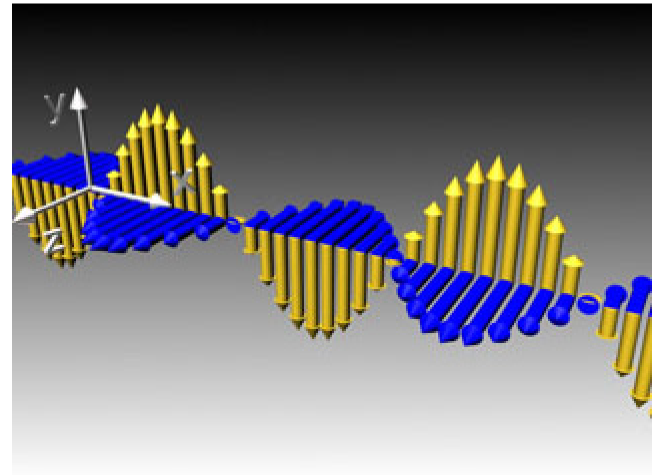
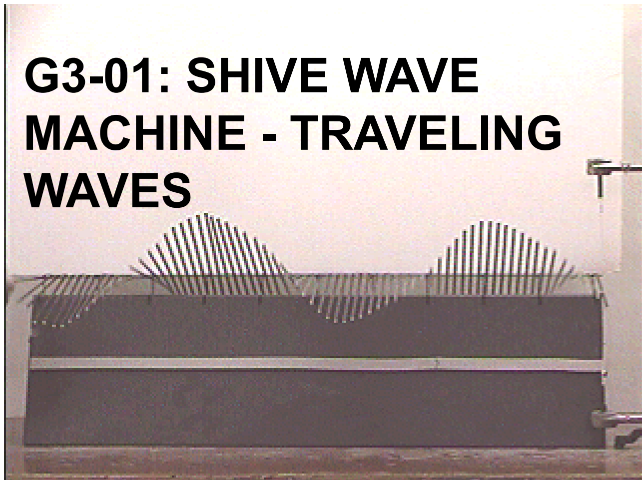
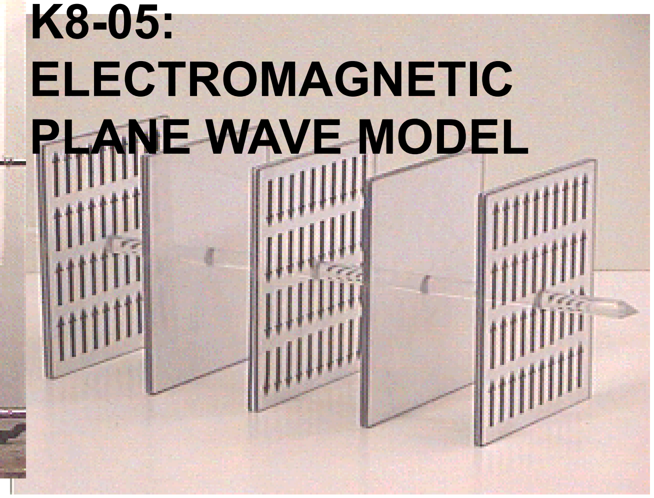
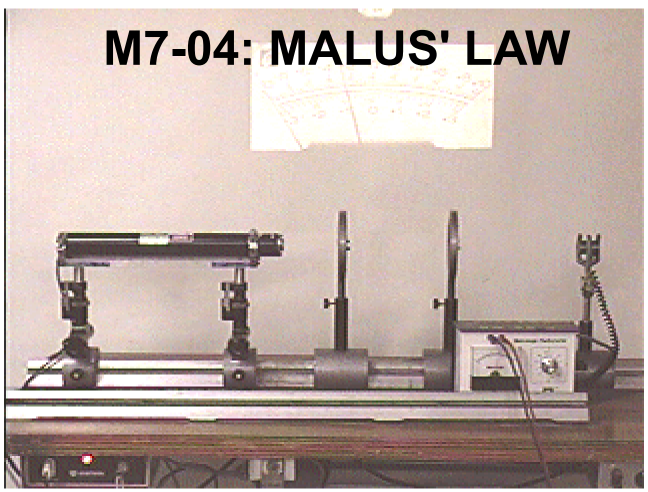
MIT simulation of EM waves

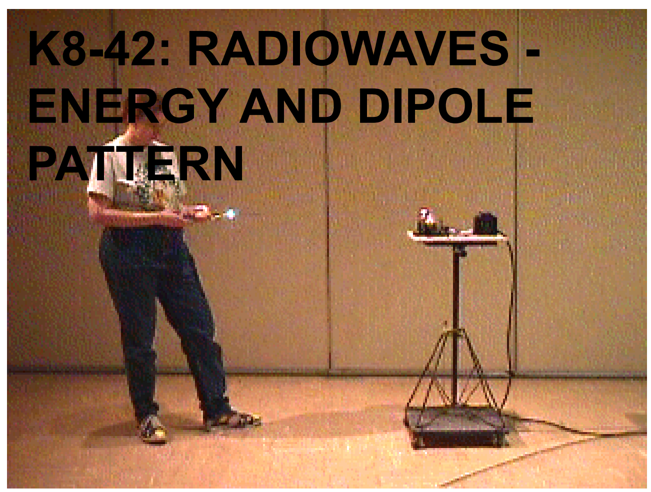
http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html











Homework set #3

- Due Tuesday by 5PM
- No late homework accepted

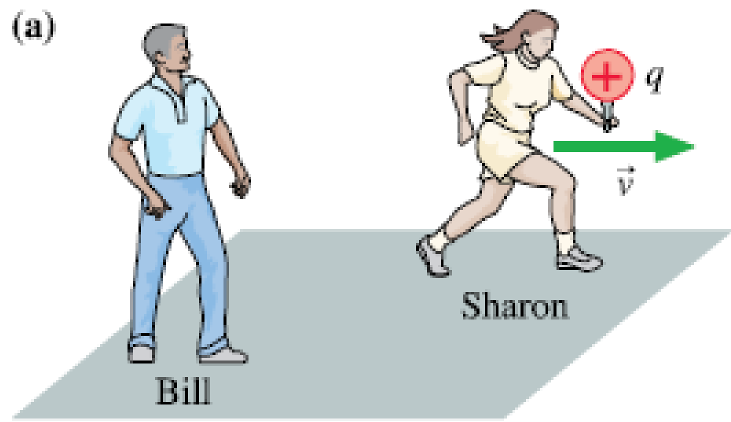
Quiz #3

- Sections 34.8-34.10, 35.0-35.5

Last time

E or B? Galilean transformation

Only consider constant velocity between reference frames!



Charge q moves with velocity $\vec{v}$ relative to Bill.

Sharon only observes an electric field from charge

Bill observes an electric field from charged particle
AND magnetic field produced by the moving charge

Both observe no net force on particle

E or B? Galilean transformation

Galilean transformation of velocity,

$$\vec{v}\,' = \vec{v} - \vec{V} \quad \text{or} \quad \vec{v} = \vec{v}\,' + \vec{V}$$

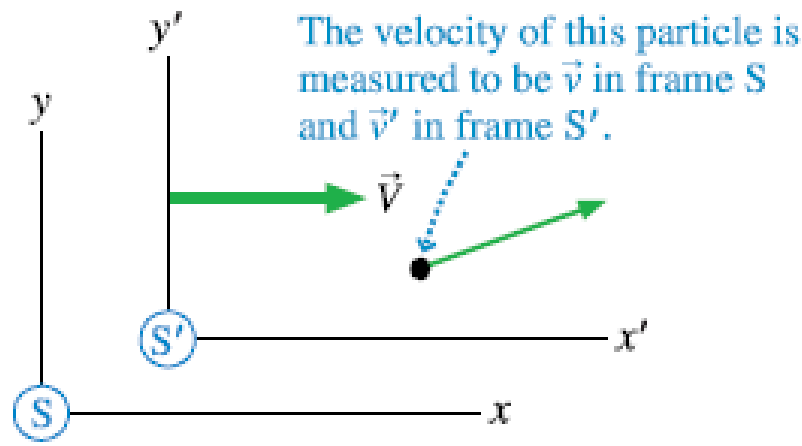
$$\frac{d\vec{v}\,'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{V}}{dt}$$

$\vec{V}$ is a *constant* velocity, so $d\vec{V}/dt = 0$.

$$\vec{a}\,' = \vec{a}$$

$$\vec{F}\,' = \vec{F}$$

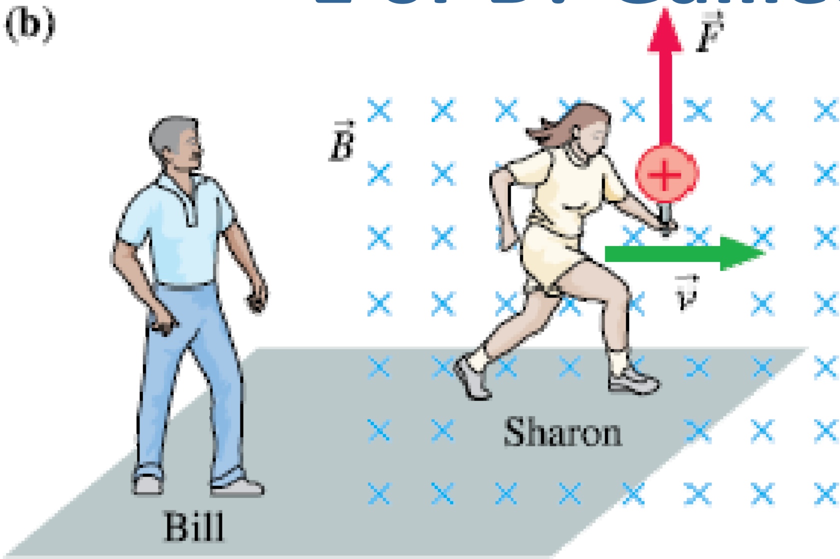
FIGURE 35.3 The particle's velocity is measured in both frame S and frame S′.



Both observers agree on the net force on the particle!

E or B? Galilean transformation

(b)



Charge q moves through a magnetic field $\vec{B}$ established by Bill.

Consider a TEST charge (to measure forces).

Bill (frame S) sets up B-field, observes charge moving at velocity → Force up:

$$\vec{F}_B = q\vec{v} \times \vec{B} = qvB \text{ up}$$

Sharon (frame S') is moving along with charge so v=0

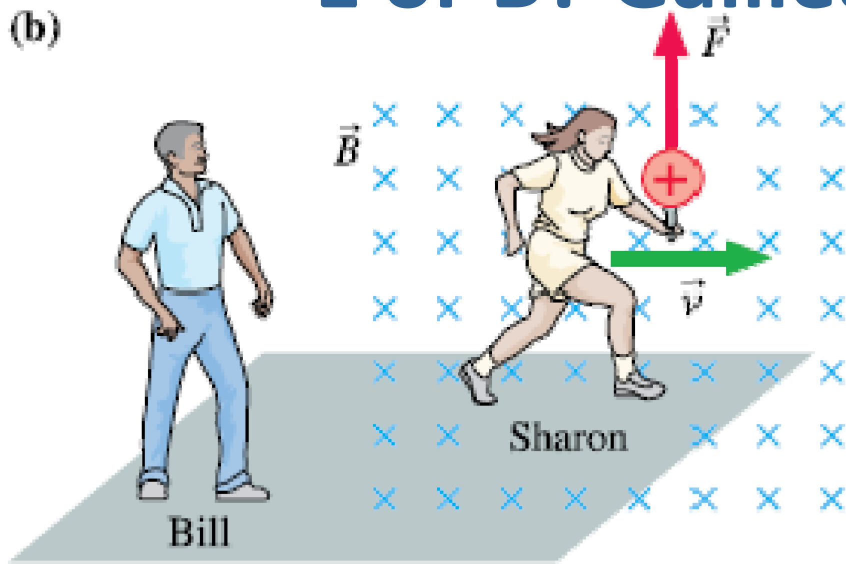
$$\vec{F}_B = q\vec{v} \times B = 0$$

0

There MUST be a force observed by Sharon since Bill observes one.

There must be an E-field in Sharon's frame that push's the charge!

E or B? Galilean transformation



Charge q moves through a magnetic field $\vec{B}$ established by Bill.

Bill (frame S) sets up B-field, observes charge moving at velocity → Force up:

$$\vec{F}_B = q\,\vec{v} \times \vec{B} = q\,v\,B \text{ up}$$

Sharon (frame S') is moving along with charge so v=0

$$\vec{F}_B = q\,\underbrace{\vec{v}}_{0} \times B = 0$$

Must have:

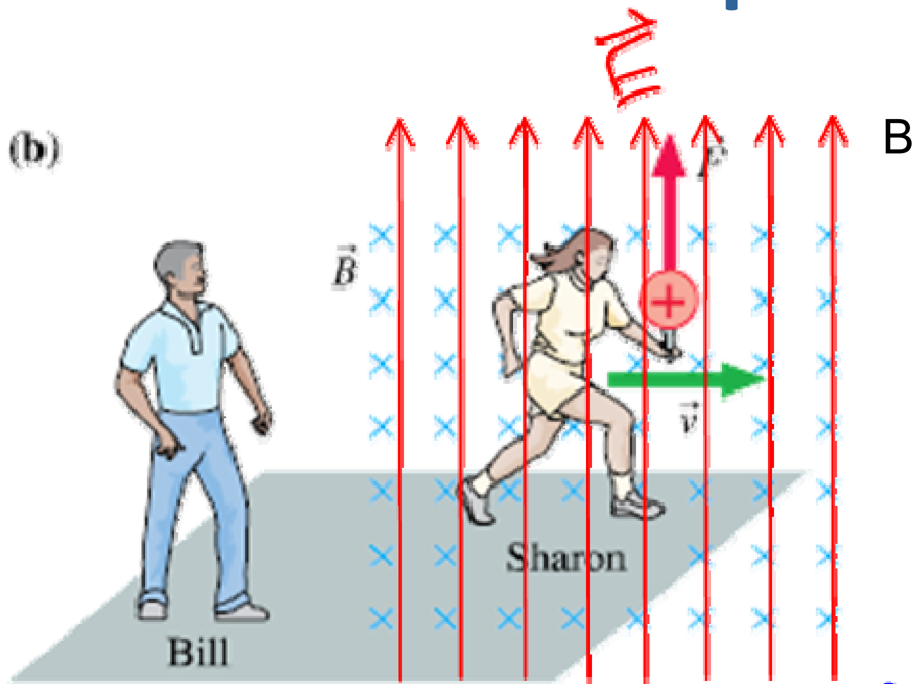
$$\vec{F}' = \vec{F}$$

Lorentz Force:

$$q\,\vec{E}' + q\,\underbrace{\vec{v}'}_{=0} \times \vec{B}' = q\,\underbrace{\vec{E}}_{=0} + q\,\underbrace{\vec{v}}_{\vec{v}\text{, velocity of Sharon}} \times \vec{B}$$

$$\Rightarrow \vec{E}' = \vec{v} \times \vec{B}$$

E or B? Two Aspects of same phenomenon



Charge q moves through a magnetic field $\vec{B}$ & $\vec{E}$
established by Bill.

Bill (frame S) Force up:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

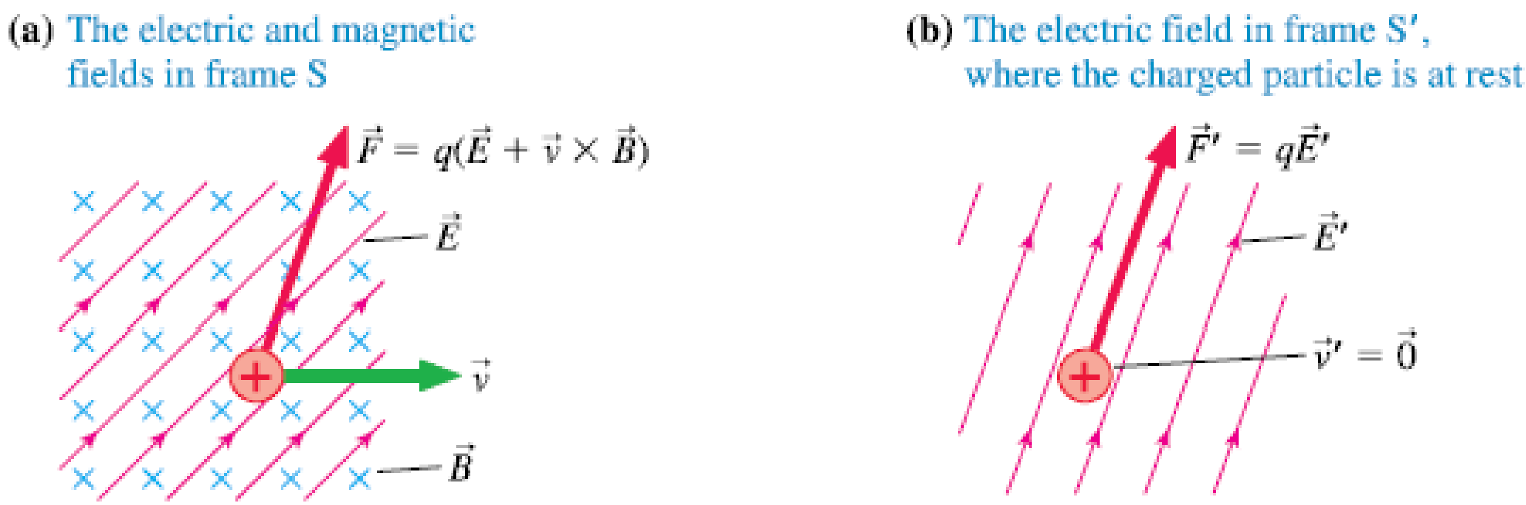
Sharon (frame S') is moving along with charge so v=0:

$$\vec{F}' = q\vec{E}'$$

$$\vec{F}' = \vec{F} \Rightarrow \vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

E or B? Two Aspects of same phenomenon

FIGURE 35.6 A charge in frame S experiences electric and magnetic forces. The charge experiences the same force in frame S′, but it is due only to an electric field.



$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B}$$

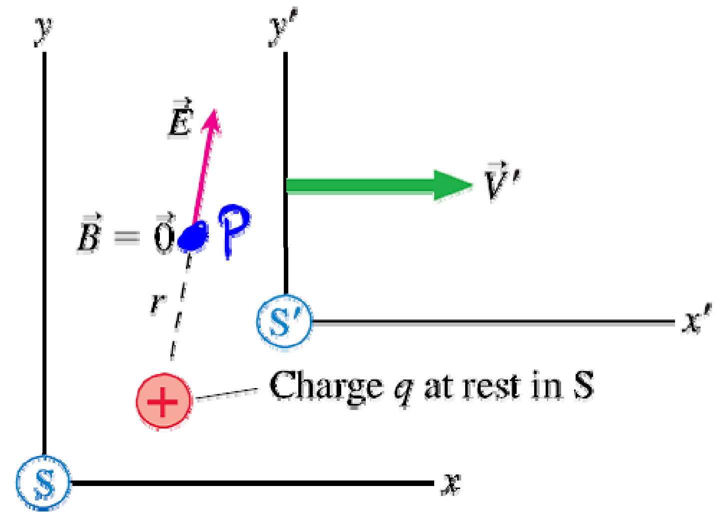
E field in frame S’ from E and B fields in frame S

How do the B-fields tranform from one frame to another?

B-field transformation: Biot Savart Law

Bill, S : $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$, $\vec{B} = 0$ @ P

(a) In frame S, the static charge creates an electric field but no magnetic field.



Sharon, S′ : Charge moves in $-\vec{V}$ creating a B-field (as well as an E-field due to pt charge)

$$\vec{E}' = \vec{E} \quad @ P$$

or Biot-Savart Law : $\vec{B}' = \frac{\mu_0}{4\pi} q \frac{-\vec{V} \times \hat{r}}{r^2}$ @ P ①

$$\Rightarrow \vec{B}' = -\mu_0 \varepsilon_0 \, \vec{V} \times \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}\right) \quad ②$$

$$\Rightarrow \vec{B}' = -(\mu_0 \varepsilon_0) \, \vec{V} \times \vec{E} \quad ③$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3 \cdot 10^8 \ \text{m/sec}$$

③ → ① ⇒ Biot-Savart Law is derivable from equation 3 !!

E-field of a pt charge transformed into different inertial reference frames yields Biot-Savart Law

E or B? Two Aspects of same phenomenon

The **Galilean field transformation** equations are

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B} \qquad \text{or} \qquad \vec{E} = \vec{E}' - \vec{V} \times \vec{B}'$$

$$\vec{B}' = \vec{B} - \frac{1}{c^2}\vec{V} \times \vec{E} \qquad \qquad \vec{B} = \vec{B}' + \frac{1}{c^2}\vec{V} \times \vec{E}'$$

where ***V*** is the velocity of frame S′ relative to frame S and where the fields are measured *at the same point in space* by experimenters *at rest* in each reference frame.

NOTE: These equations are only valid if $V << c$.

Maxwell's Equations

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

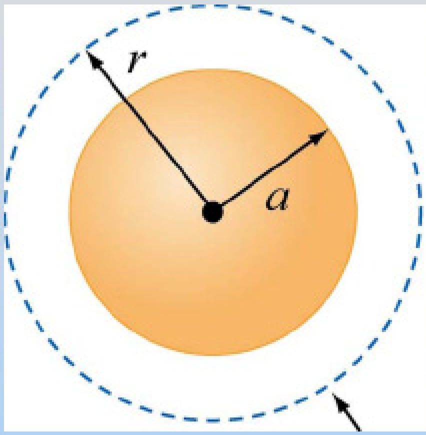
$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

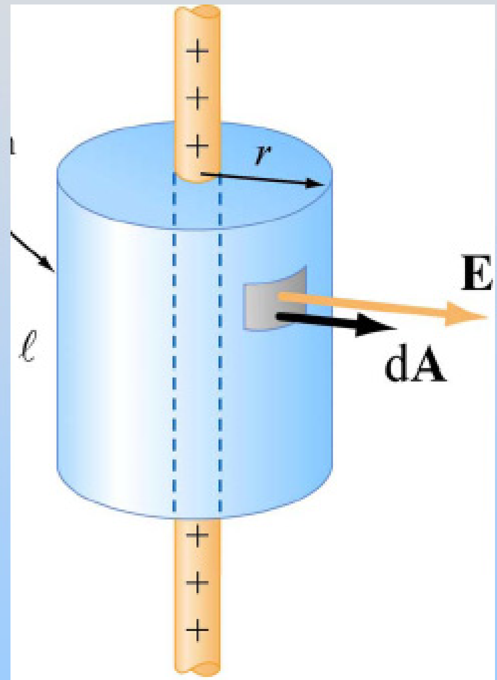
$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$ (Lorentz force Law)

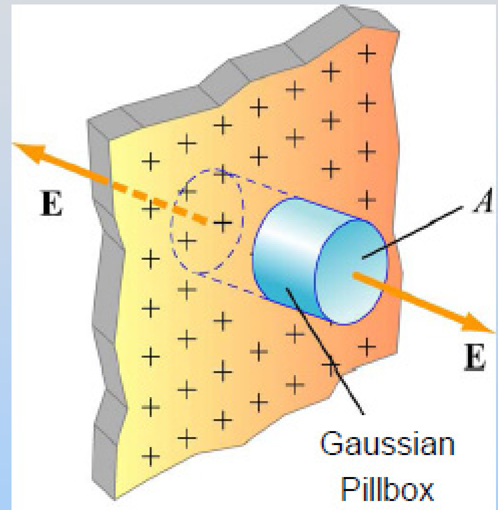
Gauss's Law: $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$



Spherical Symmetry



Cylindrical Symmetry



Planar Symmetry

$\oint \vec{E} \cdot d\vec{a} = \Phi_E \propto$ Net # of $\vec{E}$-field lines piercing surface

$= \frac{Q_{enc}}{\varepsilon_0}$

Magnetic Gauss's Law

Gauss's Law:

$$\oint \vec{E} \cdot d\vec{a} = \Phi_E \propto \text{Net \# of } \vec{E}\text{-field lines piercing closed surface}$$

$$= \frac{Q_{enc}}{\varepsilon_0}$$

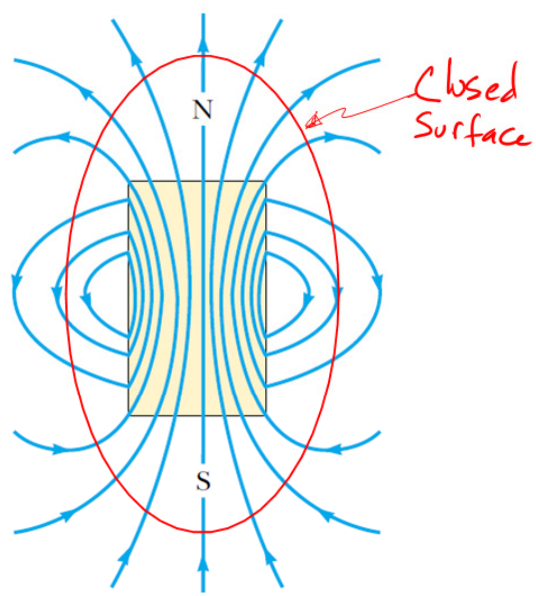
Magnetic Gauss's Law:

$$\oint \vec{B} \cdot d\vec{a} = \Phi_B \propto \text{Net \# of } \vec{B}\text{-field lines piercing closed surface}$$

$$= 0$$

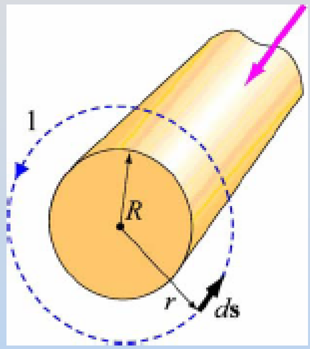
Since there are NO magnetic monopoles (only dipoles and conglomerates of dipoles),

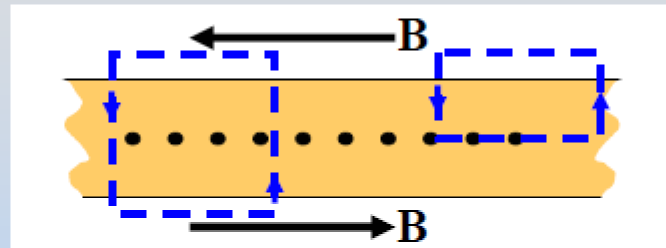
Net number of field lines piercing any closed surface is zero



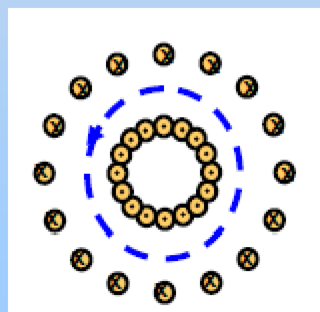
Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

Long Circular Symmetry



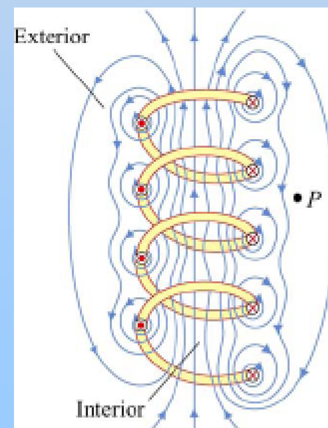


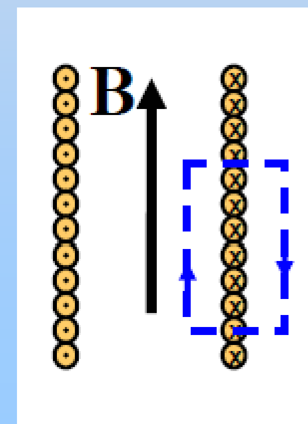
(Infinite) Current Sheet



Torus/Coax

Solenoid = 2 Current Sheets





Faraday's Law of Induction

$$\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -N \frac{d\Phi_B}{dt}$$

$$= -N \frac{d}{dt}\left(BA\cos\theta\right)$$

Moving bar, entering field

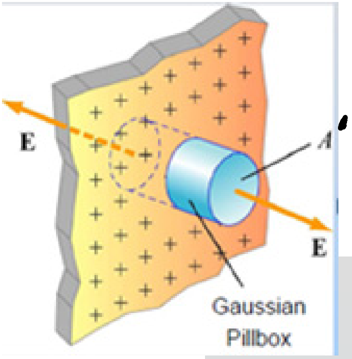
Ramp B

Rotate area in field

Lenz's Law:

Induced EMF is in direction that ***opposes*** the change in flux that caused it

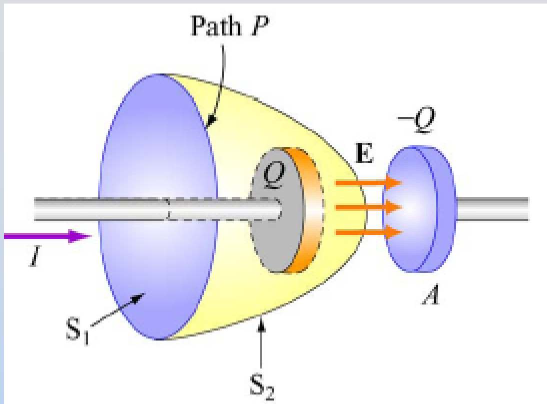
Modification to Ampere's Law



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow E A' = \frac{1}{\varepsilon_0} \frac{Q}{A} A'$$

$$\Rightarrow E = \frac{Q}{\varepsilon_0 A}$$

Displacement Current



$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 E A = \varepsilon_0 \Phi_E$$

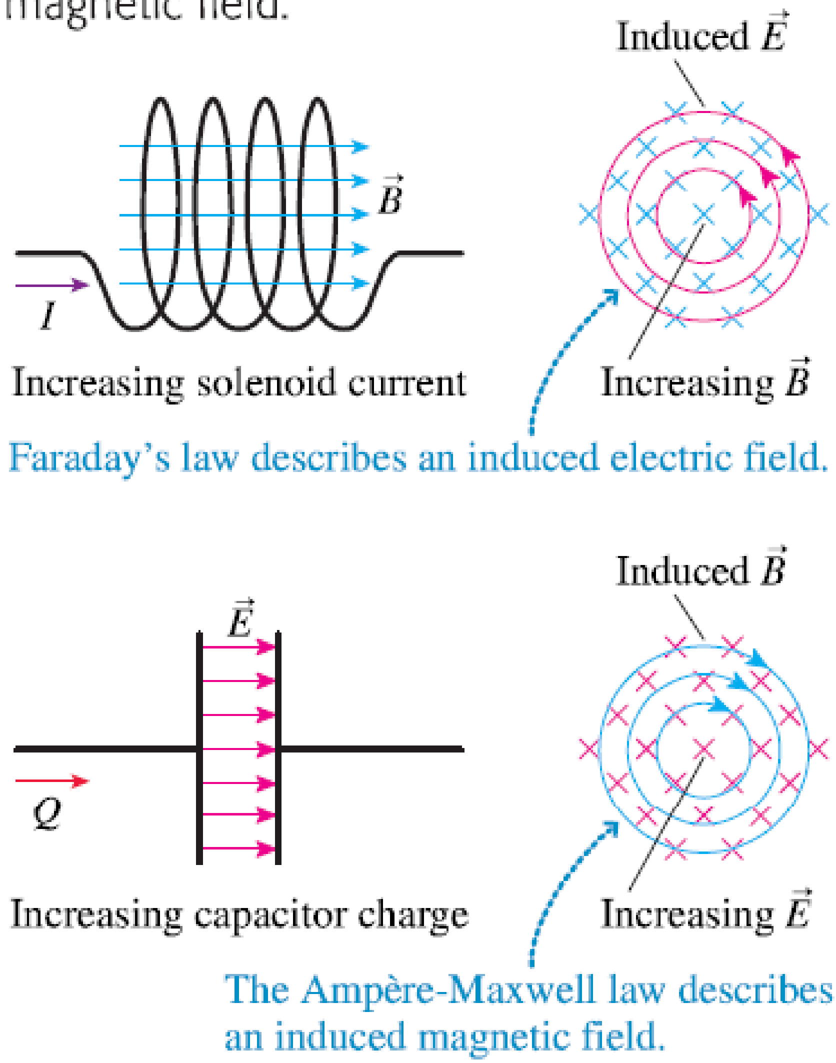
$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_d$$

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

Capacitors,
EM Waves

$$= \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

FIGURE 35.17 The close analogy between an induced electric field and an induced magnetic field.



Changing B-field induces E-field,
Lenz's law gives direction

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Changing E-field induces B-field, Opposite of Lenz's law gives B-field direction

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left[I_{enc} + \epsilon_0 \frac{d\Phi_E}{dt}\right]$$

$$= +\mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

opposite sign
=> opp. "Lenz's Law"

Maxwell's Equations

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

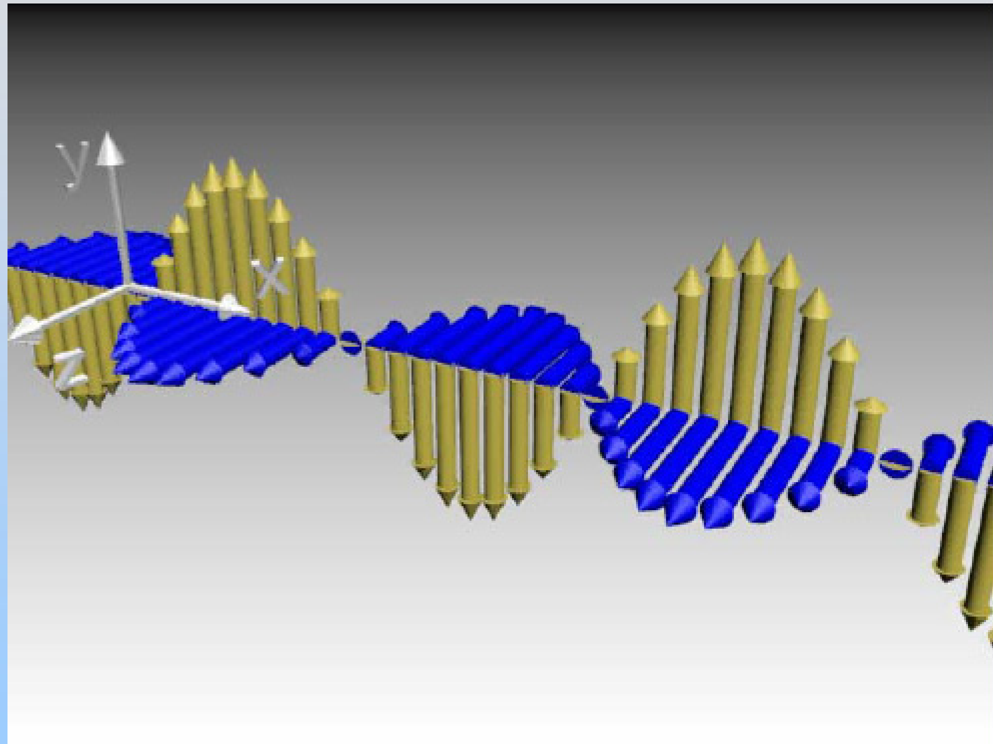
$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$ (Lorentz force Law)

Which Leads To…
EM Waves

Electromagnetic Radiation: Plane Waves



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html

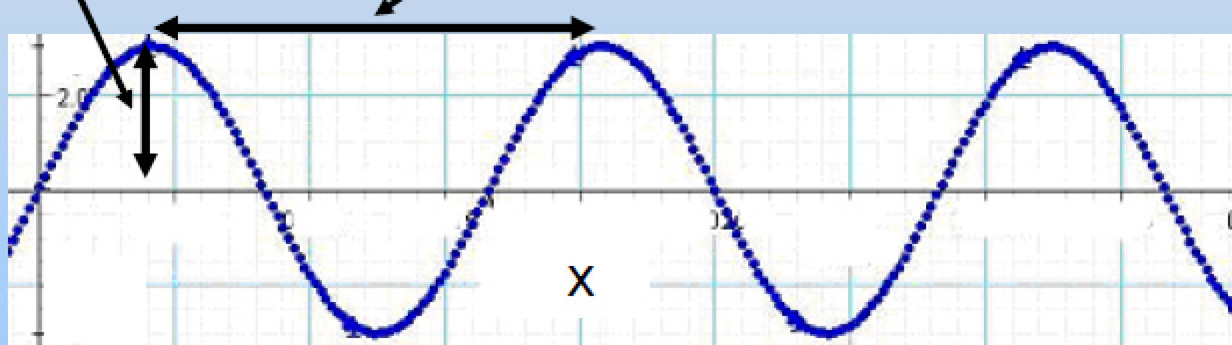
Quickly Review of Traveling Waves

Traveling Sine Wave

Now consider $f(x) = y = y_0 \sin(kx)$:

$$\text{Wavelength } (\lambda) = \frac{2\pi}{\text{wavenumber } (k)}$$

Amplitude (y_0)



What is $g(x,t) = f(x+vt)$? Travels to left at velocity v

$$y = y_0 \sin(k(x+vt)) = y_0 \sin(kx+kvt)$$

Traveling Sine Wave

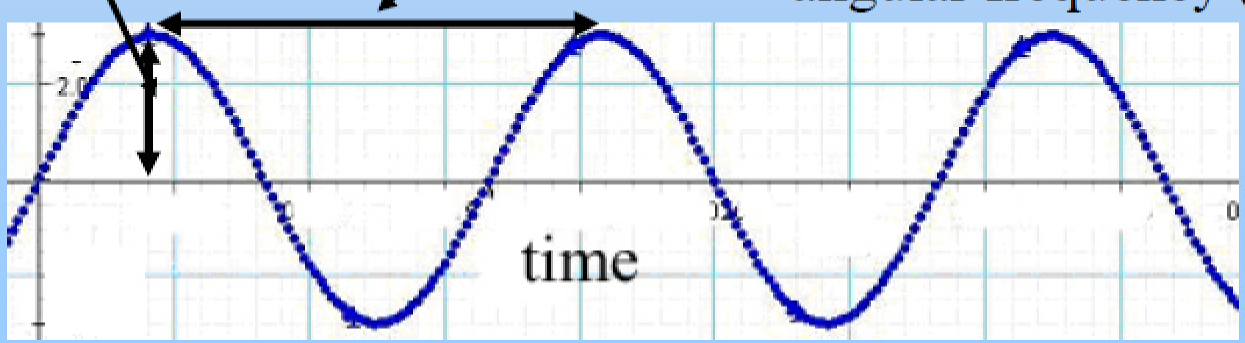
$$y = y_0 \sin\left(kx + kvt\right)$$

At x=0, just a function of time: $y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t)$

$$\text{Period } (T) = \frac{1}{\text{frequency } (f)}$$

$$= \frac{2\pi}{\text{angular frequency } (\omega)}$$

Amplitude (y_0)

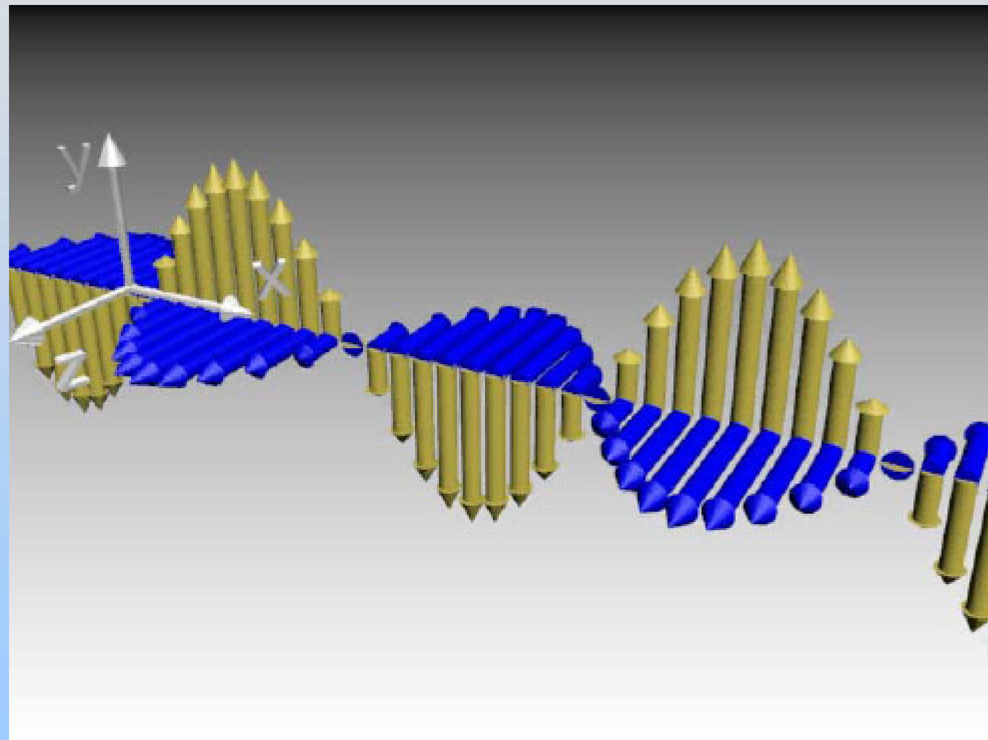


Traveling Sine Wave

$$\boxed{y = y_0 \sin(kx - \omega t)}$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \frac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- Speed of Propagation: $v = \frac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

Electromagnetic Radiation: Plane Waves



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html

Traveling E & B Waves

- Wavelength: λ
- Frequency : f

$$\boxed{\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(kx - \omega t)}$$

- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

Move along with wave $\Rightarrow$ $kx - \omega t = \text{constant}$,

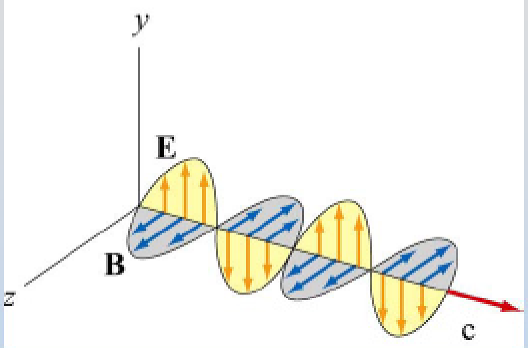
$\Rightarrow \quad x = \dfrac{\omega}{k} t + \text{constant}$

$\Rightarrow \quad \dfrac{dx}{dt} = \dfrac{\omega}{k}$

Properties of EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$



At every point in the wave and any instant of time, E and B are in phase with one another, with

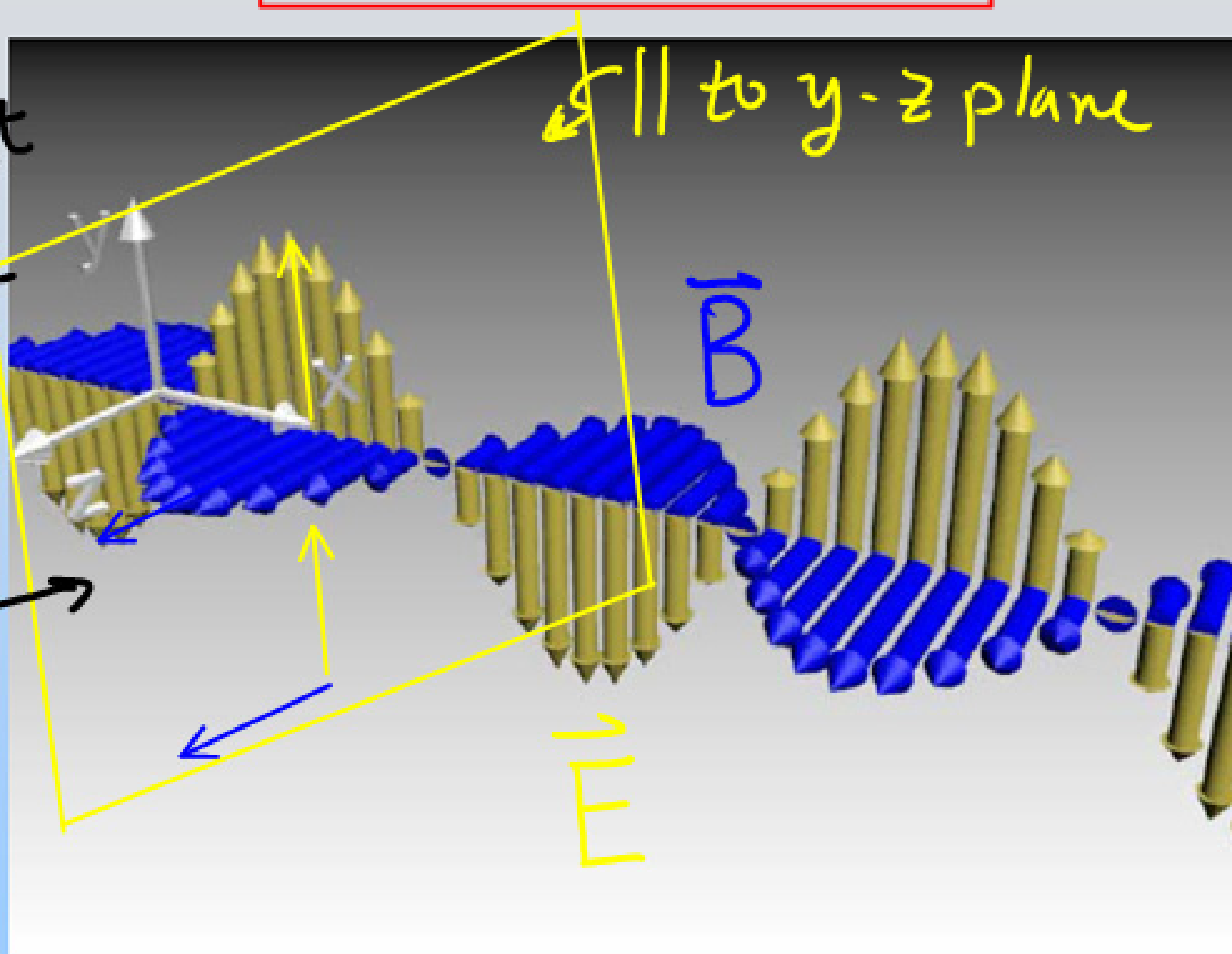
$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

Electromagnetic Radiation: Plane Waves

$\vec{E}$ & $\vec{B}$ Constant Everywhere on plane



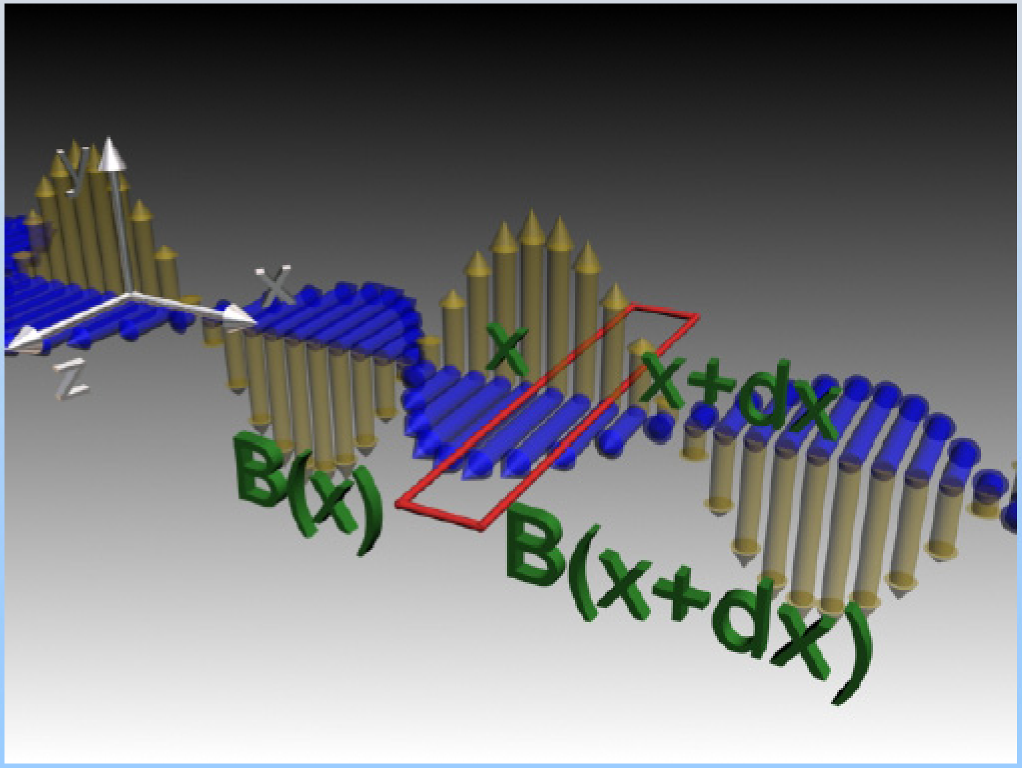
http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html

How Do Maxwell's Equations Lead to EM Waves?

Derive Wave Equation

Wave Equation

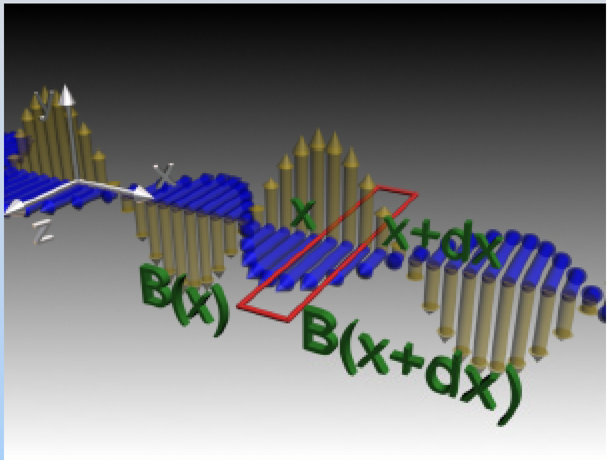
Start with Ampere-Maxwell Eq: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$



Wave Equation

Start with Ampere-Maxwell Eq: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$

Apply it to red rectangle:



$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B_z(x,t)l - B_z(x+dx,t)l$$

$$\mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \mu_0 \varepsilon_0 \left(l\, dx \frac{\partial E_y}{\partial t} \right)$$

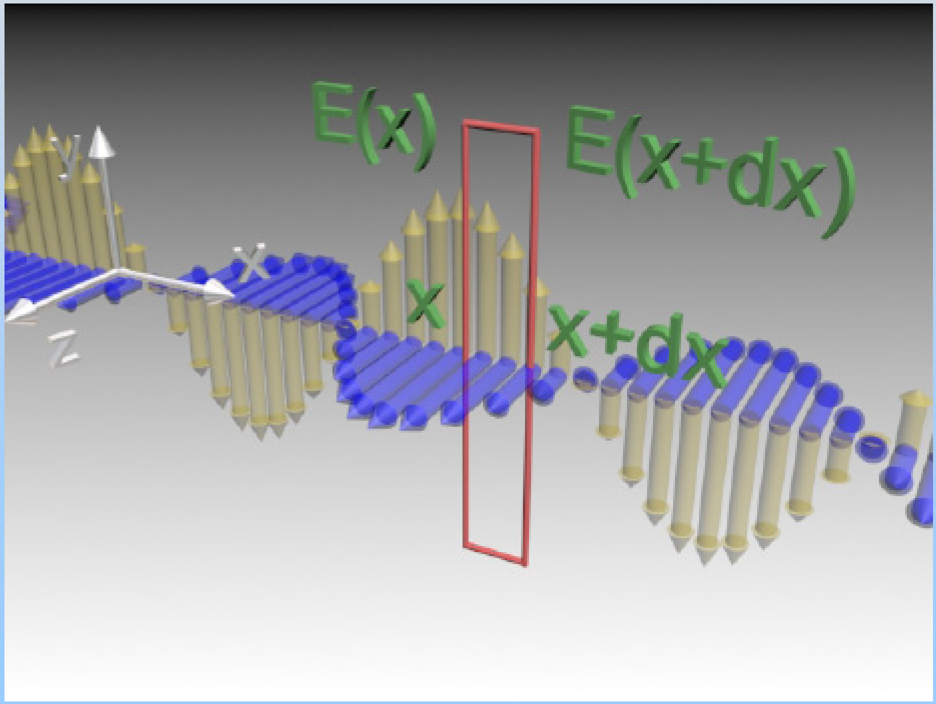
$$-\frac{B_z(x+dx,t) - B_z(x,t)}{dx} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

So in the limit that *dx* is very small:

$$\boxed{-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}}$$

Wave Equation

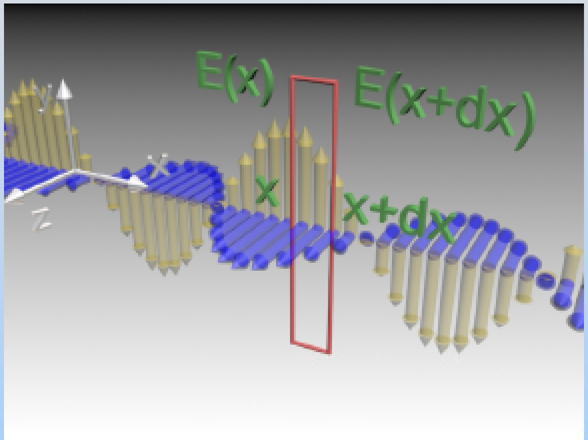
Now go to Faraday's Law $$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$



Wave Equation

Faraday's Law: $$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

Apply it to red rectangle:



$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = E_y(x+dx,t)l - E_y(x,t)l$$

$$-\frac{d}{dt}\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -l\,dx\,\frac{\partial B_z}{\partial t}$$

$$\frac{E_y(x+dx,t) - E_y(x,t)}{dx} = -\frac{\partial B_z}{\partial t}$$

So in the limit that *dx* is very small: $$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

1D Wave Equation for E

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad -\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Take *x*-derivative of 1st and use the 2nd equation

$$\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial x}\right) = \underline{\frac{\partial^2 E_y}{\partial x^2}} = \frac{\partial}{\partial x}\left(-\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = \underline{\mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

1D Wave Equation for E

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}}$$

This is an equation for a wave. Let: $E_y = f(x - vt)$

$$\left.\begin{array}{l} \dfrac{\partial^2 E_y}{\partial x^2} = f''(x - vt) \\[2ex] \dfrac{\partial^2 E_y}{\partial t^2} = v^2 f''(x - vt) \end{array}\right\} v^2 = \frac{1}{\mu_0 \varepsilon_0}$$

P30-24

$V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \cdot 10^8 \text{ m/sec}$; Light = EM wave

1D Wave Equation for B

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Take *x*-derivative of 1st and use the 2nd equation

$$\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial t}\right) = \underline{\frac{\partial^2 B_z}{\partial t^2}} = \frac{\partial}{\partial t}\left(-\frac{\partial E_y}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial E_y}{\partial t}\right) = \underline{\frac{1}{\mu_0 \varepsilon_0}\frac{\partial^2 B_z}{\partial x^2}}$$

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}}$$

Electromagnetic Radiation

Both E & B travel like waves:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

But there are strict relations between them:

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

Here, E_y and B_z are "the same," traveling along x axis

Amplitudes of E & B

Let $E_y = E_0 f\left(x - vt\right);\ B_z = B_0 f\left(x - vt\right)$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \Rightarrow -vB_0 f'\left(x - vt\right) = -E_0 f'\left(x - vt\right)$$

$$\boxed{\Rightarrow vB_0 = E_0}$$

E_y and B_z are "the same," just different amplitudes

Energy and momentum of EM radiation

u = Energy density of E + B fields

$$= u_E + u_B = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$$

For E+M waves: $B = \frac{E}{c}$ + $\frac{1}{c^2} = \epsilon_0 \mu_0$

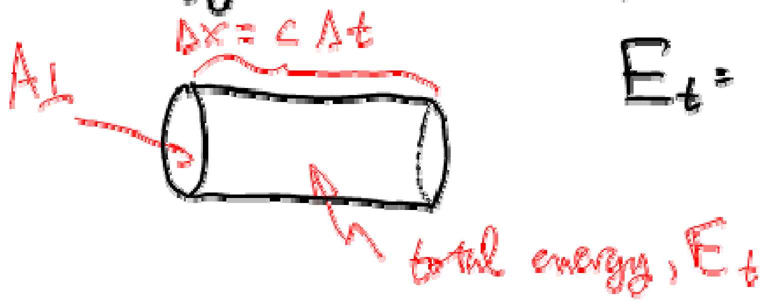
$$\Rightarrow u = \epsilon_0 E^2 \quad \text{or} \quad \frac{B^2}{\mu_0}$$

Amplitude of E-field

$$u_{avg} = \epsilon_0 \left(E^2\right)_{avg} = \frac{1}{2}\epsilon_0 E_0^2 = \epsilon_0 \frac{1}{2} c E_0 B_0 \quad ①$$

$$\int_0^T \cos^2(\omega t)\, dt = \frac{1}{2}$$

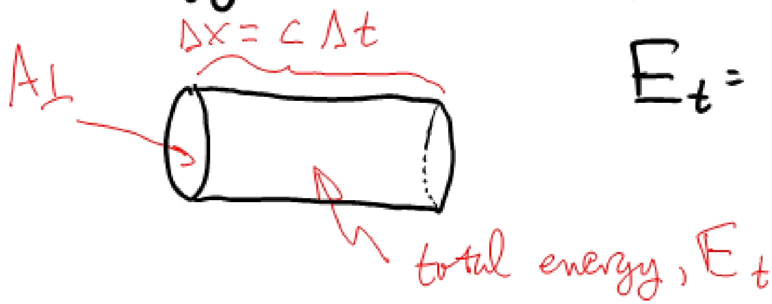
Energy flows at speed c:



$$E_t = u_{avg} A_\perp \Delta x = u_{avg} A_\perp c \Delta t$$

Energy and momentum of EM radiation

Energy flows at speed c:



$$E_t = u_{avg} A_\perp \Delta x = u_{avg} A_\perp c\,\Delta t$$

Energy moving through $A_\perp$ in time Δt = E_t

Power

Intensity, $I_{avg} \equiv \dfrac{E_t/\Delta t}{A_\perp} = u_{avg} \cdot c$ ②

① + ② $\therefore$ $I_{avg} = \dfrac{1}{2} c^2 \varepsilon_0 E_0 B_0 = \dfrac{1}{2\mu_0} E_0 B_0$

Define $\vec{S}_{avg} = \dfrac{1}{2\mu_0} \vec{E}_0 \times \vec{B}_0$, or $\boxed{\vec{S} = \dfrac{1}{\mu_0} \vec{E} \times \vec{B}}$

Magnitude gives intensity, instantaneous flow of energy direction

Energy Flow

Poynting vector: $\vec{\mathbf{S}} = \dfrac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$

- (Dis)charging C, L
- Resistor (always in)
- EM Radiation

Radiation Pressure:

Can derive from Maxwell's Equations. Complicated!

Quantum mechanics $\Rightarrow$ momentum of photon, $p = \dfrac{E}{c}$

Change in momentum of Many photons

$$\text{Force} = \frac{\Delta p}{\Delta t} = \frac{1}{c}\frac{\Delta E}{\Delta t} = \frac{\text{Power}}{c}$$

Absorb all energy

$$\Rightarrow \text{Pressure, } P = \frac{\text{Force}}{A_\perp} = \frac{1}{c}\frac{\text{Power}}{A_\perp}$$

$$P = \frac{I}{c} \quad \text{for perfect absorber}$$

$$P = \frac{2I}{c} \quad \text{for perfect reflector}$$

(Δp factor of 2 larger)

Polarization and Malus's law

FIGURE 35.27 The plane of polarization is the plane in which the electric field vector oscillates.

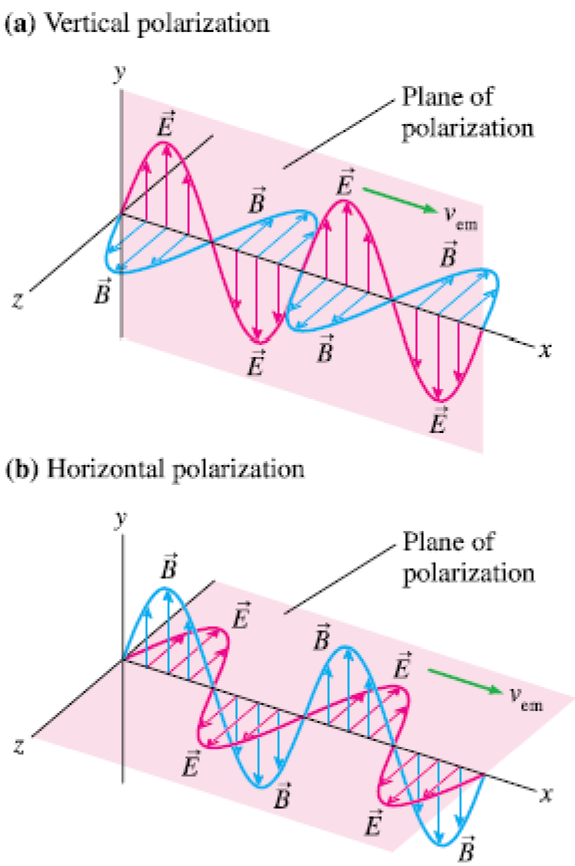
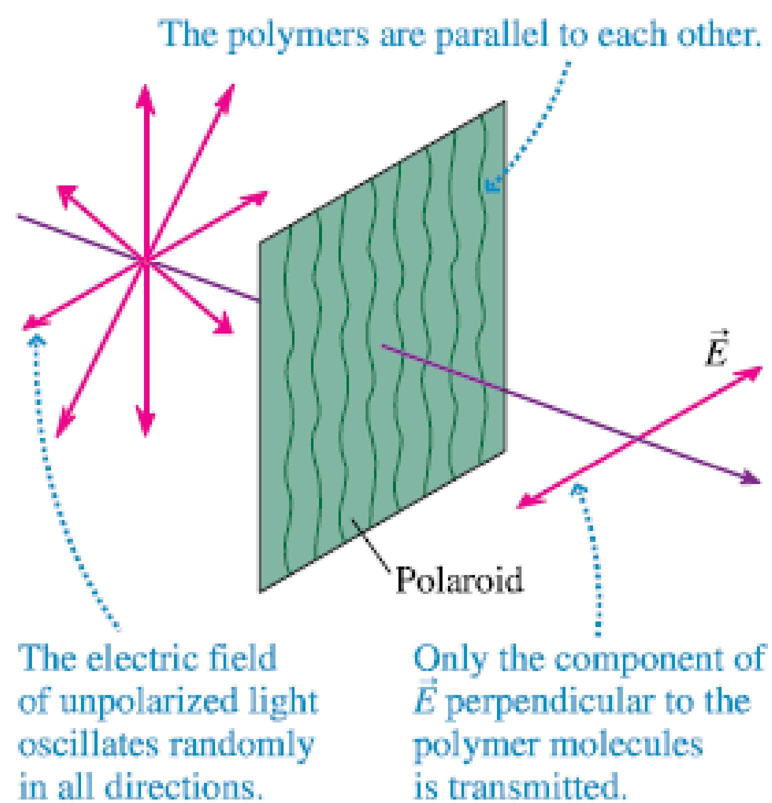
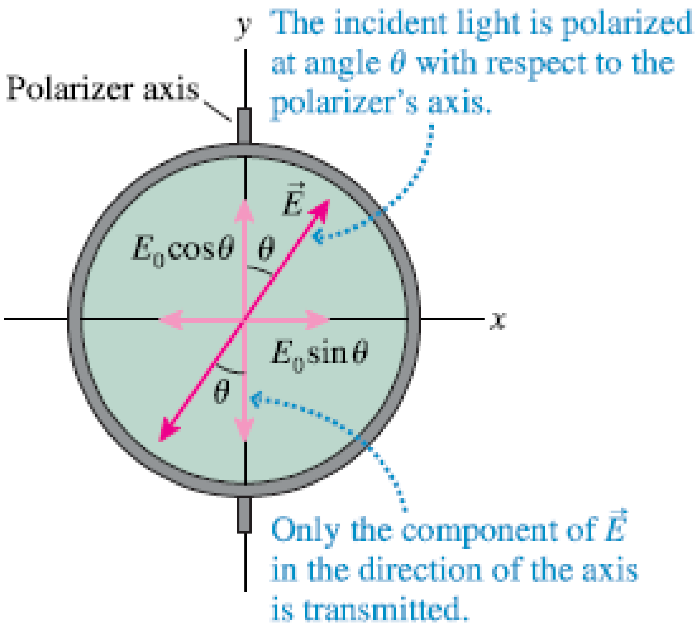


FIGURE 35.28 A polarizing filter.



Polarization and Malus's law

FIGURE 35.29 An incident electric field can be decomposed into components parallel and perpendicular to a polarizer's axis.

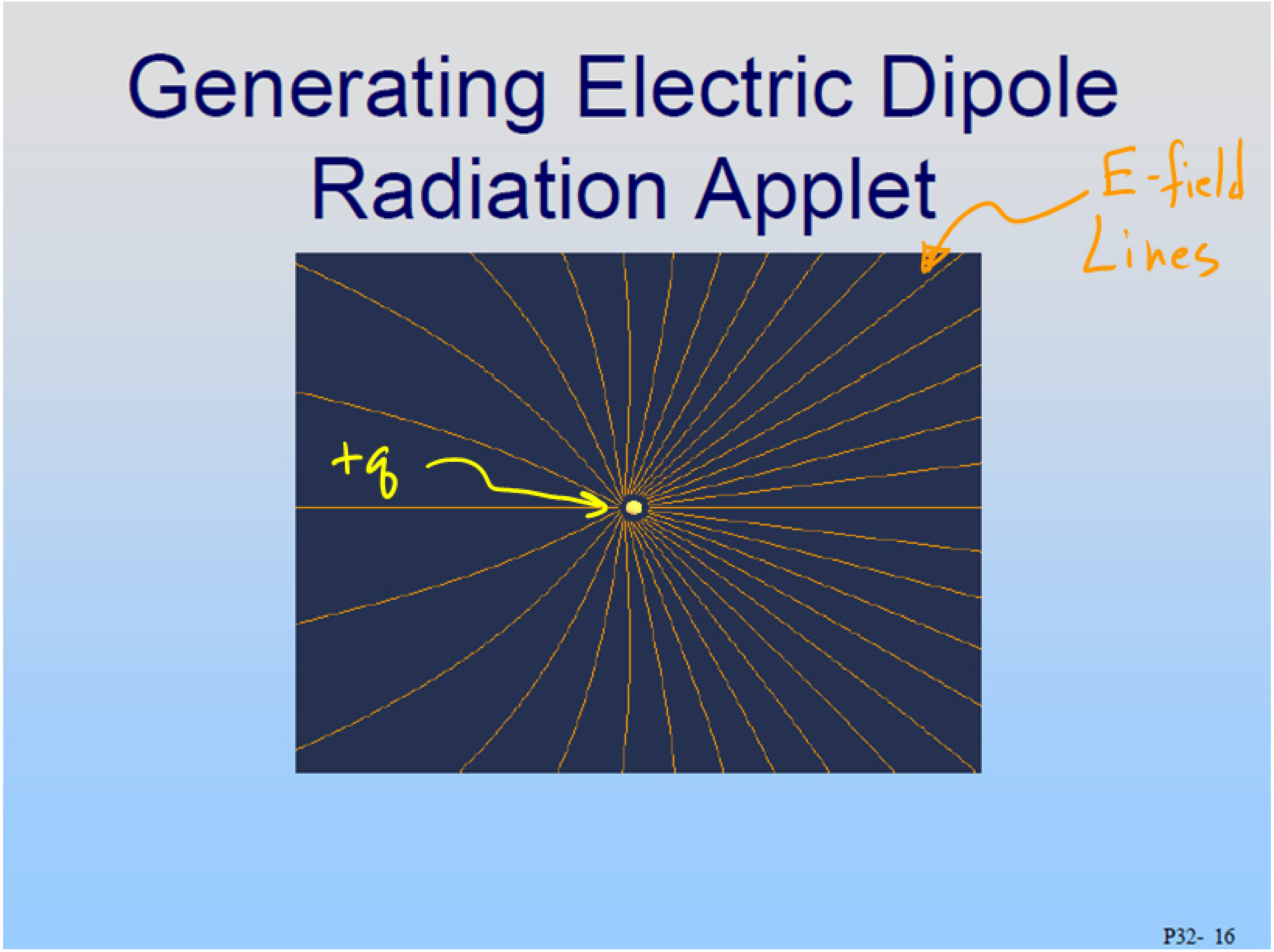


$$\vec{E}_{\text{transmitted}} = E_{\parallel}\,\hat{j} = E_0\cos\theta\,\hat{j}$$

$$I \propto E^2 \Rightarrow \frac{I_{trans}}{I_0} = \left(\frac{E_{trans}}{E_0}\right)^2$$

$$I_{\text{transmitted}} = I_0\cos^2\theta \qquad \text{(incident light polarized)}$$

Producing and Receiving EM waves

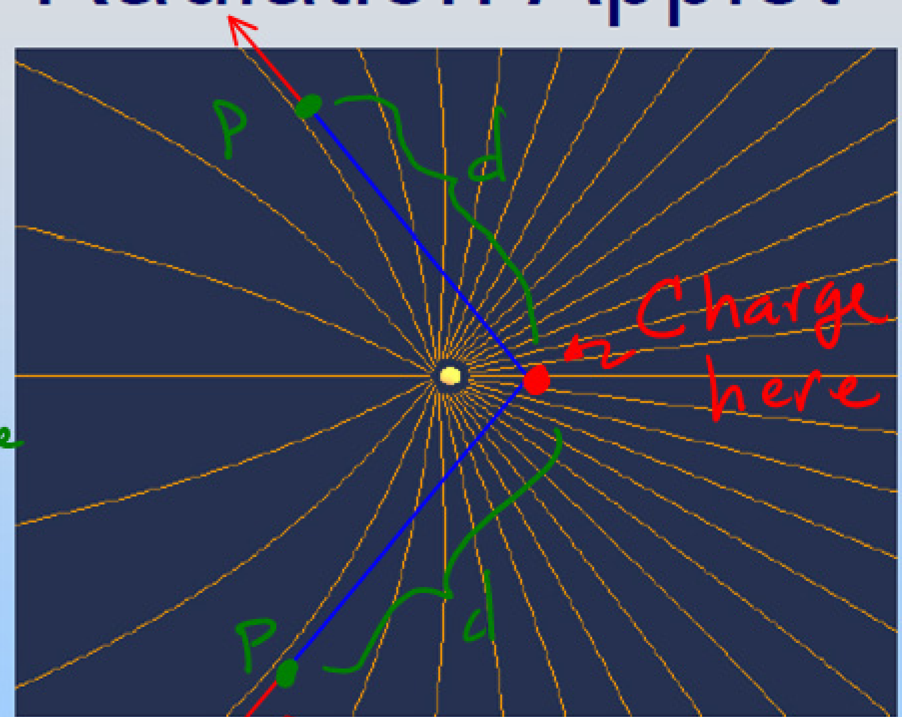


Which way is the charge moving?

Producing and Receiving EM waves

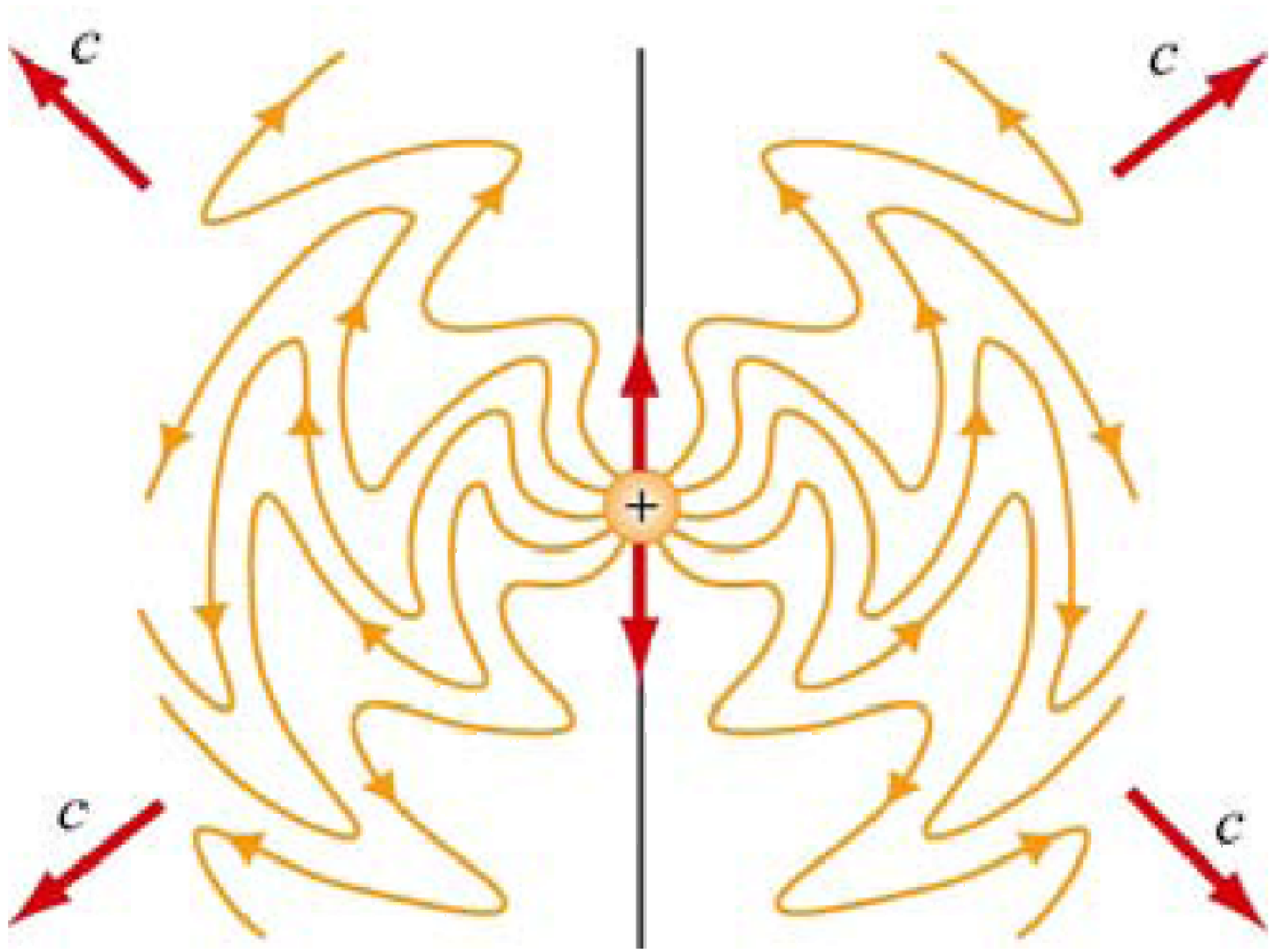
Generating Electric Dipole Radiation Applet

Takes time $\Delta t = \frac{d}{c}$ for anything @ point P to know the charge has moved from the point marked by ●



Which way is charge moving? To Left!

Producing and Receiving EM waves

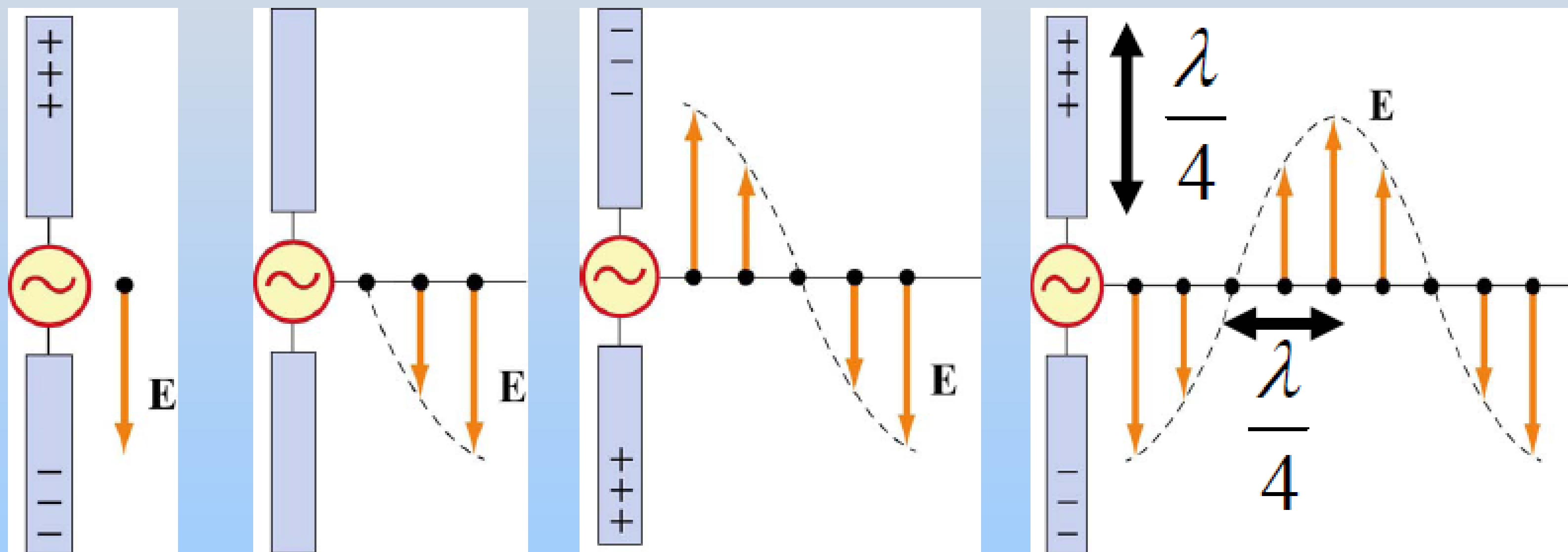


At large distances, E becomes ‘flat’ → Plane waves

Producing and Receiving EM waves

Quarter-Wavelength Antenna

Accelerated charges are the source of EM waves.
Most common example: Electric Dipole Radiation.



$t = 0$ $t = T/4$ $t = T/2$ $t = T$

Producing and Receiving EM waves

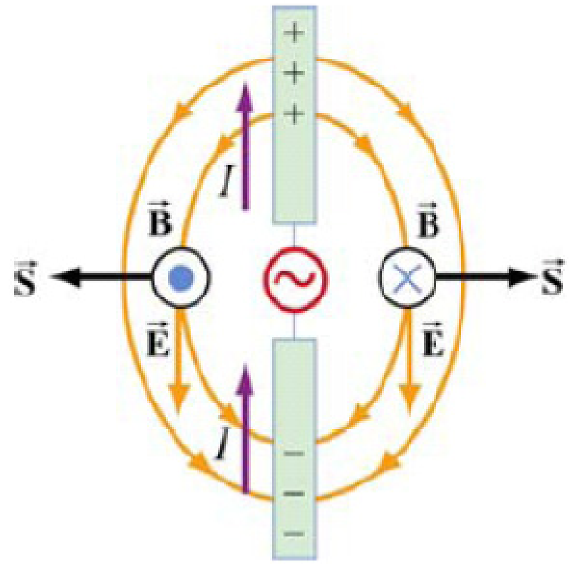


Figure 13.8.3 Electric and magnetic field lines produced by an electric-dipole antenna.

(in the "far field")

Producing and Receiving EM waves

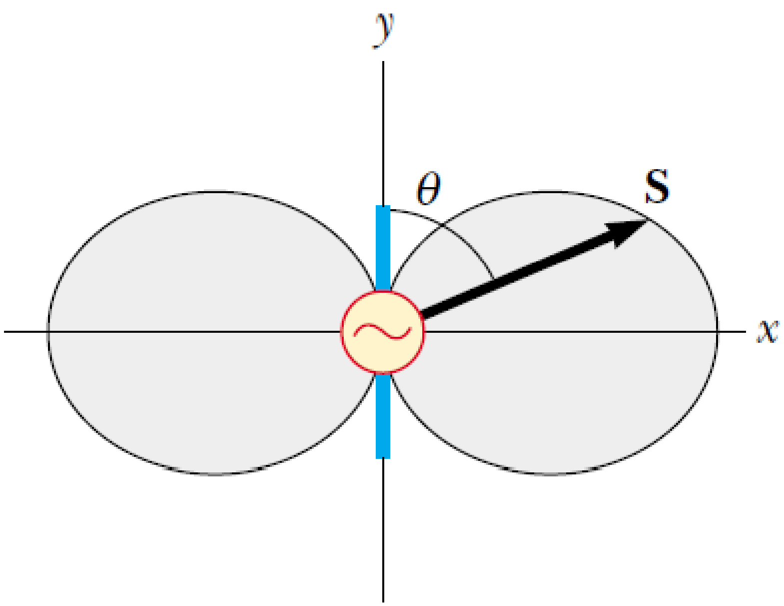


Figure 34.11 Angular dependence of the intensity of radiation produced by an oscillating electric dipole. The distance from the origin to a point on the edge of the gray shape is proportional to the intensity of radiation.

No radiation along axis of dipole:

Biot-Savart law state there is no B-field along y if there is current parallel to r-hat

$$I \propto \frac{\sin^2\theta}{r^2}$$

General derivation of EM wave equation

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Stoke's Thm: $\oint(\vec{\nabla}\times\vec{V})\cdot d\vec{a} = \int \vec{V}\cdot d\vec{s}$

Divergence Thm: $\int \vec{\nabla}\cdot\vec{V}\, d\mathcal{V} = \oint \vec{V}\cdot d\vec{a}$

$$\vec{\nabla}\times(\nabla\times\vec{V}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{V}) - \nabla^2\vec{V}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$(\nabla^2 = \vec{\nabla}\cdot\vec{\nabla})$

Consider Gauss's Law(s) in free space $(q=0, I=0)$

$$\oint \vec{E}\cdot da = \int(\vec{\nabla}\cdot\vec{E})\, d\mathcal{V} = \underbrace{\int \frac{\rho}{\varepsilon_0}\, d\mathcal{V}}_{Q_{inc}/\varepsilon_0}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

free space, $\Rightarrow \nabla\cdot\vec{E} = 0$

Also: $\oint \vec{B}\cdot d\vec{a} = \int \vec{\nabla}\cdot\vec{B}\, d\mathcal{V} = 0$

$$\Rightarrow \vec{\nabla}\cdot\vec{B} = 0$$

General derivation of EM wave equation

Faraday's Law:

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Stokes Thm $\Rightarrow$ $\oint \vec{E}\cdot d\vec{s} = \oint (\vec{\nabla}\times\vec{E})\cdot d\vec{a}$

$$\& \quad -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\oint B\cdot d\vec{a} = \oint -\frac{d\vec{B}}{dt}\cdot d\vec{a}$$

$$\therefore \quad \vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$$

Ampere - Maxwell Law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt}$$

Stokes Thm $\Rightarrow$ $\oint \vec{B}\cdot d\vec{s} = \oint (\vec{\nabla}\times\vec{B})\cdot d\vec{a}$

$$\mu_0 I_{enc} = \oint \mu_0 \vec{J}\cdot d\vec{a}$$

$$\varepsilon_0\mu_0 \frac{d\Phi_E}{dt} = \varepsilon_0\mu_0 \int \frac{d\vec{E}}{d\vec{t}}\cdot d\vec{a}$$

$$\therefore \quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \varepsilon_0\mu_0\frac{dE}{dt}$$

General derivation of EM wave equation

	Integral Form	Differential Form
Gauss's Law	$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{inc}}{\varepsilon_0}$	$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$
Gauss's Law of Magnetism	$\oint \vec{B}\cdot d\vec{a} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$
Faraday's Law	$\int \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}$
Ampere - Maxwell Law	$\int \vec{B}\cdot d\vec{s} = \mu_0 I_{inc} + \varepsilon_0\mu_0\frac{d\Phi_E}{dt}$	$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \varepsilon_0\mu_0\frac{d\vec{E}}{dt}$

Free space: $\rho = 0$, $\vec{J} = 0$ $\Rightarrow$ $\vec{\nabla}\cdot\vec{E} = 0$, $\vec{\nabla}\cdot\vec{B} = 0$

$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt}, \quad \vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0\frac{d\vec{E}}{dt}$$

General derivation of EM wave equation

General form of Wave Equation in free space:

$\nabla \cdot E = 0$

$\nabla \cdot B = 0$

$\nabla \times E = -\frac{dB}{dt}$

$\nabla \times B = \mu_0 \varepsilon_0 \frac{dE}{dt}$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} \Rightarrow \underbrace{\vec{\nabla} \times (\vec{\nabla} \times \vec{E})}_{\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{E}}_{0}) - \nabla^2 \vec{E}} = -\frac{d}{dt}(\vec{\nabla} \times \vec{B}) = -\frac{d}{dt}\left(\mu_0 \varepsilon_0 \frac{dE}{dt}\right)$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \Rightarrow \underbrace{\vec{\nabla} \times (\vec{\nabla} \times \vec{B})}_{\vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2 \vec{B}} = \mu_0 \varepsilon_0 \frac{d}{dt} \underbrace{(\vec{\nabla} \times \vec{E})}_{-\frac{dB}{dt}}$$

$$\Rightarrow \boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

Each component of $\vec{E}$ + $\vec{B}$ obey wave equation: Consider $\vec{E} = \hat{x} E_x(x,y,z,t) + \hat{y} E_y(x,y,z,t) + \hat{z} E_z(x,y,z,t)$

$$\nabla^2 \vec{E} = \hat{x} \frac{\partial^2}{\partial x^2} E_x + \hat{y} \frac{\partial^2}{\partial y^2} E_y + \hat{z} \frac{\partial^2}{\partial z^2} E_z$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = \hat{x} \frac{\partial^2}{\partial t^2} E_x + \hat{y} \frac{\partial^2}{\partial t^2} E_y + \hat{z} \frac{\partial^2}{\partial t^2} E_z$$

$$\therefore \frac{\partial^2 E_x}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad \text{etc...}$$

Same for B components

General derivation of EM wave equation

Plane wave solutions:

Assume ① $\vec{E}$ only pts. in one direction everywhere ② $|\vec{E}|$ only varies with time + position x $\Big\}$ $\vec{E} = \hat{y}\, E_y(x,t)$

$$\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{d\vec{E}}{\partial t} \quad + \quad \vec{E} = \hat{y}\, E_y(x,t) \Rightarrow \vec{B} = \hat{z}\, B_z(x,t)$$

Big Assumptions! However, superposition of plane wave solutions that have various amplitudes, ~~wavelengths~~ frequencies, + direction of travel will reproduce any possible solution
(Exactly like any time domain waveform can be thought of as a superposition of Sin's + Cos's of various Amplitudes + frequencies; Principle of Fourier Analysis!)

$$\Rightarrow \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad + \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

Looks like a wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 y}{\partial t^2}$$

wave on a string $\Big] \Rightarrow \boxed{V = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \cdot 10^8 \frac{m}{sec}}$

Same as light!!!