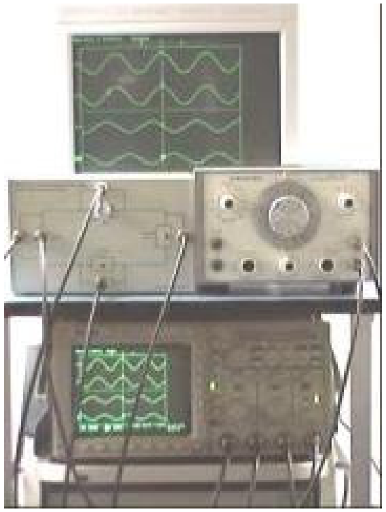
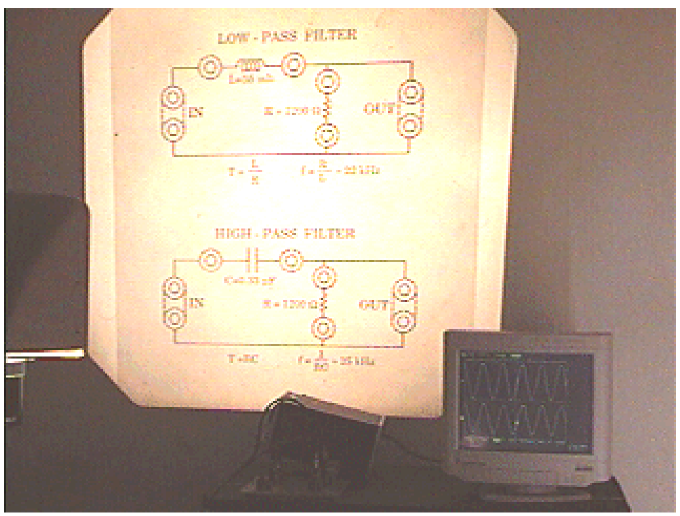


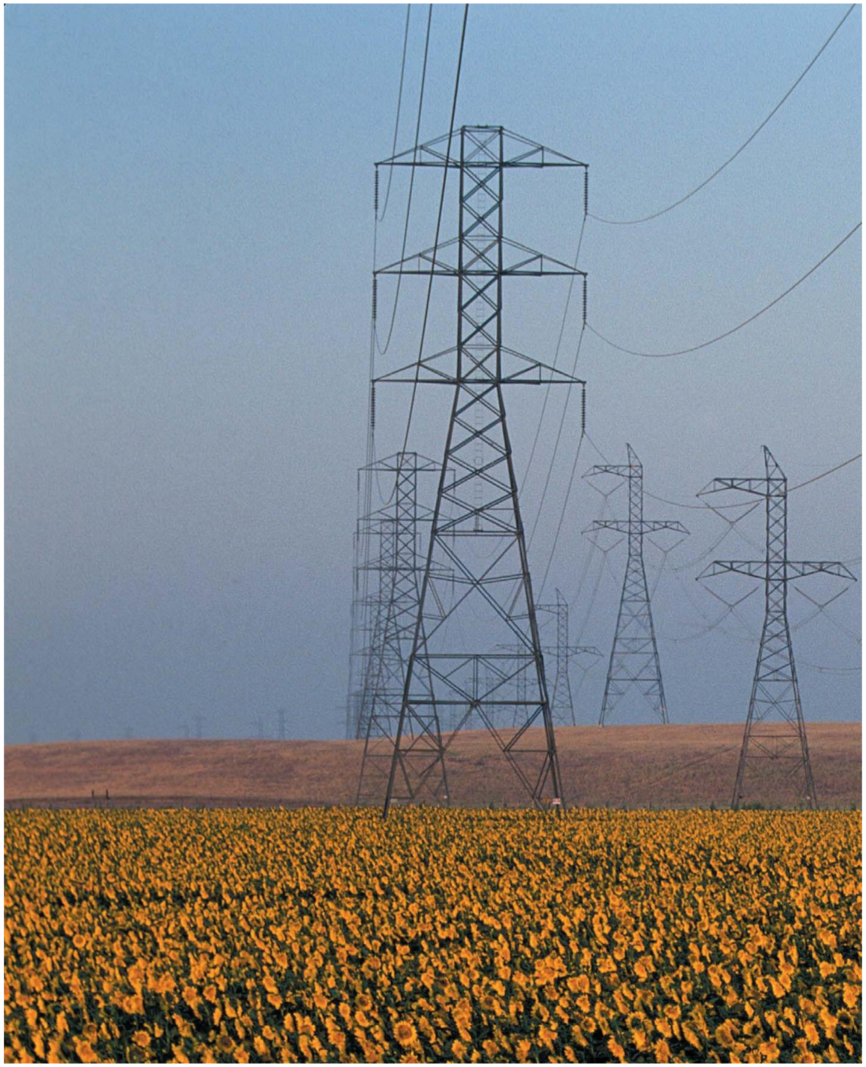
K7-27: RLC CIRCUIT - COMPLETEK7-27: RLC CIRCUIT - COMPLETE



K7-45: LOW AND HIGH PASS FILTERS

Chapter 36. AC Circuits

Today, a “grid” of AC electrical distribution systems spans the United States and other countries. Any device that plugs into an electric outlet uses an AC circuit. In this chapter, you will learn some of the basic techniques for analyzing AC circuits.



Chapter Goal: To understand and apply basic techniques of AC circuit analysis.

Chapter 36. AC Circuits

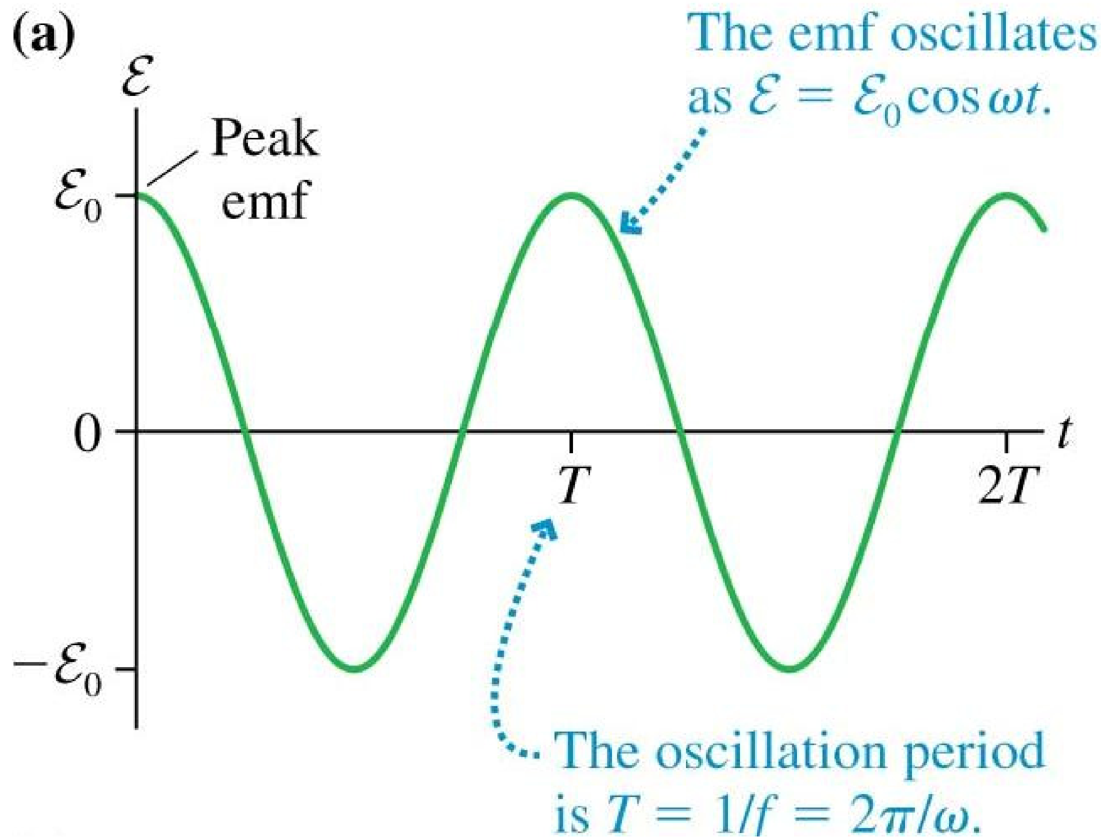
Topics:

- AC Sources and Phasors
- Capacitor Circuits
- *RC* Filter Circuits
- Inductor Circuits
- The Series *RLC* Circuit

Skip Section 36.6 – Power factor

AC Sources and Phasors

FIGURE 36.1 An oscillating emf can be represented as a graph or as a phasor diagram.



AC Sources and Phasors

FIGURE 36.1 An oscillating emf can be represented as a graph or as a phasor diagram.

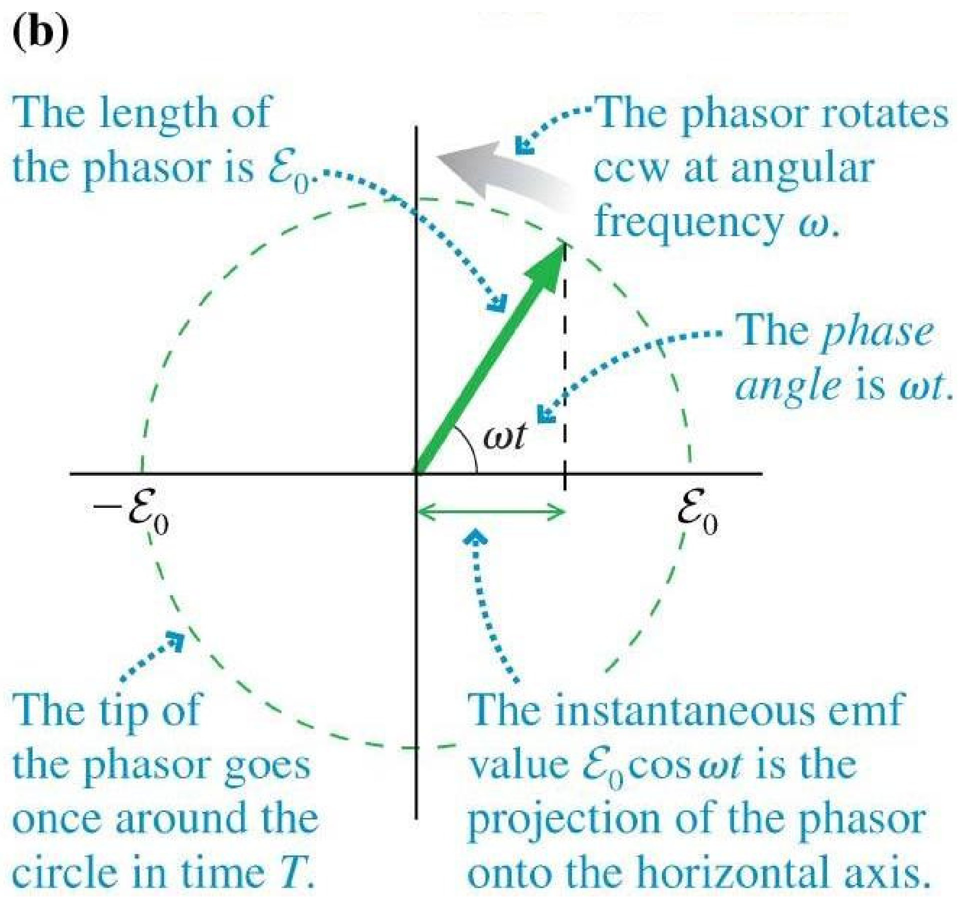
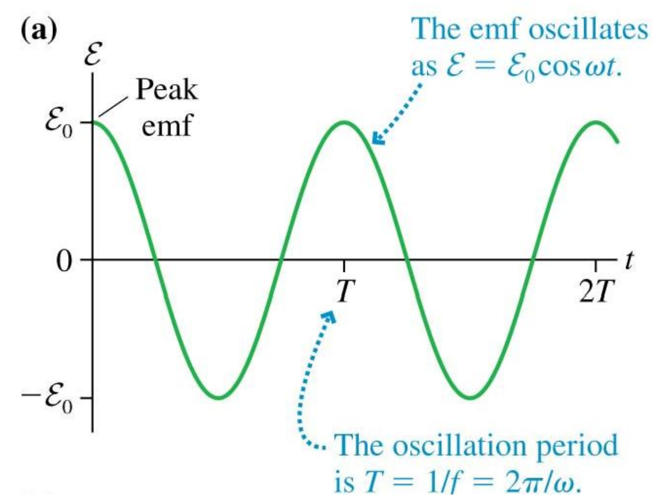


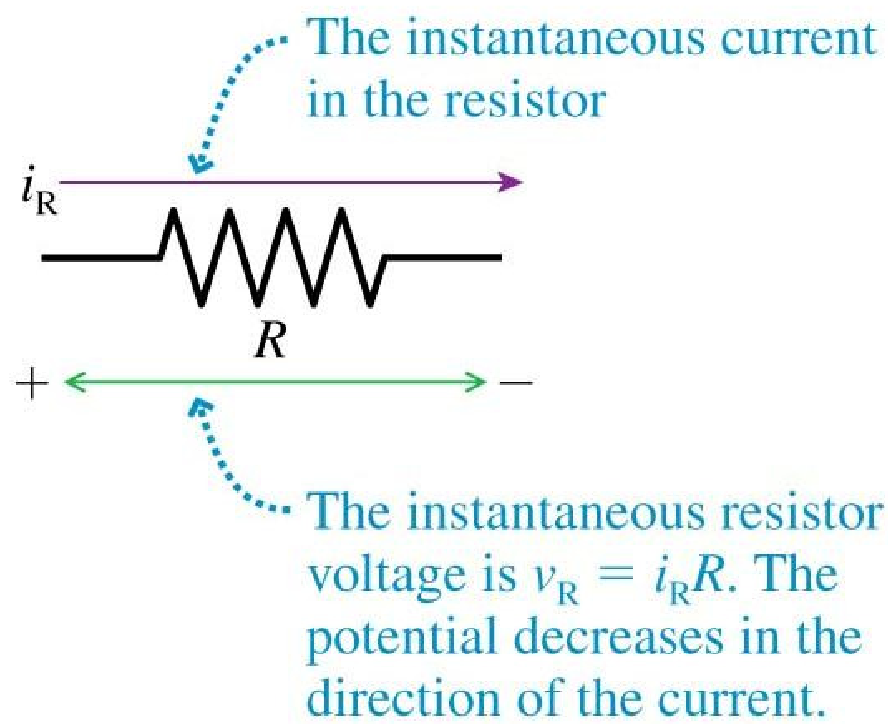
FIGURE 36.1 An oscillating emf can be represented as a graph or as a phasor diagram.



AC Circuits - Resistors

In an AC resistor circuit, Ohm's law applies to both the instantaneous *and* peak currents and voltages.

FIGURE 36.3 Instantaneous current i_R through a resistor.



AC Circuits - Resistors

The *resistor voltage* v_R is given by

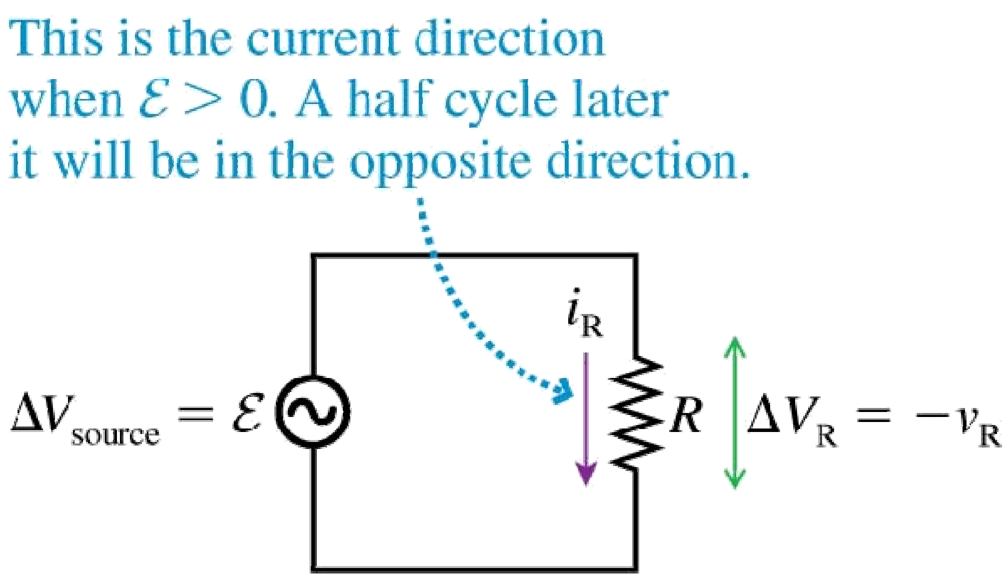
$$v_R = V_R \cos \omega t$$

where V_R is the peak or maximum voltage. The current through the resistor is

$$i_R = \frac{v_R}{R} = \frac{V_R \cos \omega t}{R} = I_R \cos \omega t$$

where $I_R = V_R/R$ is the peak current.

FIGURE 36.4 An AC resistor circuit.



AC Circuits - Resistors

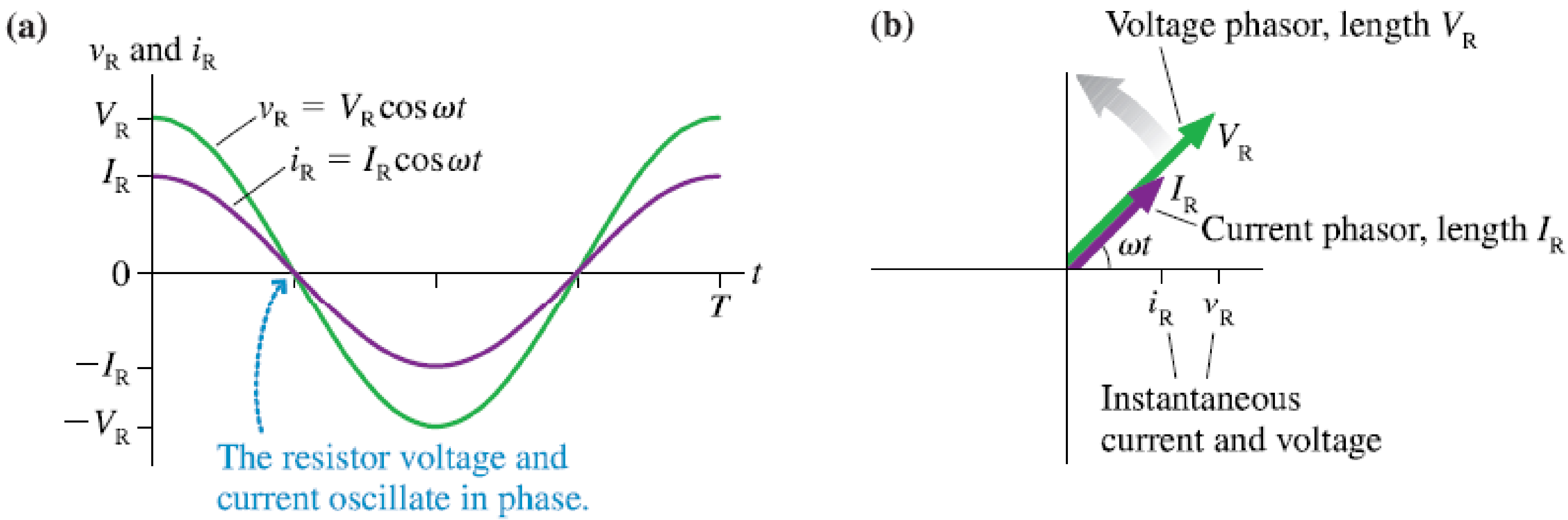
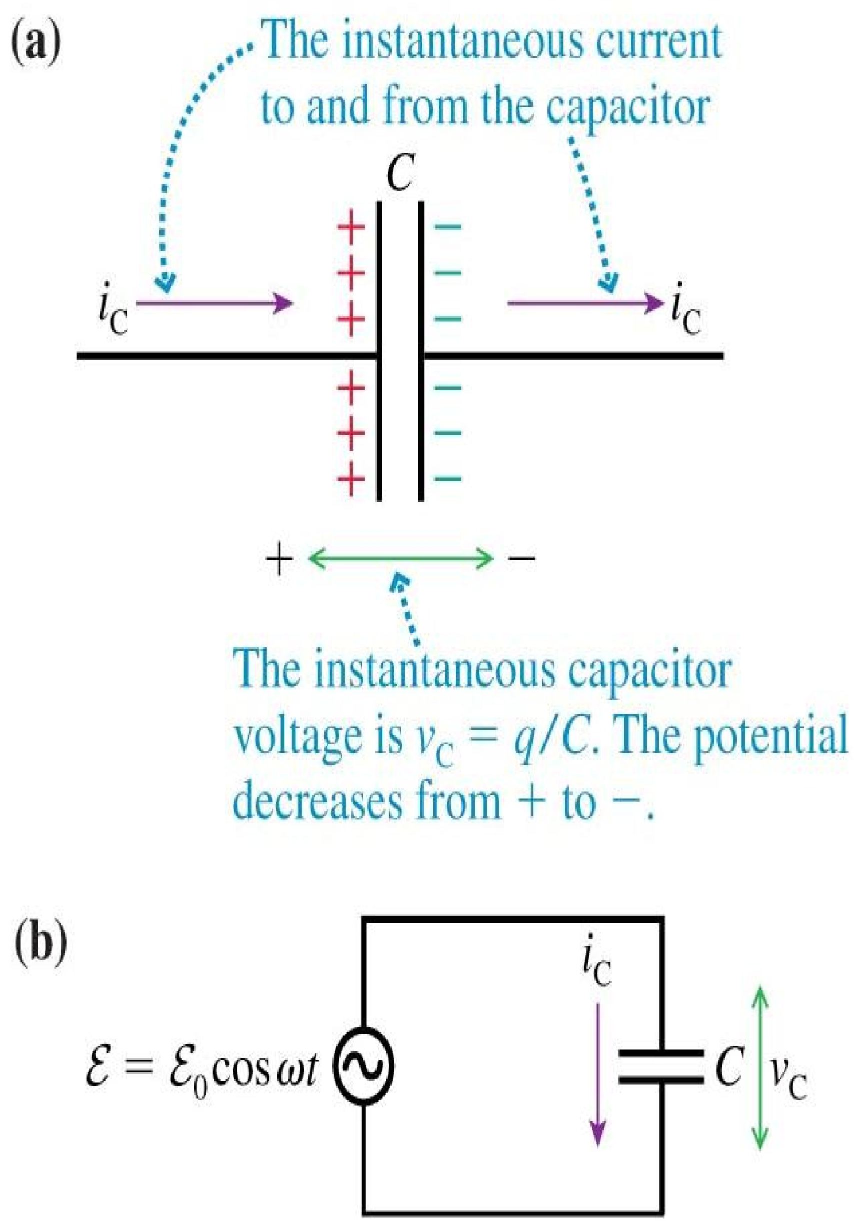


FIGURE 36.5 Graph and phasor diagram of the resistor current and voltage. The current and voltage are in phase.

AC Circuits - Capacitors

FIGURE 36.7 An AC capacitor circuit.



$$v_C = V_C \cos\omega t = q/C$$

$$\therefore q = C V_C \cos\omega t$$

$$\frac{dq}{dt} = i_C = \underbrace{V_C \omega C}_{I_C} \underbrace{(-\sin\omega t)}_{\cos(\omega t + \frac{\pi}{2})}$$

$$i_C = I_C \cos(\omega t + \pi/2)$$

The AC current to and from a capacitor *leads* the capacitor voltage by π/2 rad, or 90°.

Want "Ohm's Law":

$$I_C = V_C / X_C \Rightarrow X_C = 1/\omega C$$

AC Circuits - Capacitors
Capacitive Reactance

The capacitive reactance X_C is defined as

$$X_C \equiv \frac{1}{\omega C}$$

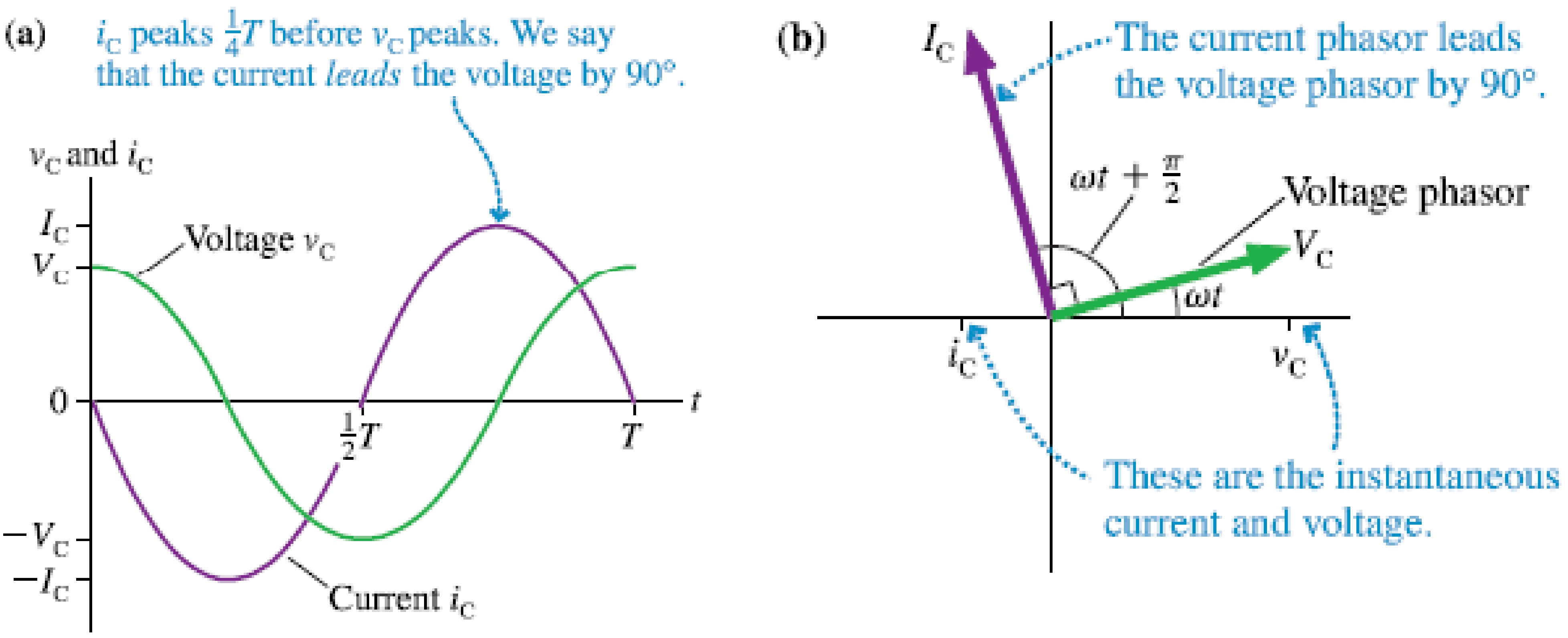
The units of reactance, like those of resistance, are ohms. Reactance relates the peak voltage V_C and current I_C:

$$I_C = \frac{V_C}{X_C} \quad \text{or} \quad V_C = I_C X_C$$

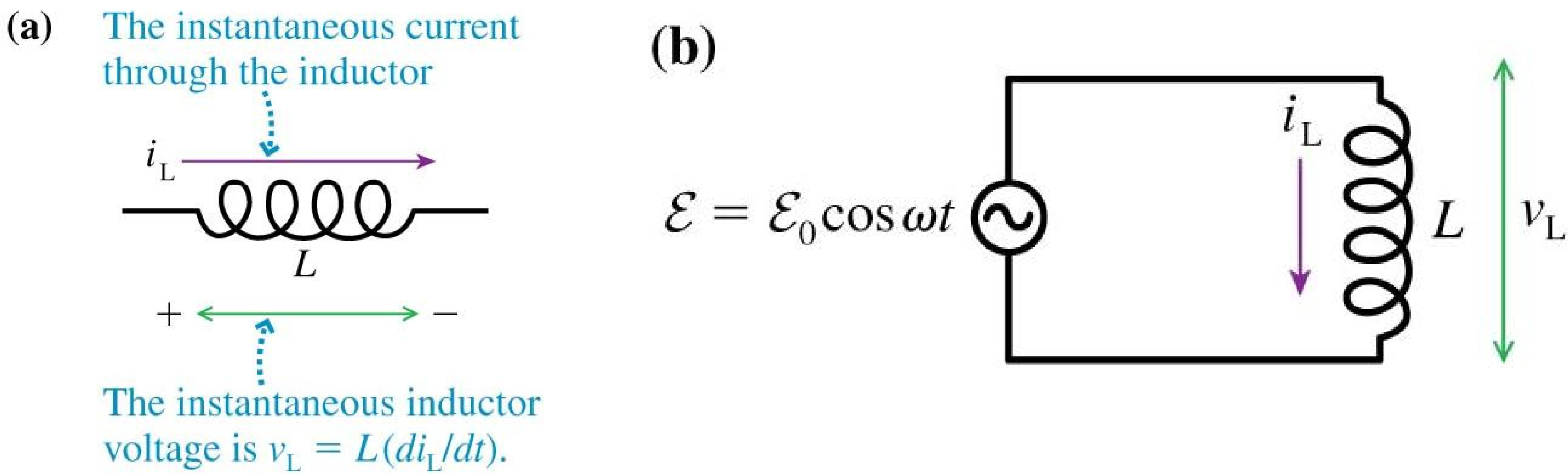
NOTE: Reactance differs from resistance in that it does *not* relate the instantaneous capacitor voltage and current because they are out of phase. That is, $v_C \neq i_C X_C$.

AC Circuits - Capacitors

ASSESS Using reactance is just like using Ohm's law, but don't forget it applies to only the *peak* current and voltage, not the instantaneous values.



AC Circuits - Inductors



$$v_L = V_L \cos\omega t = L\frac{dI_L}{dt} \Rightarrow i_L = \frac{V_L}{L}\int \cos\omega t\,dt = \frac{V_L}{\omega L}\sin\omega t = \frac{V_L}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

$$= I_L \cos\left(\omega t - \frac{\pi}{2}\right)$$

Want "Ohm's Law": $I_L = V_L / X_L \Rightarrow X_L = \omega L$

The AC current through an inductor *lags* the inductor voltage by π/2 rad, or 90°.

AC Circuits - Inductors
Inductive Reactance

The inductive reactance X_L is defined as

$$X_L \equiv \omega L$$

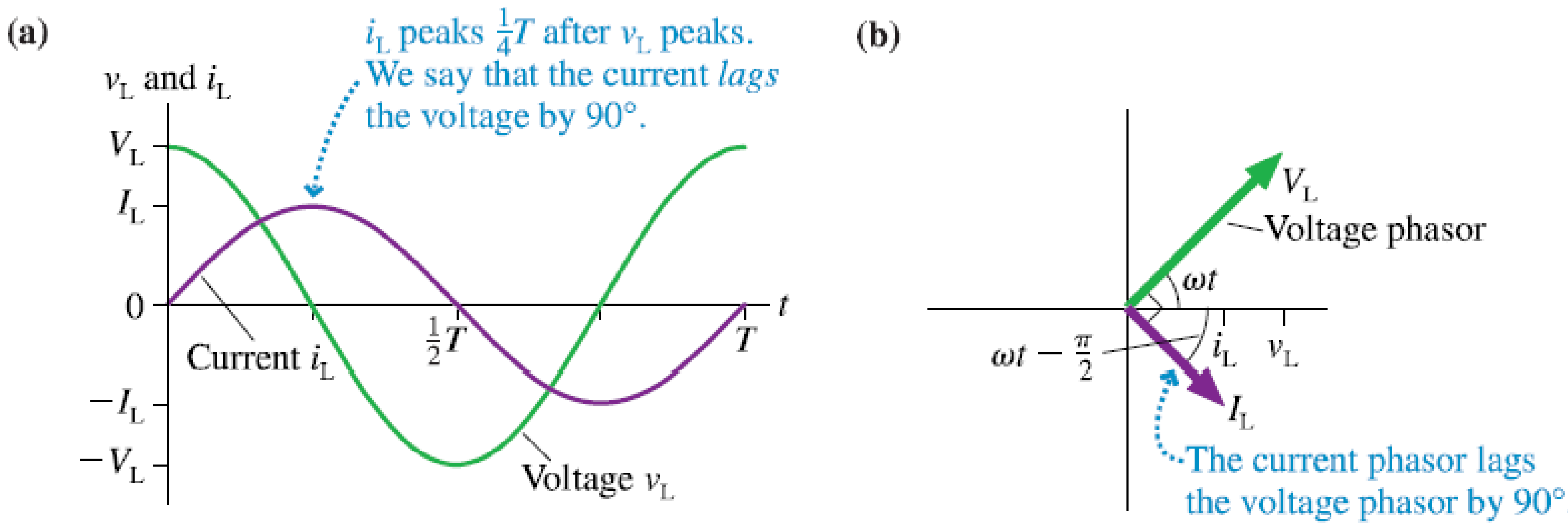
Reactance relates the peak voltage V_L and current I_L:

$$I_L = \frac{V_L}{X_L} \quad \text{or} \quad V_L = I_L X_L$$

NOTE: Reactance differs from resistance in that it does *not* relate the instantaneous inductor voltage and current because they are out of phase. That is, $v_L \neq i_L X_L$.

Inductive Reactance

$$i_L = I_L \cos(\omega t - \pi/2) \quad , \quad v_L = V_L \cos(\omega t)$$

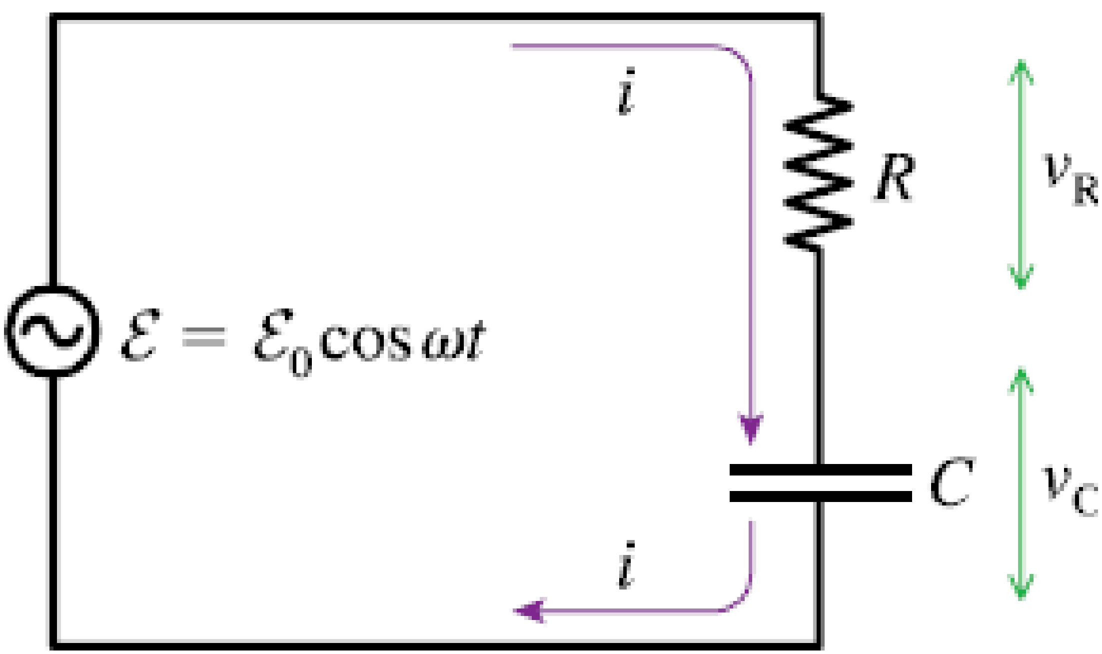


RC Filters – The concept (Fourier analysis)

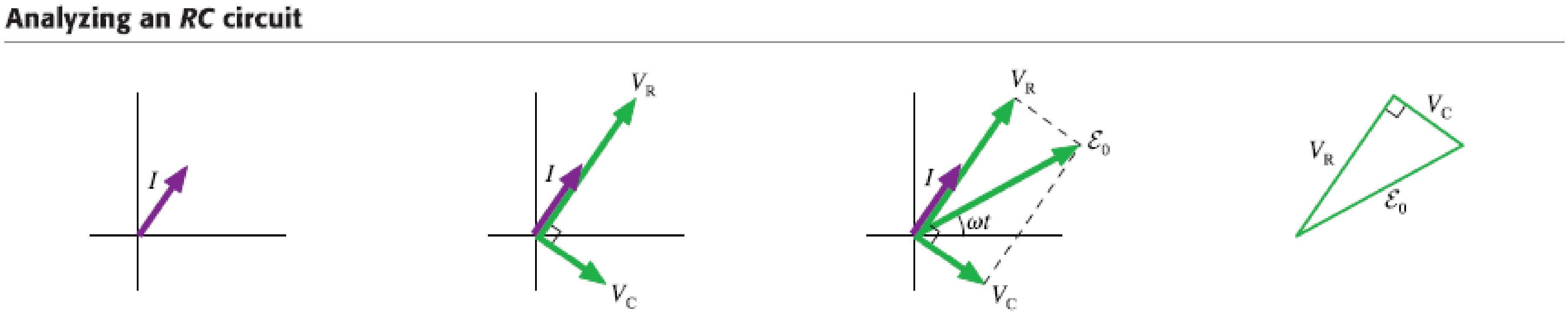
Any waveform (like voltages driving your speaker when you play music) is a sum of many sinusoidal waveforms of different amplitudes and frequencies.

The ac voltage generator depicted below for an RC circuit is idealized as ONE input frequency, but in general could be a sum of MANY waveforms (like music) with many frequencies.

Goal: Analyze the individual voltages across the resistor and capacitor when an input waveform with any frequency ω and voltage amplitude ε0 is applied across both.



RC Filters – Analysis

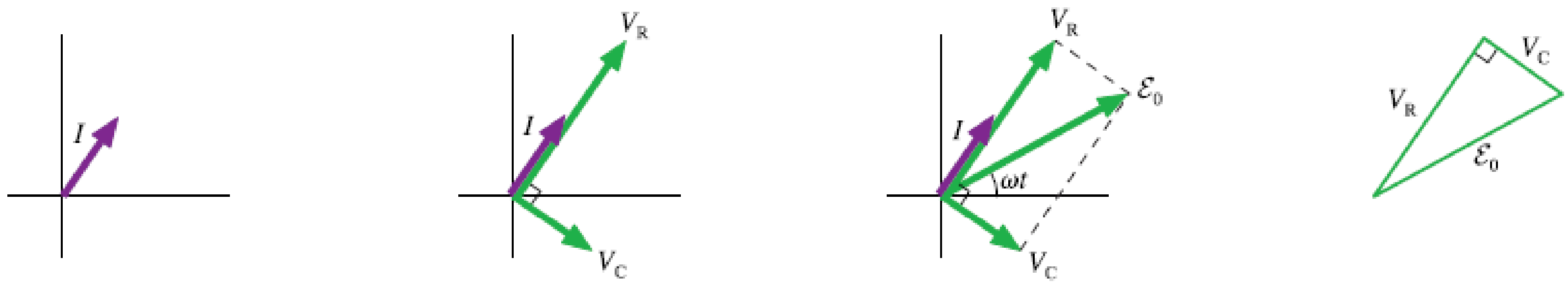


1. current is the same at all points in circuit at all time. Choose an arbitrary current vector (time).
2. Phase: V_R in phase with I, I in capacitor leads Vc
   Amplitude: For a given I peak value, we know V_R and Vc peak
3. At any instant in time, we have (Kirchoff's loop law):

$$\vec{V}_R \cdot \hat{x} + \vec{V}_C \cdot \hat{x} = \underbrace{\vec{\mathcal{E}} \cdot \hat{x}}_{\mathcal{E}_0 \cos \omega t} \Rightarrow \vec{\mathcal{E}} = \vec{V}_R + \vec{V}_C$$

RC Filters – Analysis

Analyzing an *RC* circuit



$$\mathcal{E}_0^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)I^2$$
$$= (R^2 + 1/\omega^2 C^2)I^2$$

Consequently, the peak current in the *RC* circuit is

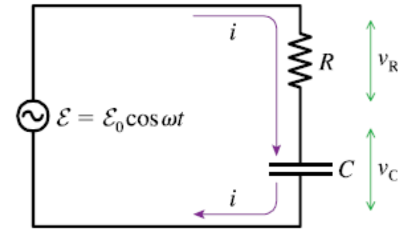
$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

Knowing I gives us the two peak voltages:

$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0/\omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

RC Filters – Analysis



$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0/\omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

Like Resitor shorted

for $w \to 0$ ($X_c \gg R$), $V_R \to 0$, $V_c \to \frac{\mathcal{E}_0}{X_c}$

Like Capacitor shorted

for $w \to \infty$ ($R \gg X_c$), $V_R \to \frac{\mathcal{E}_0}{R}$, $V_c \to 0$

for $w = \frac{1}{RC}$ ($R = X_c$), $V_R \to \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{R}$, $V_c \to \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{X_c}$

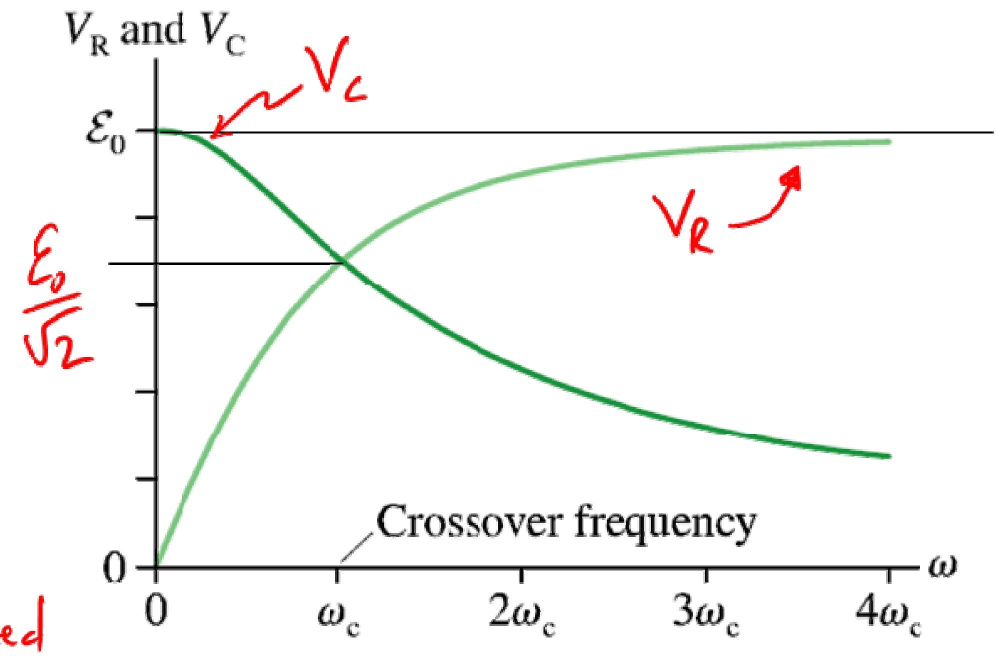
So $V_R = V_c$

$w_c \equiv \frac{1}{RC}$, cross-over frequency

RC Filters – Analysis

$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0/\omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

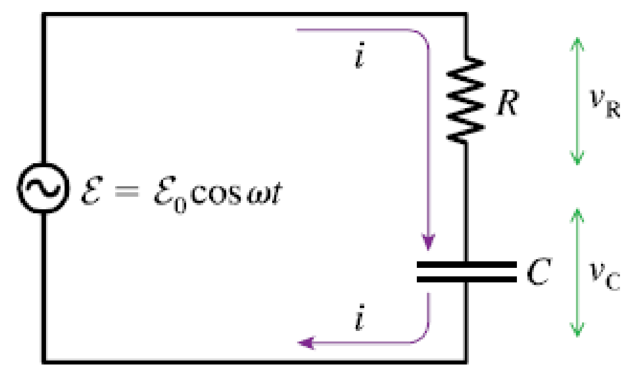


Like Resitor shorted

for $\omega \to 0$ ($X_c >> R$), $V_R \to 0$, $V_c \to \frac{\mathcal{E}_0}{X_c}$

Like Capacitor shorted

for $\omega \to \infty$ ($R >> X_c$), $V_R \to \frac{\mathcal{E}_0}{R}$, $V_c \to 0$



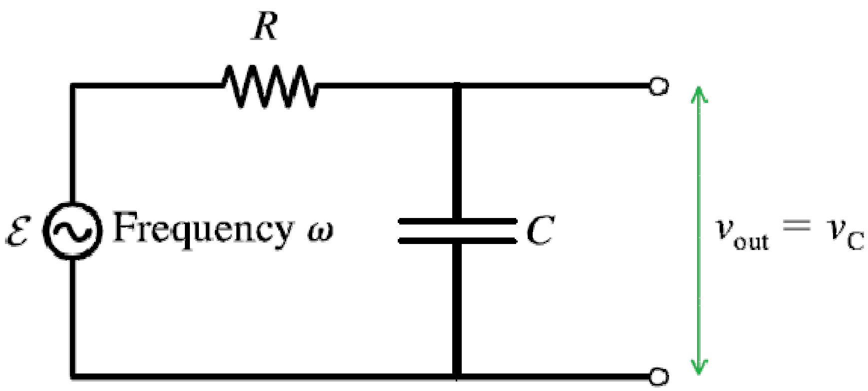
for $\omega = \frac{1}{RC}$ ($R = X_c$), $V_R \to \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{R}$, $V_c \to \frac{1}{\sqrt{2}} \frac{\mathcal{E}_0}{X_c}$

So $V_R = V_c$

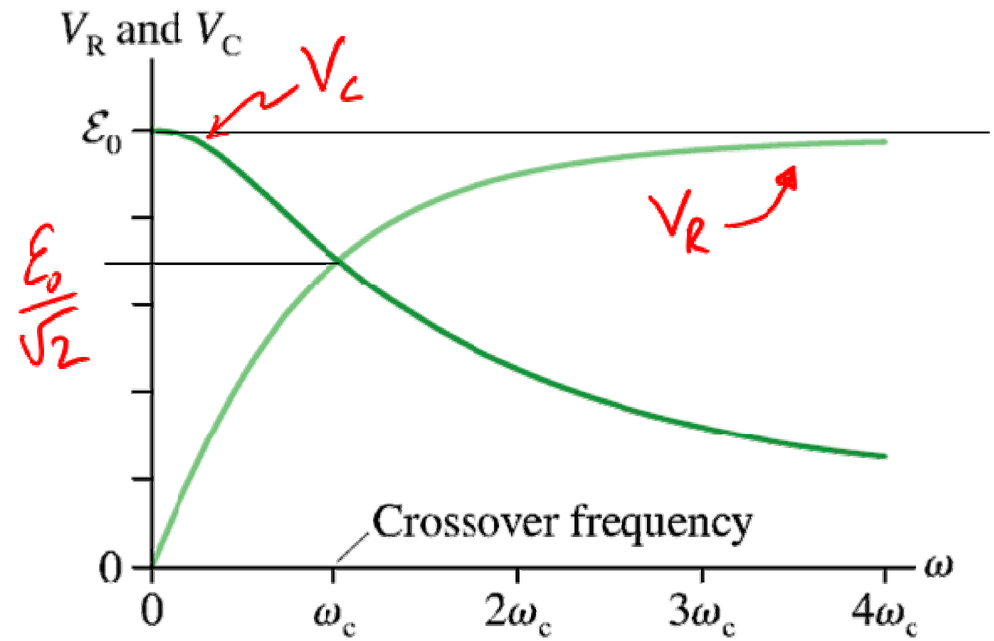
$\omega_c \equiv \frac{1}{RC}$, cross-over frequency

RC Filters – Analysis

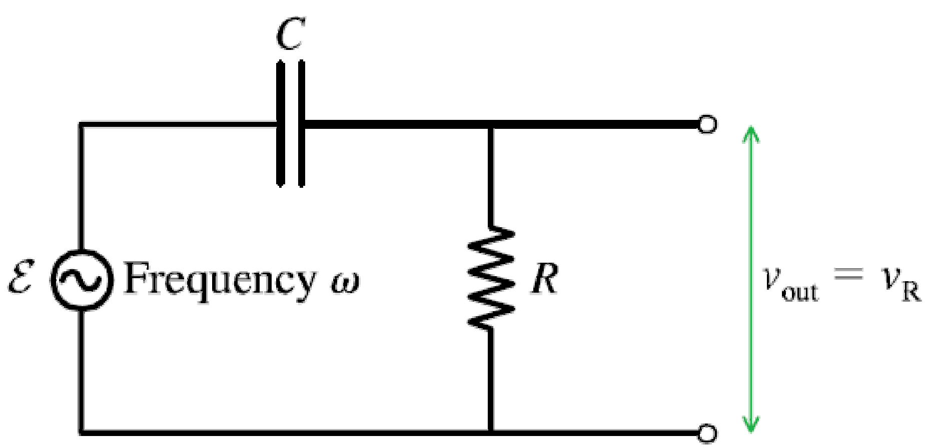
(a) Low-pass filter



Transmits frequencies $\omega < \omega_c$ and blocks frequencies $\omega > \omega_c$.



(b) High-pass filter



Transmits frequencies $\omega > \omega_c$ and blocks frequencies $\omega < \omega_c$.

- **Capacitor like a short at high frequencies since:**

$$X_C = \frac{1}{\omega C} \longrightarrow 0 \text{ at High } \omega$$

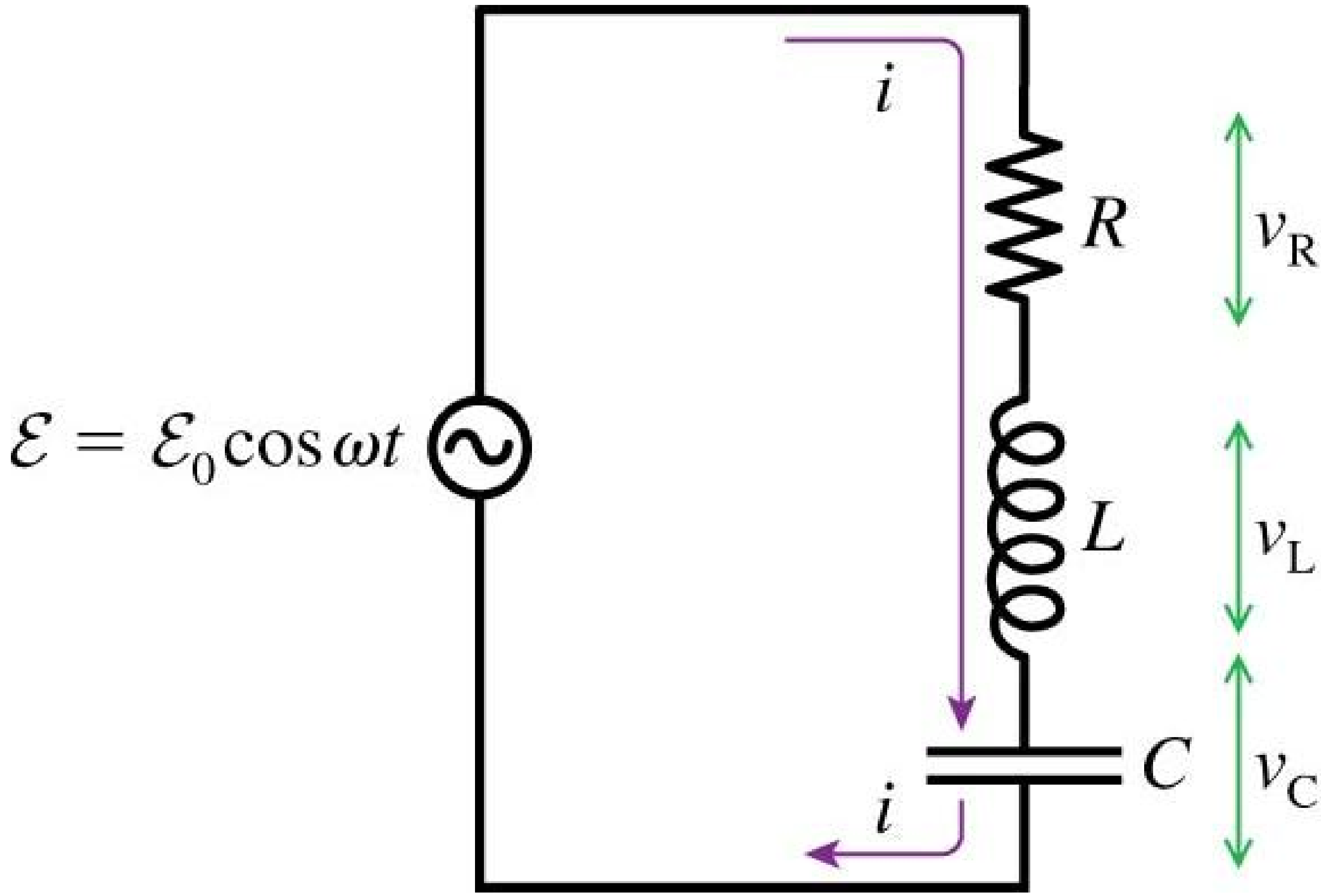
- **Voltage across Capacitor dominates at low frequencies since:**

$$X_C = \frac{1}{\omega C} \Rightarrow X_C \gg R \text{ as } \omega \rightarrow 0$$

- **If you input music, voltage across resistor would be like treble and voltage across capacitor would be like bass. Build your own speaker cross-over for woofer and tweeter.**

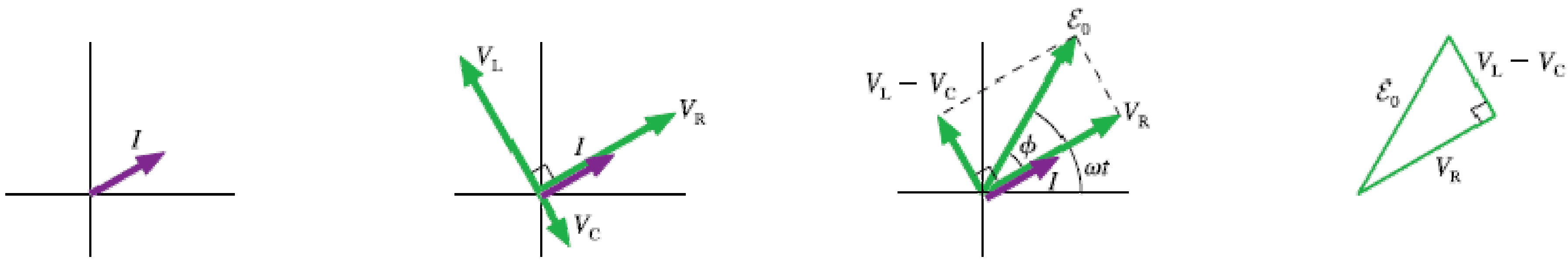
LRC filters – Analysis

FIGURE 36.17 A series *RLC* circuit.



LRC Filters – Analysis

Analyzing an *RLC* circuit



1. current is the same at all points in circuit at all time. Choose an arbitrary current vector (time).

2. Phase: Resistor VR in phase with I, I in capacitor leads Vc, I in inductor lags VL
   Amplitude: For a given I peak value, we know VR, Vc and VL peak

3. At any instant in time, we have (Kirchoff's loop law):

$$\vec{V}_R \cdot \hat{x} + \underbrace{\vec{V}_L}_{} \cdot \hat{x} + \underbrace{\vec{V}_C}_{} \cdot \hat{x} = \underbrace{\vec{\mathcal{E}} \cdot \hat{x}}_{\mathcal{E}_0 \cos \omega t}$$

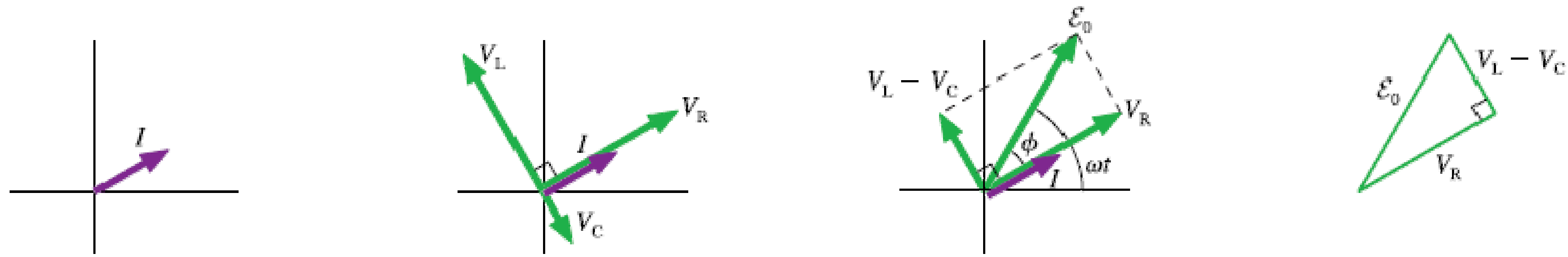
Point in opposite directions

Assume $|V_L| > |V_C|$

$$\Rightarrow \vec{V}_R + (|V_L| - |V_C|)\,\hat{V}_L = \vec{\mathcal{E}}$$

LRC Filters – Analysis

Analyzing an *RLC* circuit



$$\vec{V}_R + (|V_L| - |V_C|)\hat{V}_L = \vec{\mathcal{E}}$$

$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = [R^2 + (X_L - X_C)^2]I^2 \qquad (36.23)$$

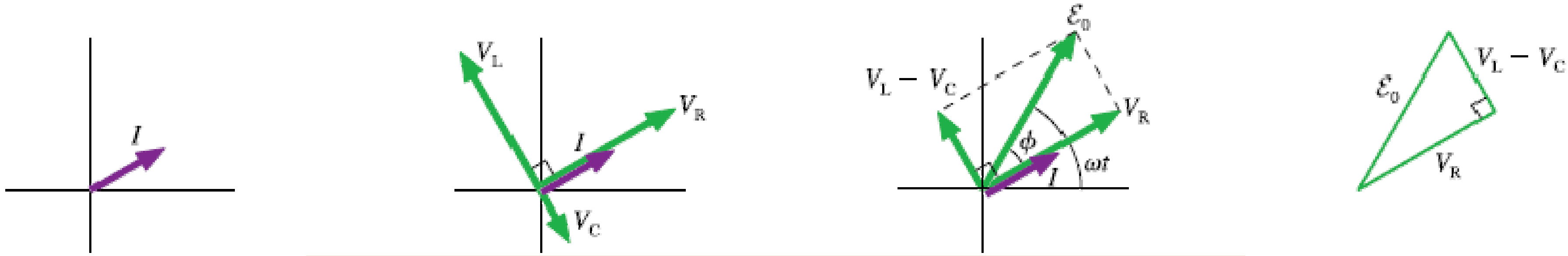
where we wrote each of the peak voltages in terms of the peak current I and a resistance or a reactance. Consequently, the peak current in the *RLC* circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \qquad (36.24)$$

The three peak voltages, if you need them, are then found from $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$.

RC Filters – Analysis

Analyzing an *RLC* circuit



$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Want an "ohm's Law" form, so let:

$$I = \frac{\mathcal{E}_0}{Z} \Rightarrow Z, \text{impedance} = \sqrt{R^2 + (X_L - X_c)^2}$$

where $X_L = \omega L$, $X_c = \frac{1}{\omega C}$

LRC Filters – Analysis

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Consider voltage across resistor:

$$V_R = IR = (\mathcal{E}_0 R) \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

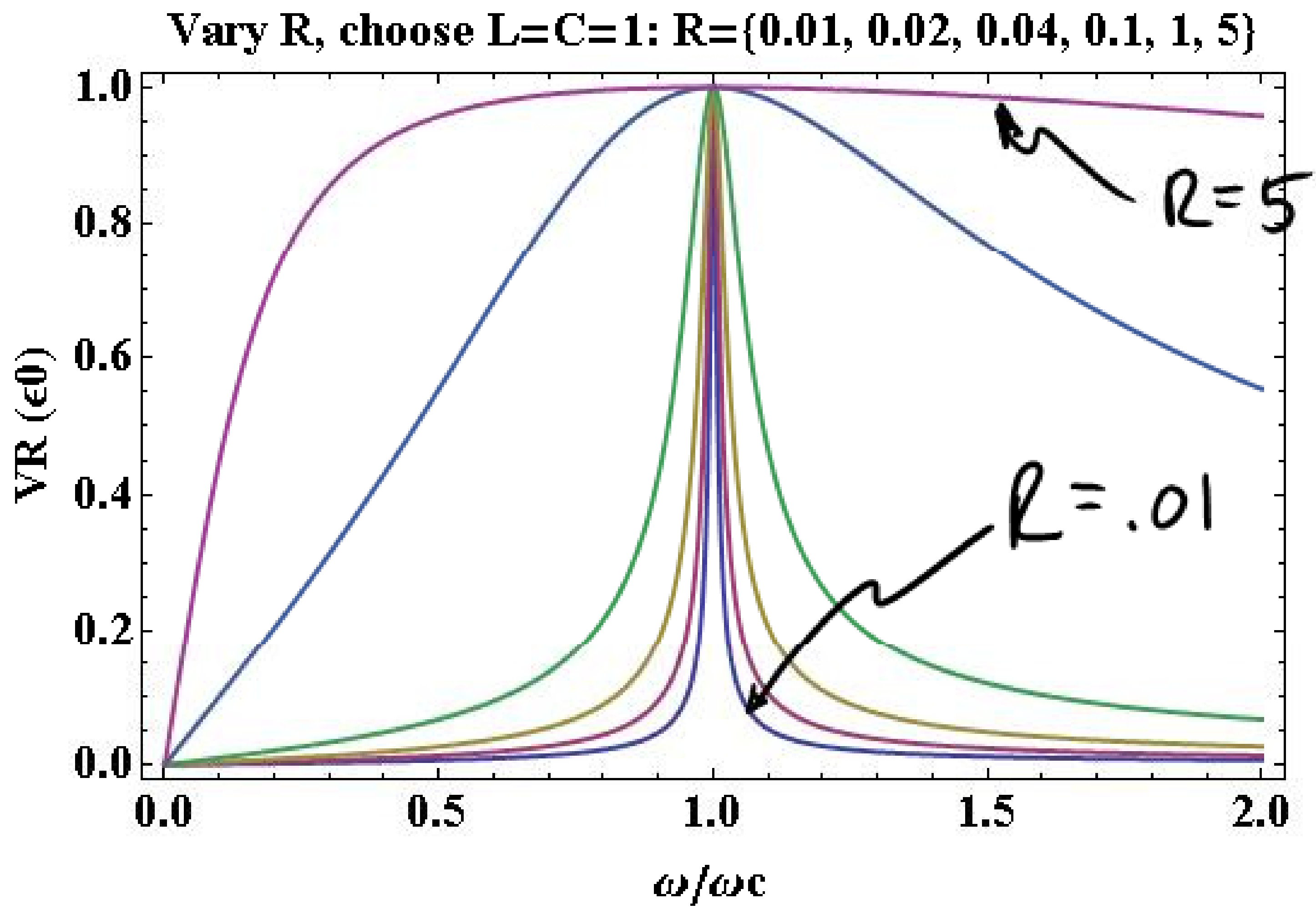
V_R is max when $\omega L = \frac{1}{\omega C}$, or $\omega = \sqrt{\frac{1}{LC}} \equiv \omega_c$

$\Rightarrow V_{max} = \mathcal{E}_0$

V_R decreases by increasing or decreasing ω away from ω_c

LRC Filters – Analysis

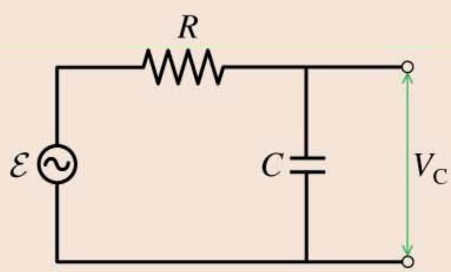
$$V_R = (\mathcal{E}_0 R)\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



Basic circuit elements

Element	i and v	Resistance/ reactance	I and V	Power
Resistor	In phase	R is fixed	$V = IR$	$V_{rms}I_{rms}$
Capacitor	i leads v by 90°	$X_C = 1/\omega C$	$V = IX_C$	0
Inductor	i lags v by 90°	$X_L = \omega L$	$V = IX_L$	0

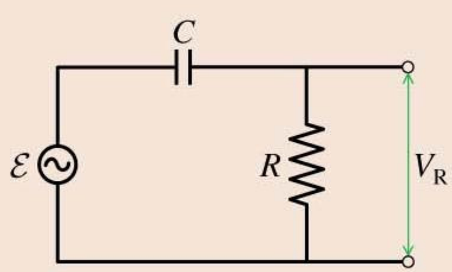
RC filter circuits



$$V_C = \mathcal{E}_0 X_C/\sqrt{R^2 + X_C^2}$$

$$V_C \rightarrow \mathcal{E}_0 \text{ as } \omega \rightarrow 0$$

A **low-pass filter** transmits low frequencies and blocks high frequencies.

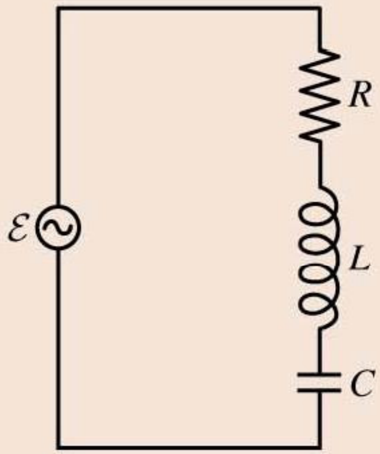


$$V_R = \mathcal{E}_0 R/\sqrt{R^2 + X_C^2}$$

$$V_R \rightarrow \mathcal{E}_0 \text{ as } \omega \rightarrow \infty$$

A **high-pass filter** transmits high frequencies and blocks low frequencies.

Series *RLC* circuits



$I = \mathcal{E}_0/Z$ where Z is the **impedance**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V_R = IR \qquad V_L = IX_L \qquad V_C = IX_C$$

When $\omega = \omega_0 = 1/\sqrt{LC}$ (the **resonance frequency**), the current in the circuit is a maximum $I_{max} = \mathcal{E}_0/R$.