Law: $S(\lambda) = \frac{2\pi c}{\lambda^4} kT$, k = Boltzmann Constant $= 1.38 \times 10^{-23}$ J/K

More proof of quantization of energy: Planck's Black-body radiation

Experimental Data



$S(\lambda)$

Classical Theory

λ

Classical Law: $S(\lambda) = \frac{2\pi c}{\lambda^4} kT$, k = Boltzmann Constant $= 1.38\times10^{-23}$ J/K

Under the assumption that the atoms that form the walls of the cavity can exist only in states ~~states~~ of definite (quantized) energy; states with intermediary energies are forbidden:

$$S(\lambda) = \frac{2\pi c}{\lambda^4} \frac{hc/\lambda}{e^{hc/kT} - 1}$$

Planck's radiation Law

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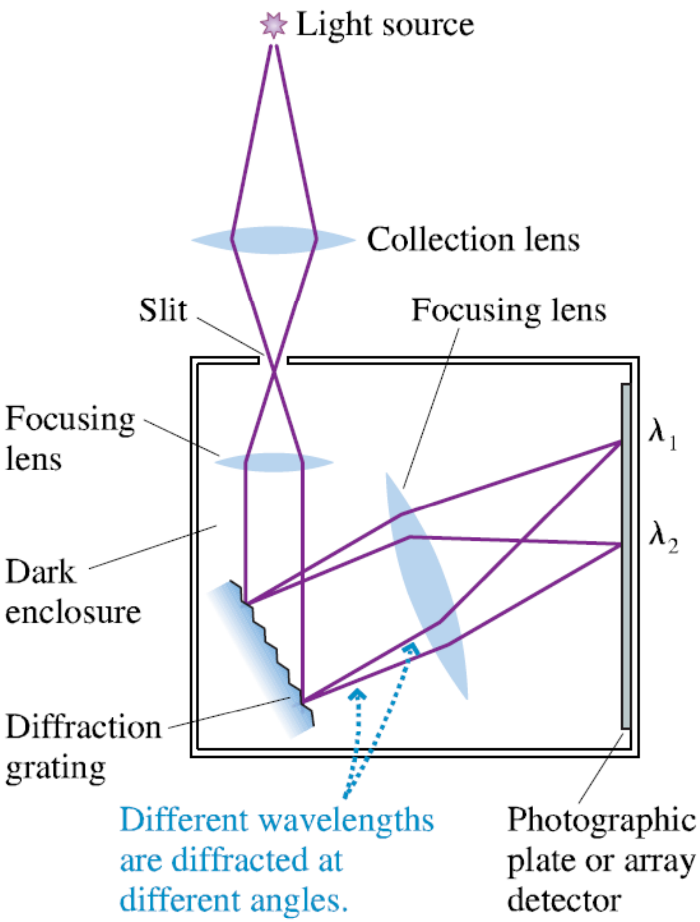
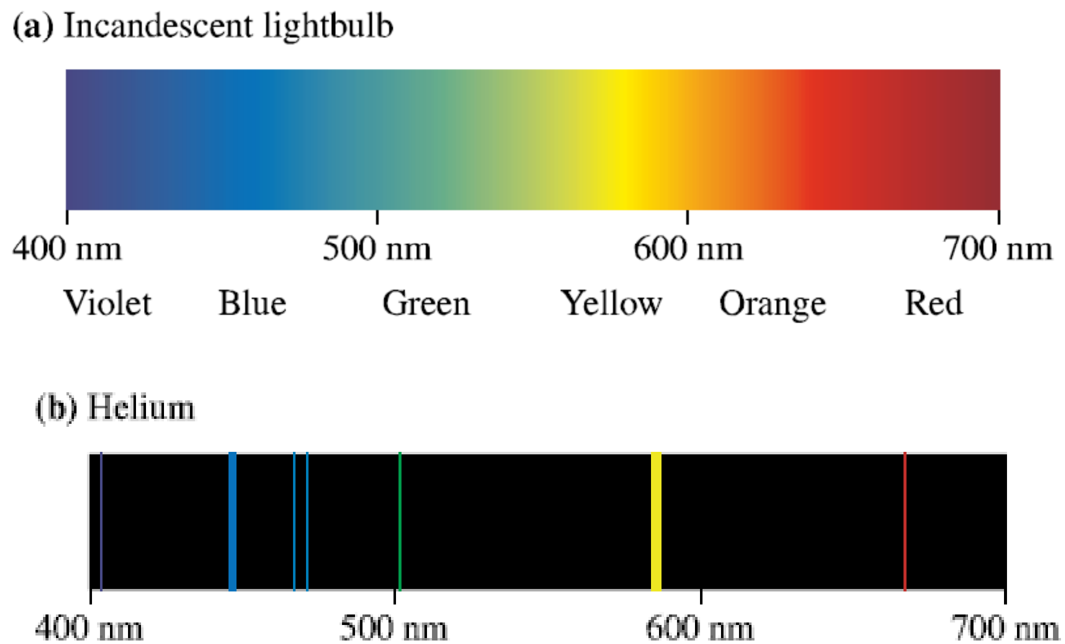
More proof of quantization of energy: Hydrogen atom (see chapter 25)

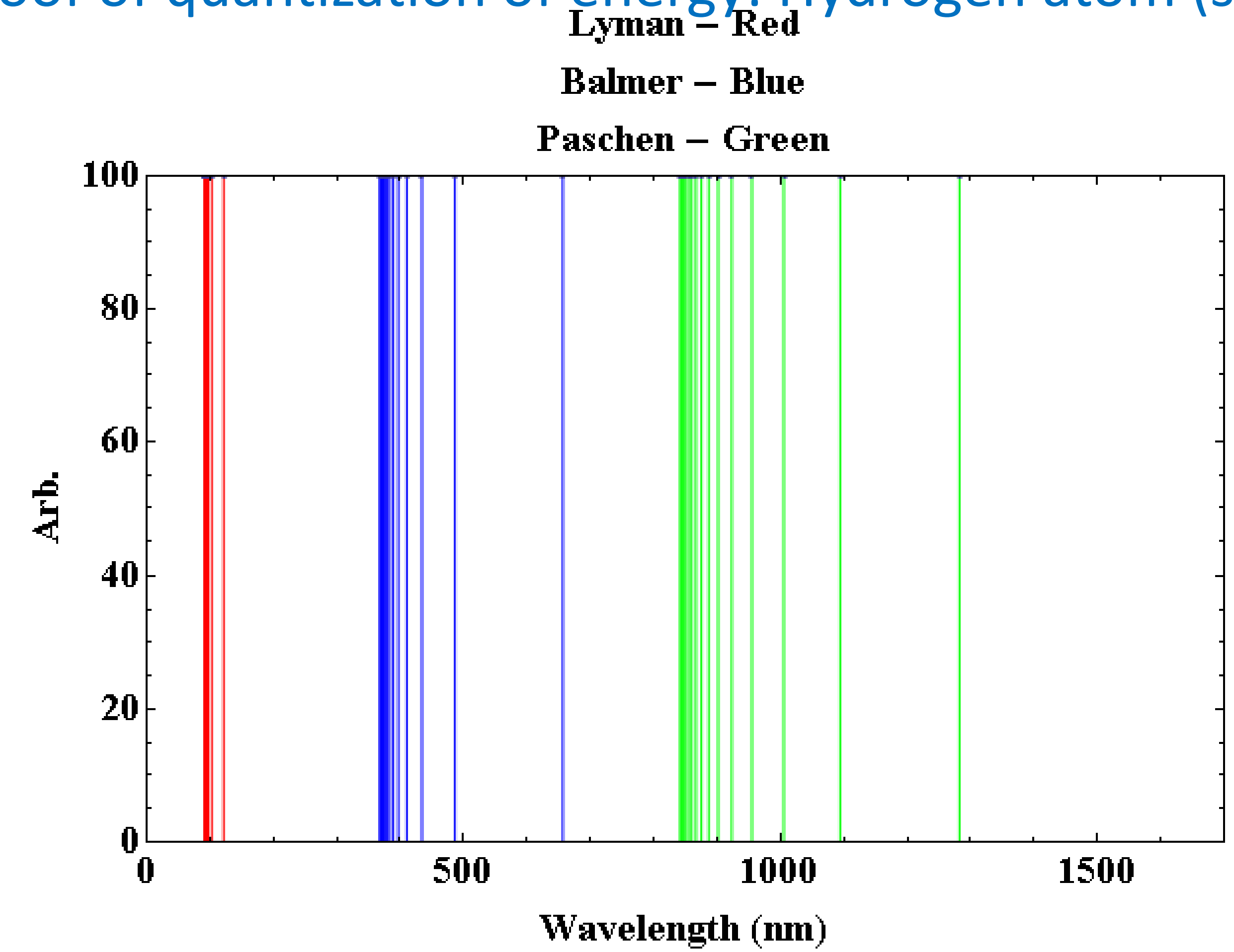
FIGURE 25.1 A diffraction spectrometer for the accurate measurement of wavelengths.





- Hot objects emit light at all frequencies (sun, light bulbs). Called "Planck's Black Body" radiation spectrum --- more about this later.
- Gas discharge tube --- different elements emit different discrete spectrum
- Classical Newtonian mechanics can not explain **discrete** spectrum!

More proof of quantization of energy: Hydrogen atom (see chapter 25)



$\lambda f = c$

$\Rightarrow f \propto \frac{1}{\lambda}$

$$\lambda = \frac{91.18 \text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \qquad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \quad \vdots & \end{cases}$$

$n = m + 1, m + 2, \ldots$

More proof of quantization of energy: Hydrogen atom (see chapter 25)

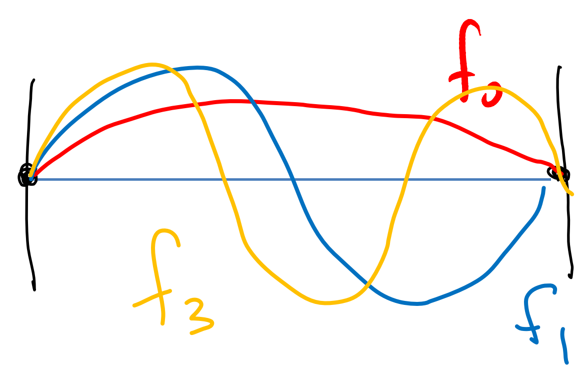
$$\lambda = \frac{91.18\ \text{nm}}{\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)} \qquad \begin{cases} m = 1 & \text{Lyman series} \\ m = 2 & \text{Balmer series} \\ m = 3 & \text{Paschen series} \\ \quad \vdots & \end{cases}$$

$$n = m + 1, m + 2, \ldots$$

Newtonian mechanics: everything is continuously variable
Energy, position, wavelength, frequency, Force, velocity, etc..

Traveling waves are also continuous: Amplitude, phase, frequency, wavelength

Standing waves discrete : $f = n f_0$

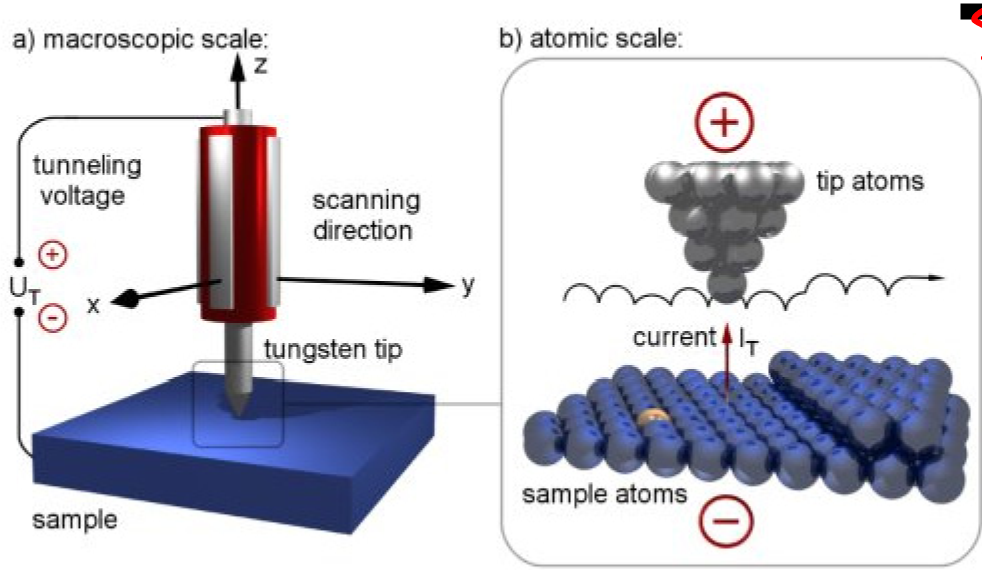


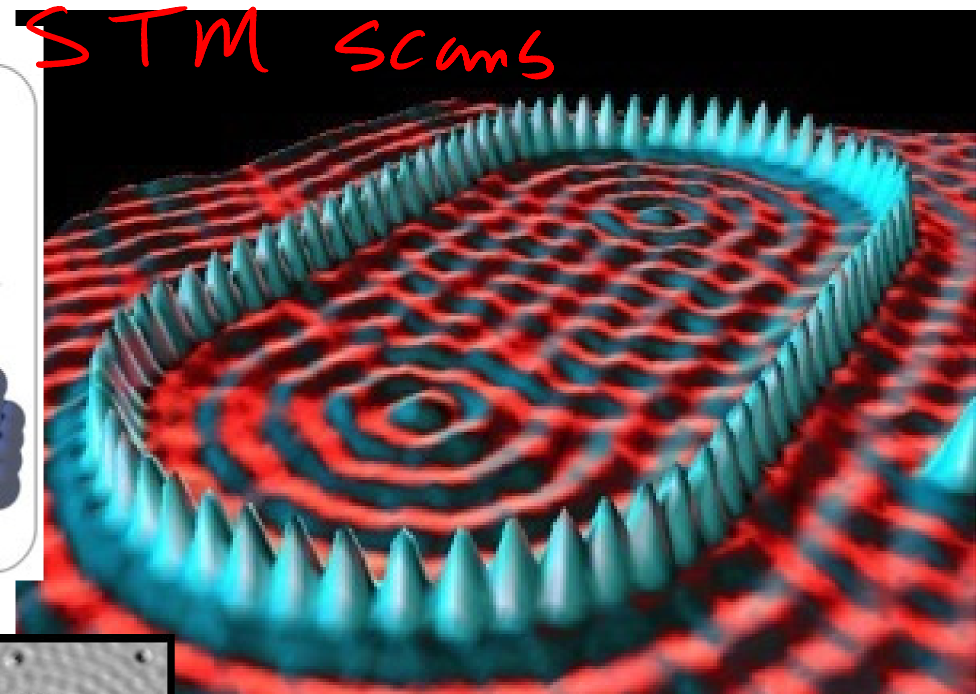
De Broglie wavelength for matter – Like Einstein's photon postulate

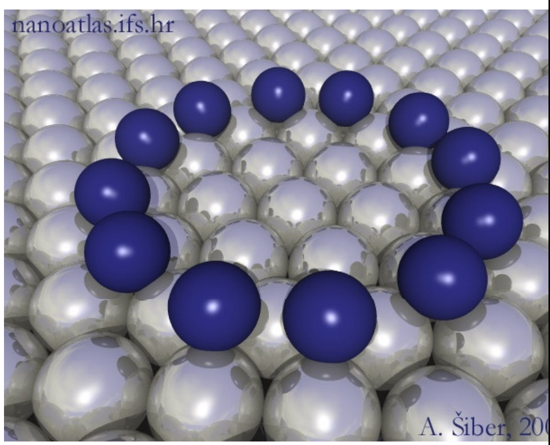
De Broglie: $p_{ph} = \frac{h}{\lambda}$ Einstein

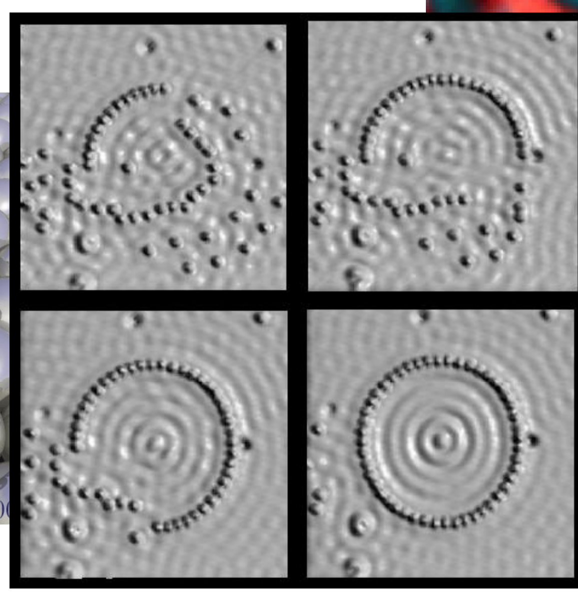
For matter, postulate: $p = \frac{h}{\lambda} = \gamma_p m u$

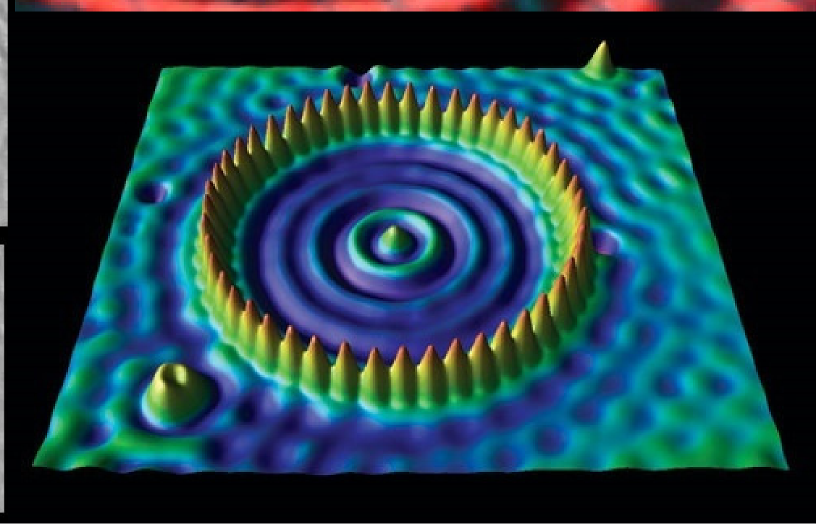
Matter waves confined --- Quantum Corral (particle(s) in a box)











Matter wave confined --- Review of standing waves Chapter 21

Sum of two opp. traveling waves (same Amplitude, freq., & wavelength) produces a standing wave

FIGURE 21.4 The superposition of two sinusoidal waves traveling in opposite directions.

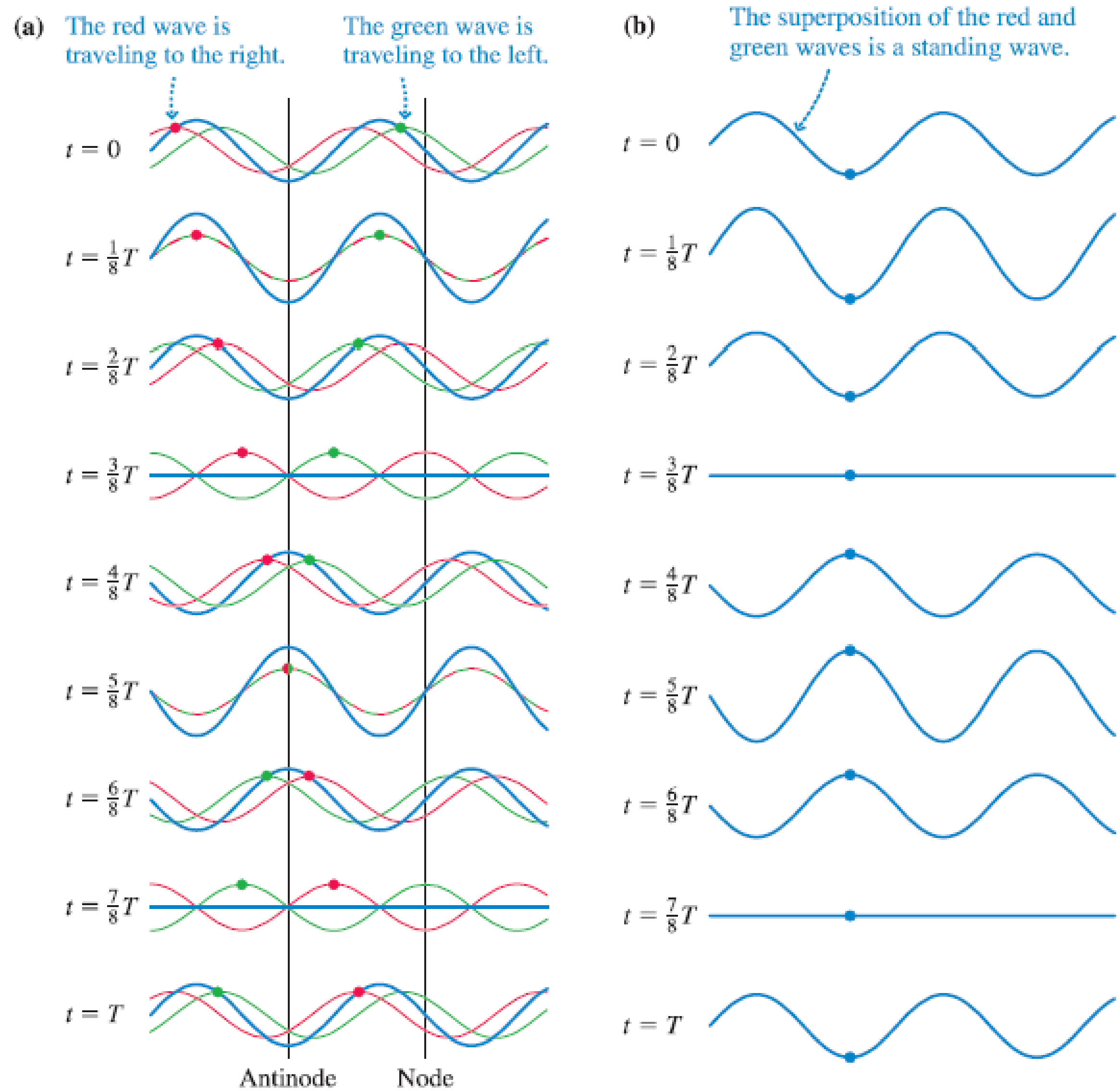
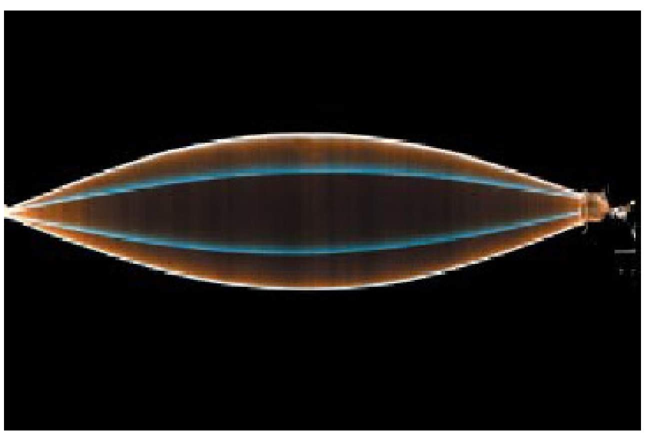
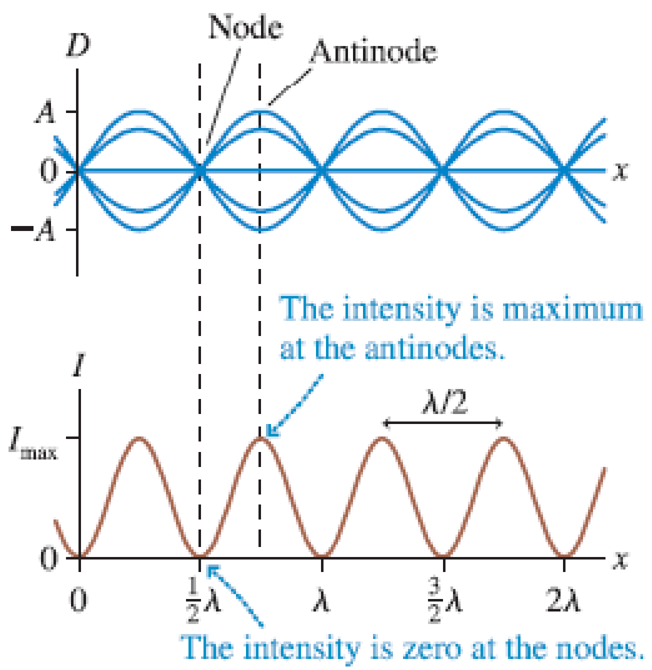


FIGURE 21.3 A vibrating string is an example of a standing wave.



Matter wave confined --- Review of standing waves Chapter 21

FIGURE 21.6 The intensity of a standing wave is maximum at the antinodes, zero at the nodes.



A sinusoidal wave traveling to the right along the x-axis with angular frequency $\omega = 2\pi f$, wave number $k = 2\pi/\lambda$, and amplitude a is

$$D_R = a\sin(kx - \omega t) \tag{21.2}$$

An equivalent wave traveling to the left is

$$D_L = a\sin(kx + \omega t) \tag{21.3}$$

$$D(x, t) = D_R + D_L = a\sin(kx - \omega t) + a\sin(kx + \omega t) \tag{21.4}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

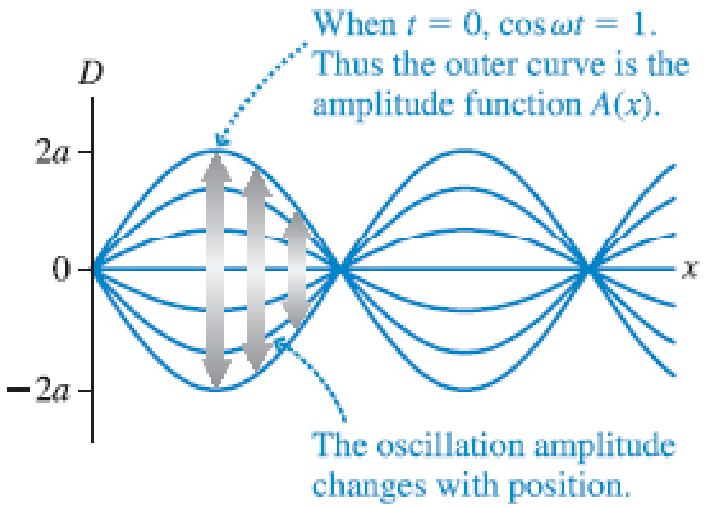
$$\begin{aligned} D(x, t) &= a(\sin kx\cos\omega t - \cos kx\sin\omega t) + a(\sin kx\cos\omega t + \cos kx\sin\omega t) \\ &= (2a\sin kx)\cos\omega t \end{aligned} \tag{21.5}$$

$$D(x, t) = A(x)\cos\omega t$$

$$A(x) = 2a\sin kx$$

Matter wave confined --- Review of standing waves Chapter 21

FIGURE 21.7 The net displacement resulting from two counter-propagating sinusoidal waves.

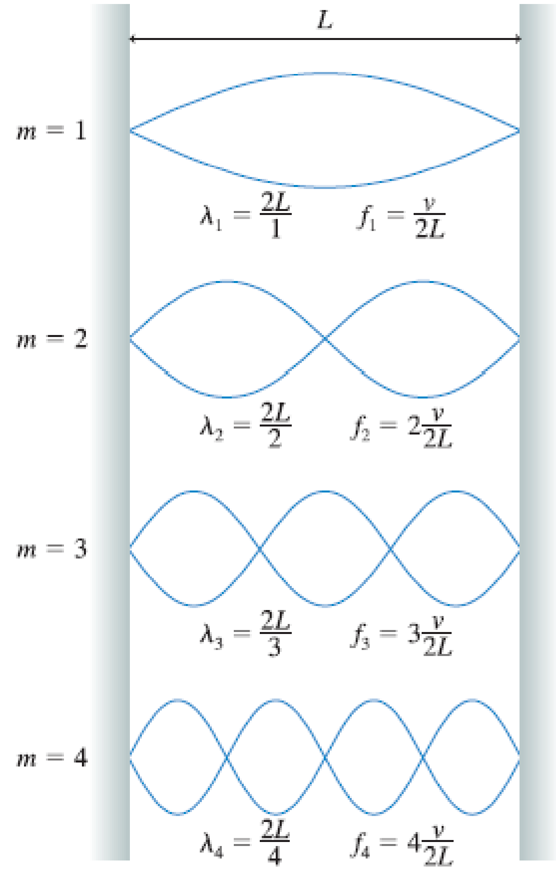


$$D(x, t) = A(x)\cos\omega t$$

$$A(x) = 2a\sin kx$$

Matter wave confined --- Review of standing waves Chapter 21

$$D(x,t) = A(x)\cos\omega t \qquad A(x) = 2a\sin kx$$



Consider wave where $A(x) = 0$ @ $x = 0$ & L

$A(0) = 0 \Rightarrow \sin k0 = 0$ ✓

$A(L) = 0 \Rightarrow \sin kL = 0 \Rightarrow kL = m\pi$

& $k = 2\pi/\lambda$

$$\Rightarrow \frac{2\pi}{\lambda_m} L = m\pi \Rightarrow \boxed{\lambda_m = \frac{2L}{m}}$$

$$\lambda f = v \Rightarrow \lambda_m = \frac{v}{f_m} = \frac{2L}{m}$$

$$\Rightarrow f_m = \frac{v}{2L}\, m$$

or $\boxed{f_m = m f_1}$ where $f_1 = \frac{v}{2L}$

Matter wave confined --- particle in a box

In Chapter 21, we found that the wavelength of a standing wave is related to the length L of the confining region by

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots \tag{25.10}$$

The particle must also satisfy the de Broglie condition $\lambda = h/p$. Equating these two expressions for the wavelength gives

$$\frac{h}{p} = \frac{2L}{n} \tag{25.11}$$

Solving Equation 25.11 for the particle's momentum p, we find

$$p_n = n\left(\frac{h}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots \tag{25.12}$$

The particle's energy, entirely kinetic energy, is related to its momentum by

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{25.13}$$

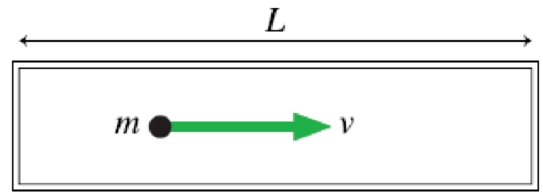
only for $v << c$!

If we use Equation 25.12 for the momentum, we find that the particle's energy is restricted to the discrete values

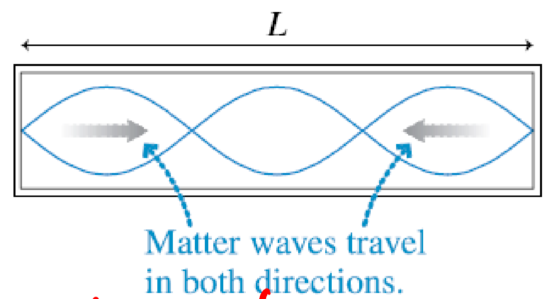
$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots \tag{25.14}$$

FIGURE 25.16 A particle of mass m confined in a box of length L.

(a) A classical particle of mass m bounces back and forth between the ends.



(b) Matter waves moving in opposite directions create standing waves.



If we use Equation 25.12 for the momentum, we find that the particle's energy is restricted to the discrete values

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1, 2, 3, 4, \ldots \qquad (25.14)$$

- Only specific discrete energies are allowed: "Quantization" of energy
  "n" is "Quantum number"
  "E_n" is the nth "Energy Level"

- Minimum energy: $E_1 = \dfrac{h^2}{8mL^2}$

the confined particle is always in motion

$E_n = n^2 E_1$

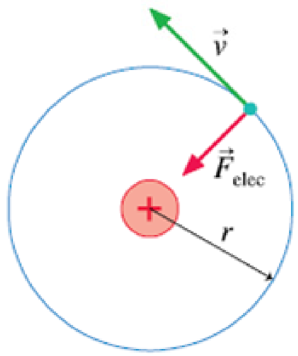
what is waving?!

Bohr model of atom: Semiclassical approach to hydrogen

Bohr Model of Atom Extends Rutherford's model:

1. There exists "stationary states" that have specific quantized energies.
2. The lowest Energy state is "stable" (can not decay). "Transitions" between quantized states occurs by the atom absorbing or emitting the difference in energy between states (includes emitting + absorbing photons)
3. Atoms naturally decay from high energy states to low energy states

FIGURE 39.20 A Rutherford hydrogen atom. The size of the nucleus is greatly exaggerated.



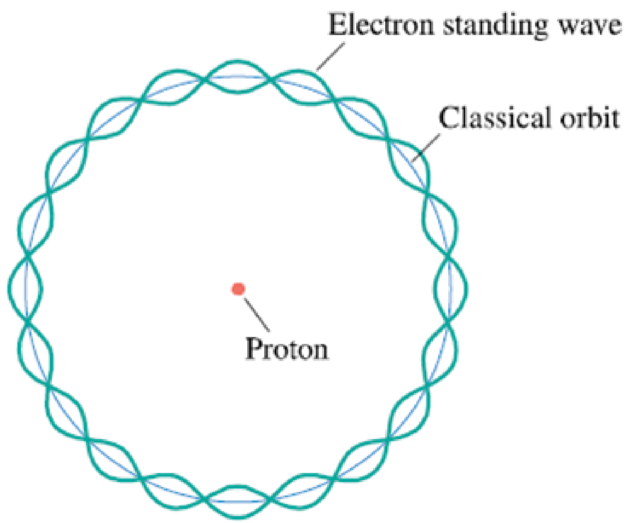
$$F = m\frac{v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \qquad (1)$$

$$\& \quad p = mv = \frac{h}{\lambda} \qquad (2)$$

Stationary state:

$$2\pi r = n\lambda \qquad (3)$$

FIGURE 39.21 An $n = 10$ electron standing wave around the orbit's circumference.



$$(3) \Rightarrow \quad 2\pi r = n\underbrace{\lambda}_{=\frac{h}{mv}\text{ from (2)}} = n\frac{h}{mv} \qquad (4)$$

$$\Rightarrow \quad v^2 = \left[\frac{n(h/2\pi)}{mr}\right]^2 \underset{\text{from (1)}}{=} \frac{e^2}{m4\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r = \left(\frac{n\hbar}{e}\right)^2 \frac{4\pi\varepsilon_0}{m}$$

$$\Rightarrow \quad r_n = n^2 \underbrace{\frac{4\pi\varepsilon_0}{m}\left(\frac{\hbar}{e}\right)^2}_{= a_B = .5\ \text{Å}}$$

$$r_n = n^2 \underbrace{\frac{4\pi\varepsilon_0}{m}\left(\frac{\hbar}{e}\right)}_{\equiv a_B = .5\,\text{Å}}$$, the radius of orbits is quantized

From (4) : $2\pi r = n\frac{h}{mv} \Rightarrow v_n = \frac{n}{r_n}\ \frac{\hbar}{m}$

$$\Rightarrow v_n = \frac{1}{n}\ \underbrace{\frac{\hbar}{m a_B}}$$

$v_1 = 2.19 \cdot 10^6\ m/s$, not relativistic

Total Energy:

$$\frac{e^2}{4\pi\varepsilon_0} = \frac{1}{a_B}\cdot\frac{\hbar^2}{m}$$

$$E = KE + PE = \frac{1}{2} m v_n^2 + \frac{\overbrace{-e^2}}{4\pi\varepsilon_0 r_n}$$

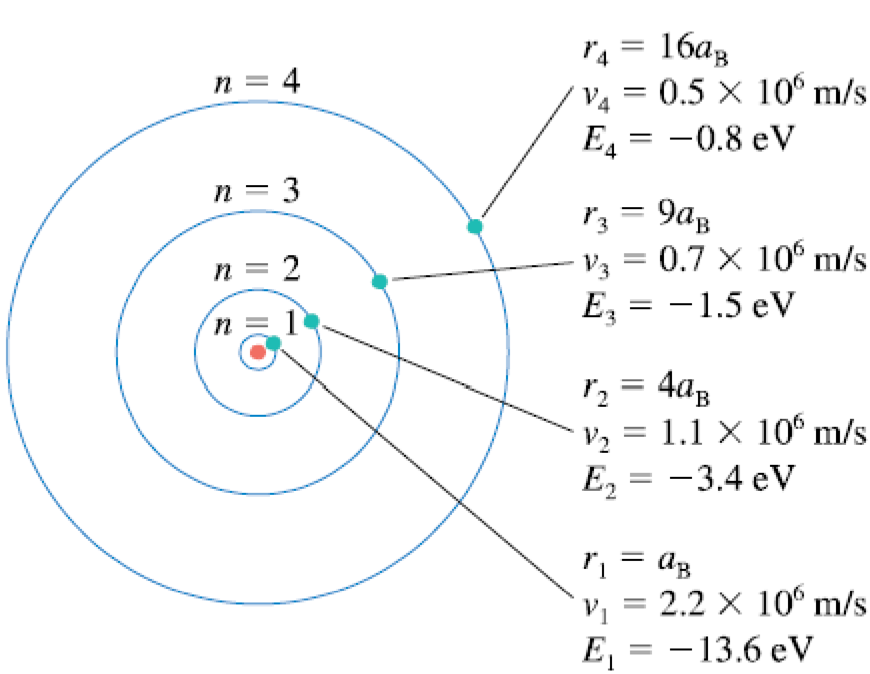
$$= \frac{1}{2} m \left(\frac{1}{n}\ \frac{\hbar}{m a_B}\right)^2 - \frac{\hbar^2}{m a_B}\cdot\left(\frac{1}{n^2 a_B}\right)$$

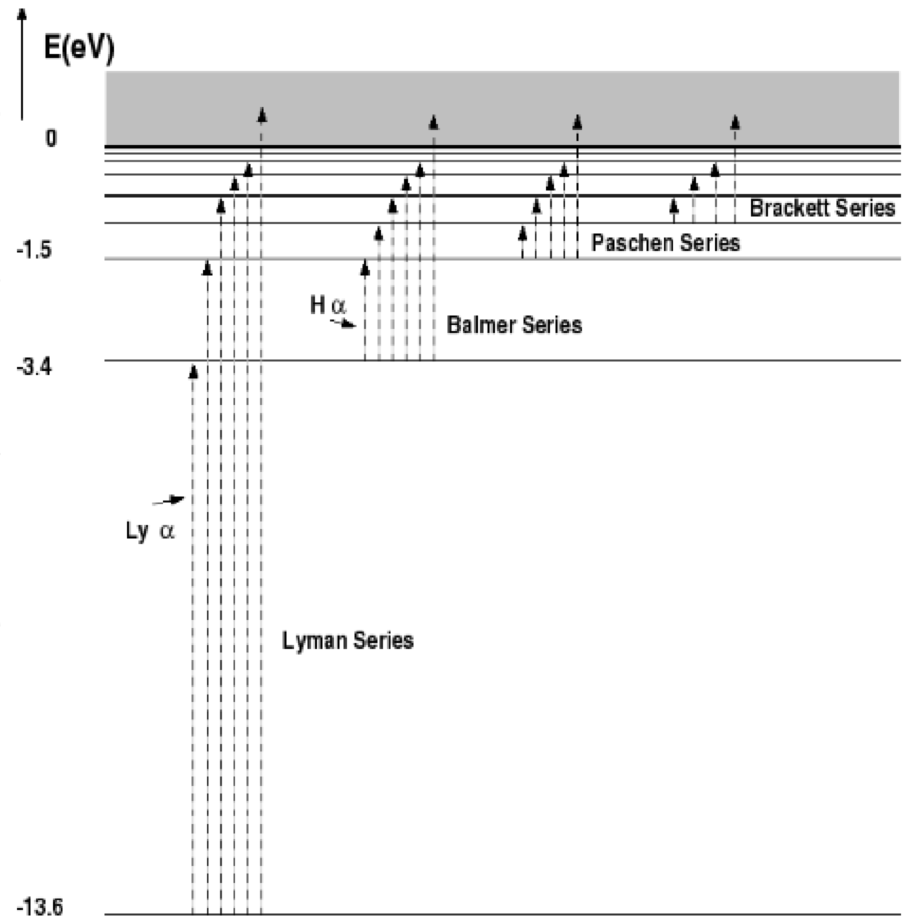
$$= \underbrace{-\frac{1}{2}\left(\frac{\hbar}{m a_B}\right)^2}_{\equiv E_1 = -13.6\ eV} \frac{1}{n^2}$$

$\Rightarrow E_n = -\frac{13.60\ eV}{n^2}$; $1\ Ry \equiv 13.60\ eV$
Rydberg

Bohr model of atom: Semiclassical approach to hydrogen

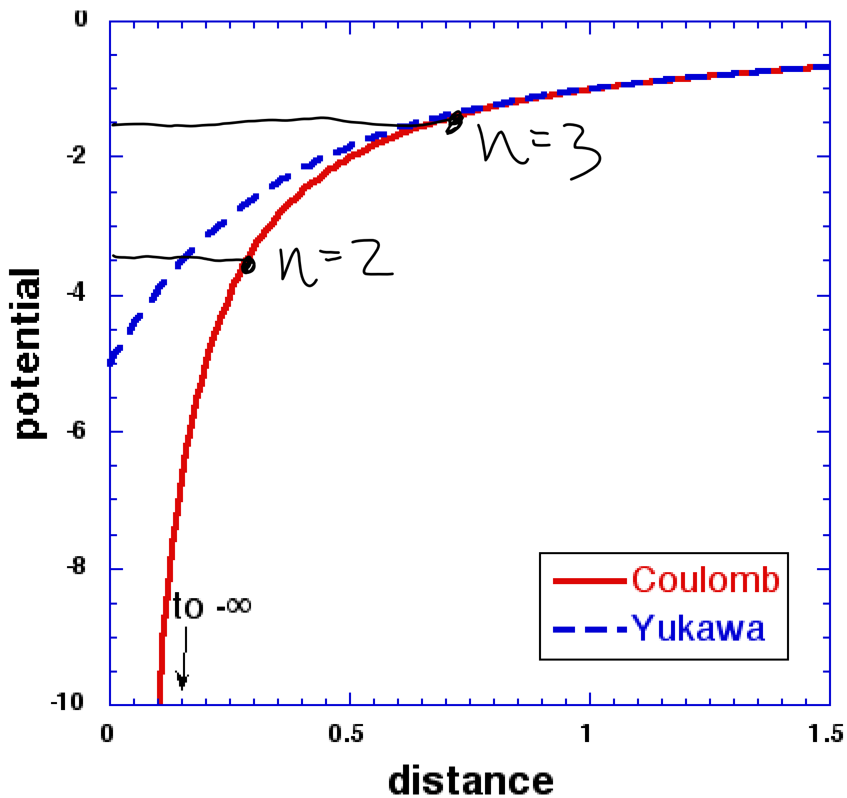
FIGURE 39.22 The first four stationary states, or allowed orbits, of the Bohr hydrogen atom drawn to scale.



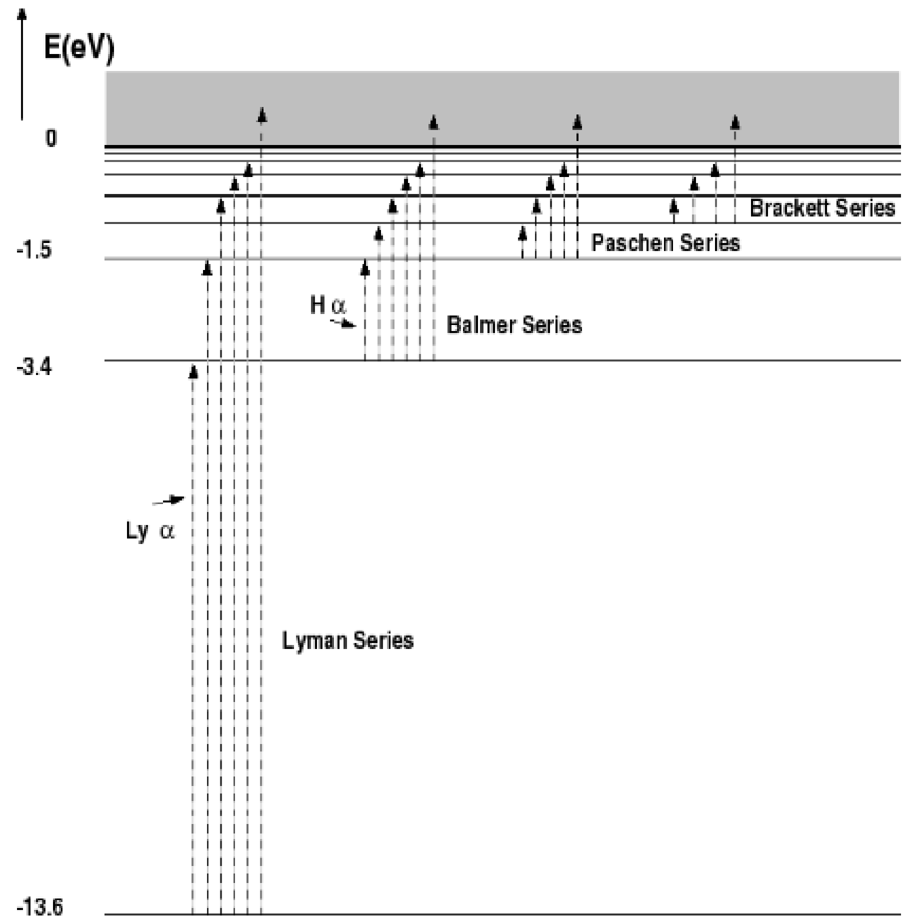


$$E_n = -\frac{Ry}{n^2}$$

Bohr model of atom: Semiclassical approach to hydrogen



Energy Level Diagram



absorption & emission transitions

$|E_n|$ is the **binding energy**

$|E_1|$ of the ground state is called the **ionization energy**

Bohr model of atom: Semiclassical approach to hydrogen

$$E_n = -\frac{R_y}{n^2}$$

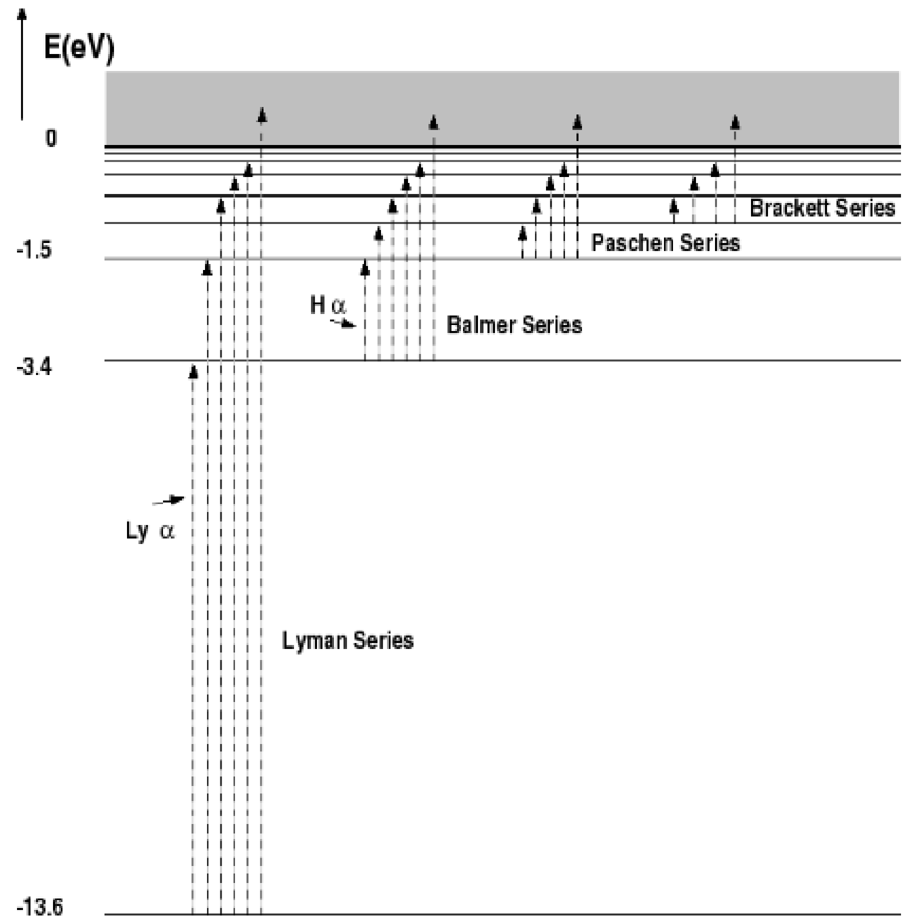
$$hf = \Delta E \Rightarrow \frac{hc}{\lambda} = E_m - E_n$$

$$= R_y\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \lambda = \frac{hc/R_y}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} = \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

$$\lambda_0 = 91.18 \text{ nm}$$

Energy Level Diagram



absorption & emission transitions

Hydrogen-Like ions: $\dfrac{e^2}{4\pi\varepsilon_0 r} \longrightarrow \dfrac{Ze^2}{4\pi\varepsilon_0 r}$

or $e^2 \longrightarrow Ze^2$

$$a_B = \frac{4\pi\varepsilon_0}{m}\left(\frac{\hbar}{e}\right)^2 \longrightarrow \frac{a_B}{Z}$$

$$r_n = n^2 a_B \longrightarrow r_n = n^2 \frac{a_B}{Z}$$

$$V_n = \frac{1}{n} \underbrace{\frac{\hbar}{m a_B}}_{V_1} \longrightarrow V_n = \frac{1}{n} Z V_1$$

$$E_n = \underbrace{-\frac{1}{2}\left(\frac{\hbar}{m a_B}\right)^2}_{\equiv E_1 = -1\,Ry} \frac{1}{n^2} \longrightarrow E_n = -Z^2 \frac{Ry}{n^2}$$

$$\lambda = \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} \sim \frac{1}{\Delta E} \longrightarrow \lambda = \frac{1}{Z^2} \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

Bohr model of atom: Semiclassical approach to hydrogen
Quantization of angular momentum

Quantization of angular momentum

Stationary wave condition:

$$2\pi r = n\lambda \quad , \quad \& \quad p = \frac{h}{\lambda} = mv$$

$$\Rightarrow 2\pi r = n\frac{h}{mv}$$

$$\Rightarrow \underbrace{mvr}_{} = n\hbar$$

$\vec{L} = m\vec{v}\times\vec{r} = mvr$ for circular orbit

$$L_n = n\hbar$$