K2-04: FARADAY'S EXPERIMENT - EME SET - 20, 40, 80 TURN COILS

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K2-43: LENZ'S LAW - PERMANENT MAGNET AND COILS

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K4-08: MAGNETOELECTRIC GENERATOR WITH INDUCTOR

Coursemail

- If you are not receiving the class e-mails, please give me an e-mail address
- Class e-mails are posted on the website – see the “Solutions/Class e-mails tab” link

Homework set #2

- Due Tuesday by 5PM
- No late homework accepted

Quiz #2

- Next week during discussion

Sections 34.8-34.10 will be covered on Hwk and Quiz #3

Last time

Biot-Savart Law examples:

Found B-fields for

1. Center of current arc → Circle,
2. On axis of current loop

$$\vec{B}_{center} = \frac{\mu_0 I}{2a}$$

Current loops are “magnetic dipoles”

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{x^3} \text{ on axis} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{x^3}$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{B} \qquad \vec{\tau}_E = \vec{p} \times \vec{E}$$

$$U_B = -\vec{\mu} \cdot \vec{B} \qquad U_E = -\vec{p} \cdot \vec{E}$$

Ampere’s Law:

“derived” Ampere’s Law for 2-D loops and infinite current carrying wire

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through}$$

Used Ampere’s Law to find B-fields for:

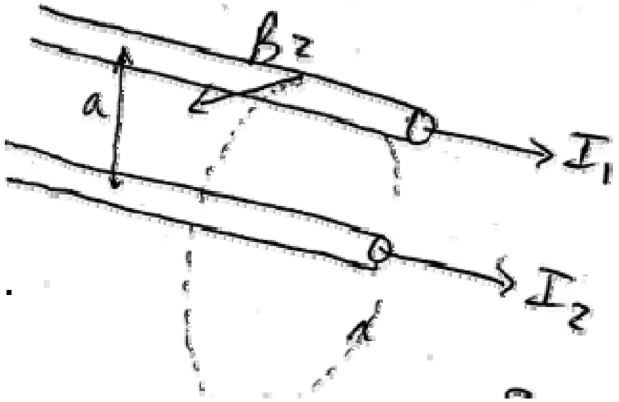
1. Infinite straight wire (inside and outside of wire)
2. Long solenoid – B uniform inside
3. Torus

Last time

Force between two straight wires:

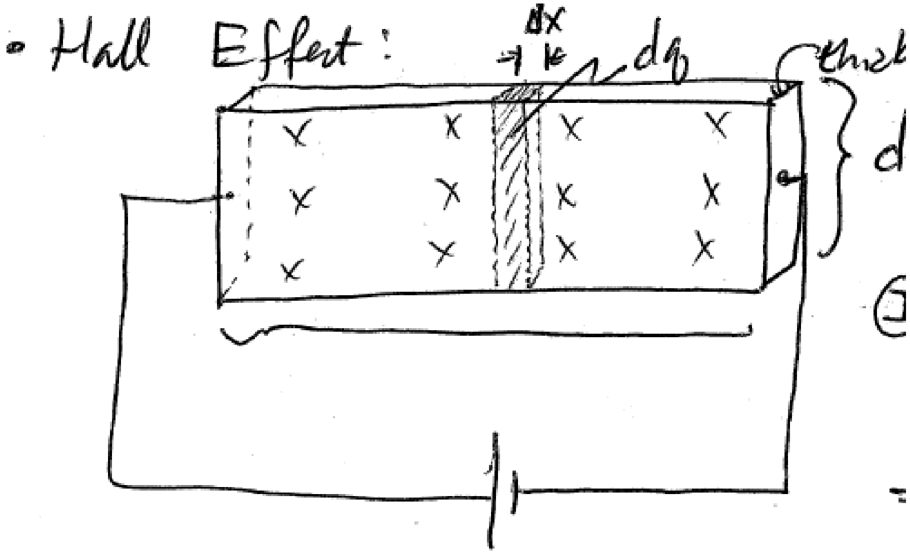
current in from an infinite wire produces a B-field, and the second wire with a current in it feels a force when placed inside the B-field

$$F_1 = I_1 l_1 \frac{\mu_0 I_2}{2\pi a} = \boxed{\frac{\mu_0}{2\pi a} I_1 I_2}$$



Parallel currents attract, Opposite currents repel.

DC Hall Effect



If charges positive, "+" charges on top

If charges negative, "-"charges on top

Hall voltage gives sign and density of charge carriers

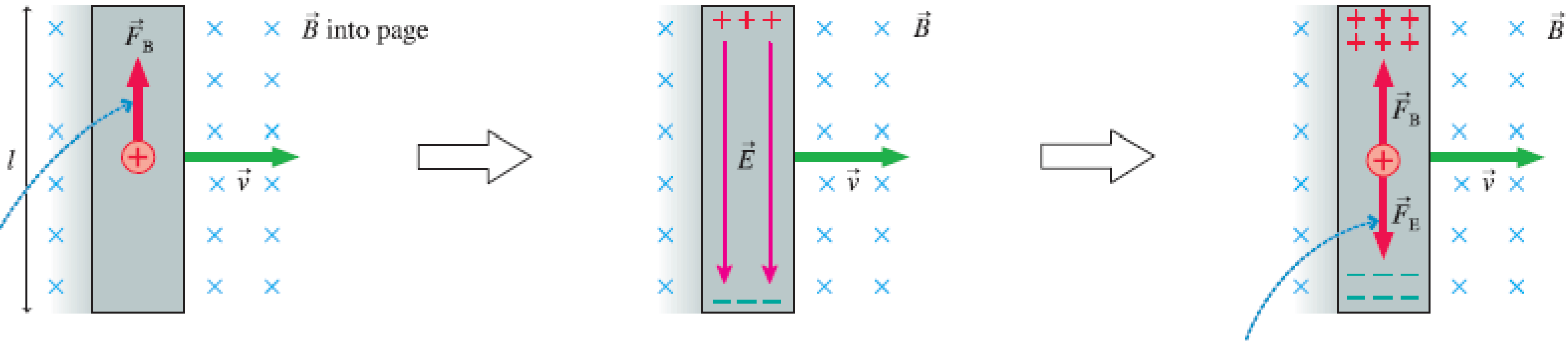
$$\Delta V_H = B\cdot d\left[\frac{I}{n q_0 d z}\right] = \boxed{\frac{IB}{n q_0 z}}$$

Chapter 34. Electromagnetic Induction

Electromagnetic induction is the scientific principle that underlies many modern technologies, from the generation of electricity to communications and data storage.

Before we begin topic of magnetic induction, apply what we learned from chapter 33: Motional emf

FIGURE 34.2 The magnetic force on the charge carriers in a moving conductor creates an electric field inside the conductor.



Charge carriers in the wire experience an upward force of magnitude $F_B = qvB$. Being free to move, positive charges flow upward (or, if you prefer, negative charges downward).

The charge separation creates an electric field in the conductor. $\vec{E}$ increases as more charge flows.

The charge flow continues until the downward electric force $\vec{F}_E$ is large enough to balance the upward magnetic force $\vec{F}_B$. Then the net force on a charge is zero and the current ceases.

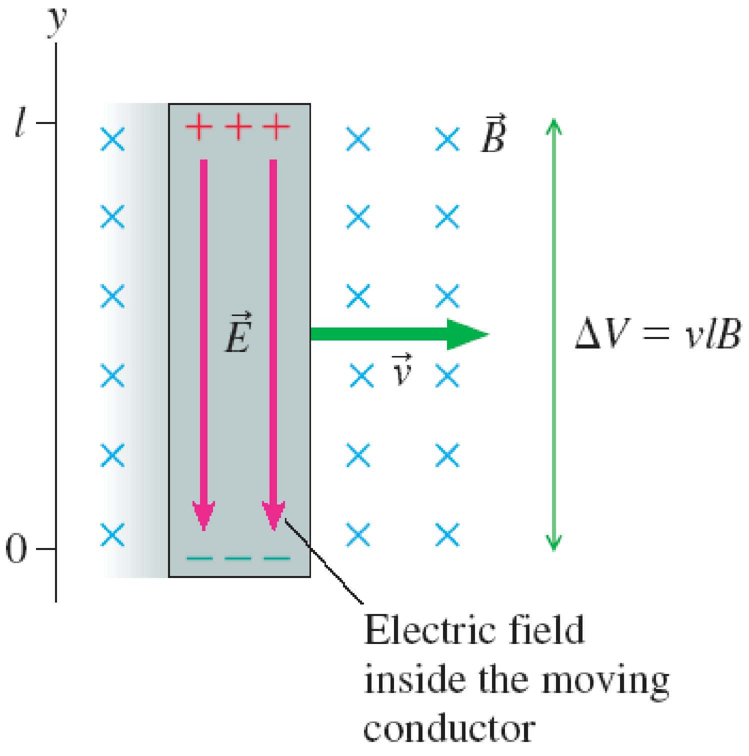
$$F_B = qvB$$

$$F_E = qE$$

$$E = vB$$

FIGURE 34.3 Two different ways to generate an emf.

(a) Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.



$F_B = F_E \Rightarrow qvB = qE$ or

$E = vB$

For uniform E-field:

$|\Delta V| = \int \vec{E} \cdot d\vec{s} = E\ell$

or $\Delta V = vB\ell$

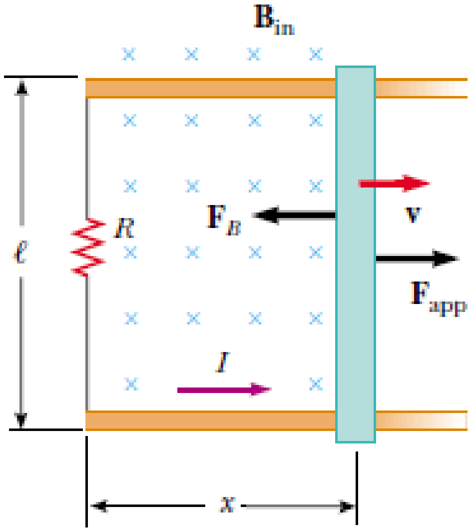
Motional emf

The motional emf of a conductor of length *l* moving with velocity *v* perpendicular to a magnetic field *B* is

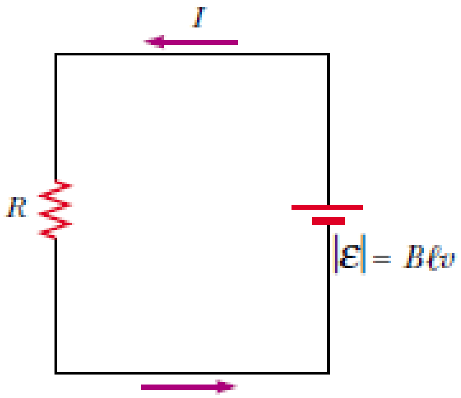
$$\mathcal{E} = vlB$$

A simple Generator

$$\mathcal{E} = vlB \qquad I = \frac{|\mathcal{E}|}{R} = \frac{B\ell v}{R}$$



(a)



$\vec{F}_B = I\vec{\ell}\times\vec{B} = I\ell B$ to left

$$\Rightarrow |F_B| = \frac{B\ell v}{R}\ell B = \frac{(B\ell)^2 v}{R}$$

For Constant velocity: $|F_B| = |F_{app}|$

Recall: $W = \int \vec{F}_{app}\cdot d\vec{x}$

$$\text{Power, } P = \frac{dW}{dt} = \int \vec{F}_{app}\cdot\frac{d\vec{x}}{dt} = \int \vec{F}_{app}\cdot\vec{v}$$

In our Example, $\vec{F}_{app} \parallel \vec{v}$ & $|\vec{F}_{app}|$ & $|\vec{v}|$ Constant

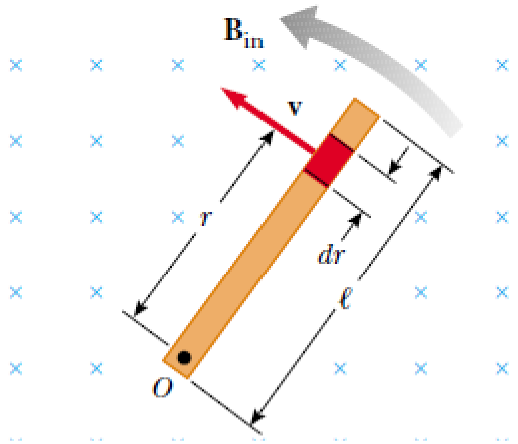
$$\therefore \quad P = F_{app}v = \frac{(B\ell v)^2}{R}$$

Power through Resistor:

$$P = I^2R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{(B\ell v)^2}{R}$$

Mechanical power put into circuit = Electrical power put into circuit

Rotating conducting bar in magnetic field



A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation, as shown in Figure 31.11. Find the motional emf induced between the ends of the bar.

Let dq be charge within element dr

In equilibrium:

$$F_E = F_B \Rightarrow \delta q\, dE = \delta q\, vB$$

Where dE is electric field across dr (|| to bar!)

$$v = r\omega \Rightarrow dE = r\omega B$$

Recall for uniform E-field: $|\Delta V| = Ed$

$$\therefore\ dV = dE\, dr = \omega B r\, dr$$

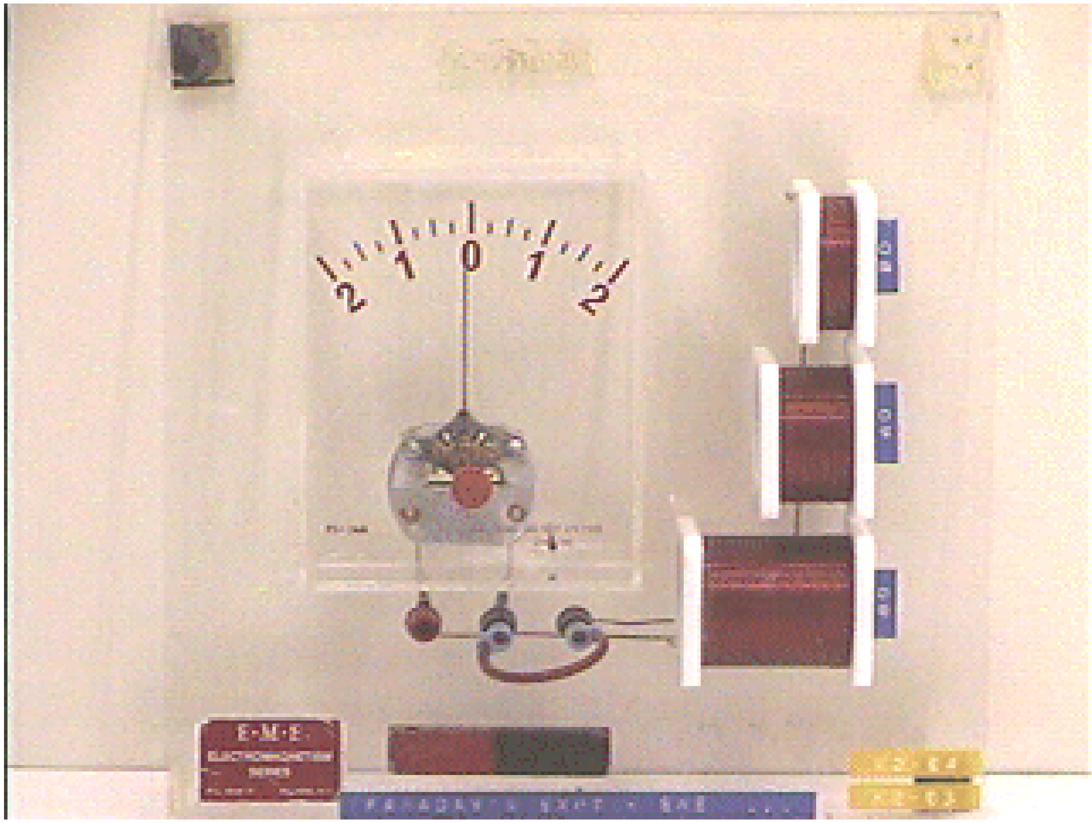
$$\Rightarrow \Delta V = \omega B \int_0^{\ell} r\, dr = \frac{\ell^2}{2}\omega B$$

Faraday’s Discovery

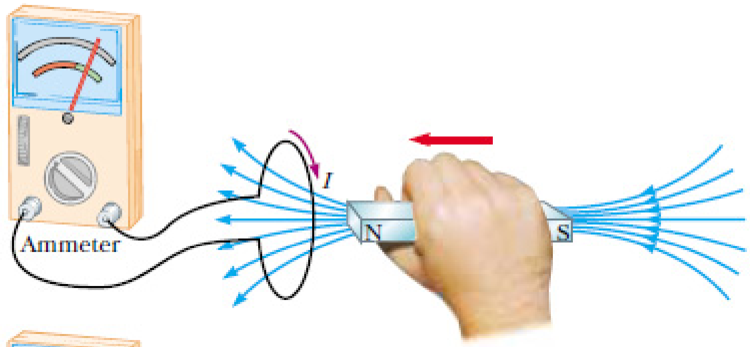
Faraday found that there is a current in a coil of wire if and only if the magnetic field passing through the coil is *changing.* This is an informal statement of *Faraday’s law.*

A more formal definition will follow involving the magnetic flux through areas….

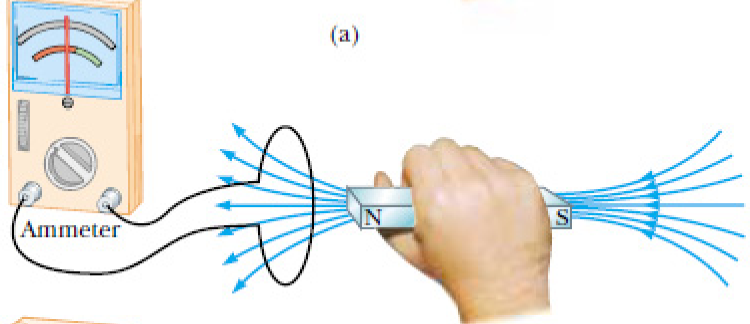
K2-04: FARADAY'S EXPERIMENT - EME SET - 20, 40, 80 TURN COILS



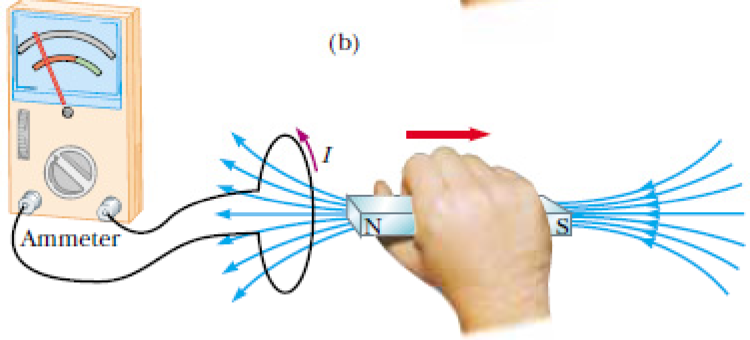
K2-04: FARADAY'S EXPERIMENT - EME SET - 20, 40, 80 TURN COILS



(a)



(b)



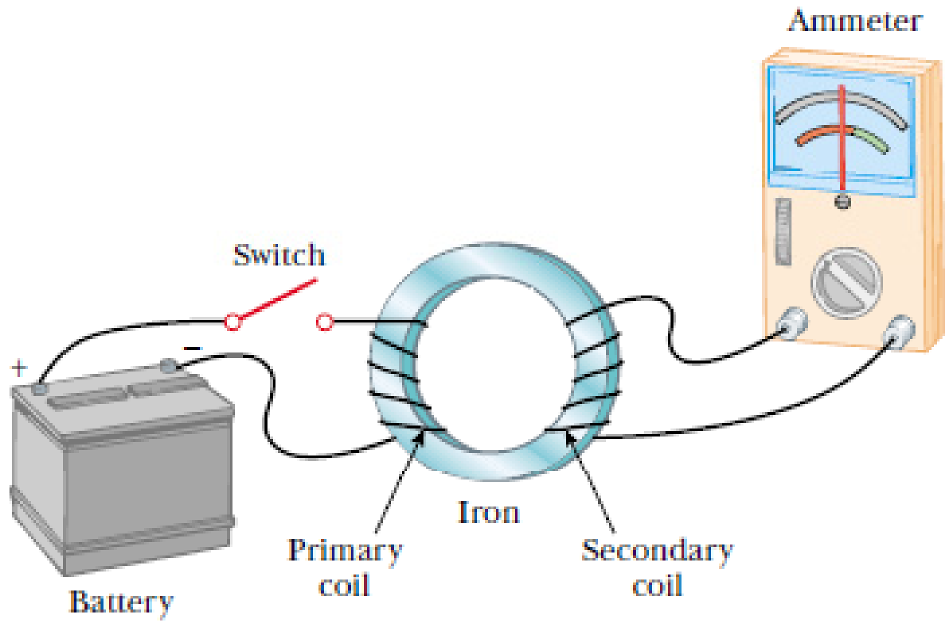
(c)

Only when B-field through loop is changing does a current flow through loop.

Faster the movement, the more current.

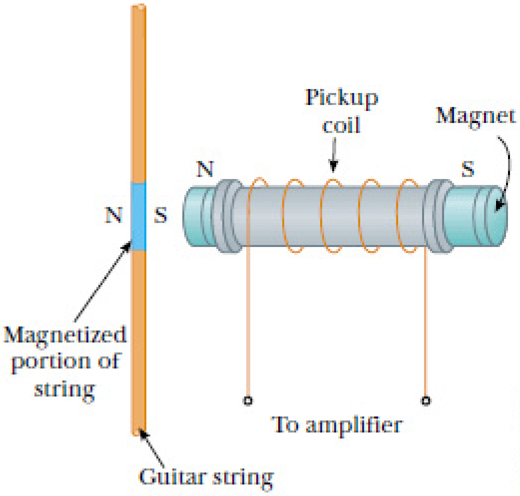
Current changes direction when either motion is reversed or polarity of magnet is reversed

Deflection depends proportionately on number of turns, N.

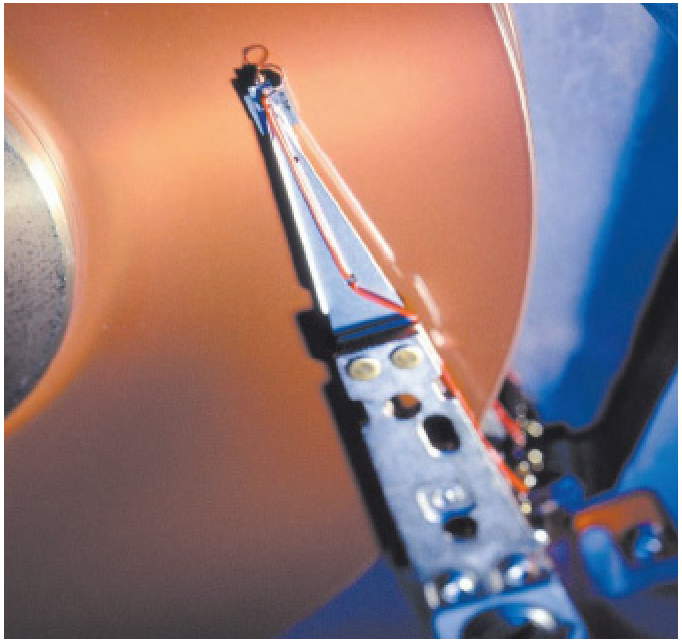


Primary coil produces a B-field which goes through the secondary coil.

Only when B-field through secondary coil is changing (produced by a changing current in primary coil) does a current flow through loop.



Charles D. Winters

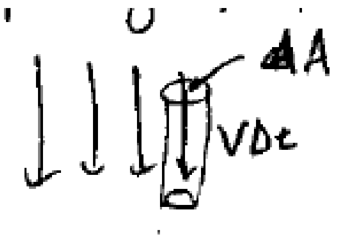


Magnetic data storage encodes information in a pattern of alternating magnetic fields. When these fields move past a small *pick-up coil,* the changing magnetic field creates an induced current in the coil. This current is amplified into a sequence of voltage pulses that represent the 0s and 1s of digital data.

Concept of Flux: Photon flux through a hoop

This concept should be familiar from Gauss's Law: $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$

Consider photons coming from the sun straight down:



avg # density of photons = n

So, avg # in cylinder is the avg # which will pass through ΔA in time Δt

Φ = # of photons passing through area ΔA in time Δt = $\frac{\Delta N}{\Delta t}$

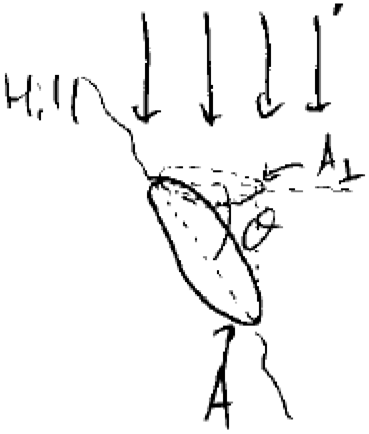
$$\Delta N = n \cdot \Delta A \, v \, \Delta t \Rightarrow \Phi_{\Delta A} = n \, v \, \Delta A$$

Consider big hoop on the ground of Area A

$$\Rightarrow \Phi_A = n \, v \, A$$

Concept of Flux: Photon flux through a hoop

Consider photons coming from the sun straight down through
A hoop at some angle theta:



Less photon through hoop even though it is same area, So orientation matters!

the flux through A @ angle same as flux through $A_\perp$,

$A_\perp =$ cross sectional area

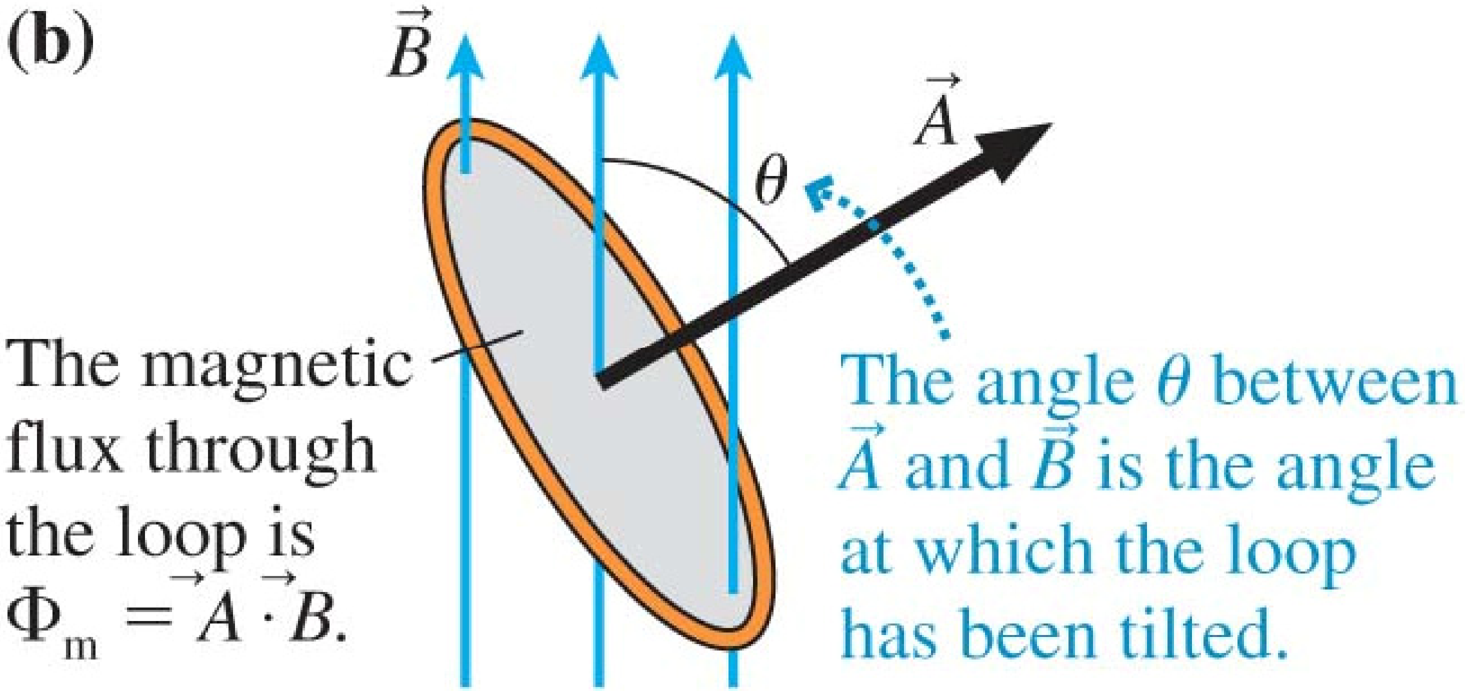
$= |A| \cos\theta$

So $\Phi_{A\,@\,\text{angle}\,\theta} = \Phi_{A_\perp} = n\,v\,A\cos\theta$

$= n(\vec{V}\cdot\vec{A})$

where $\vec{A}$ is "normal" to loop:

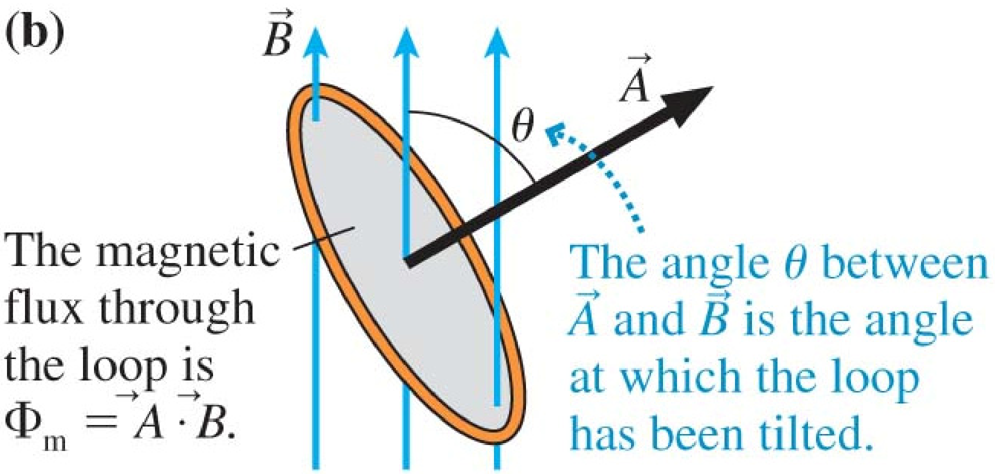
Magnetic flux can be defined in terms of an area vector



The magnetic flux through the loop is $\Phi_m = \vec{A} \cdot \vec{B}$.

The angle θ between $\vec{A}$ and $\vec{B}$ is the angle at which the loop has been tilted.

For constant magnetic field and a 2-D loop.
Definition of Magnetic Flux



The magnetic flux through the loop is $\Phi_m = \vec{A} \cdot \vec{B}$.

The angle θ between $\vec{A}$ and $\vec{B}$ is the angle at which the loop has been tilted.

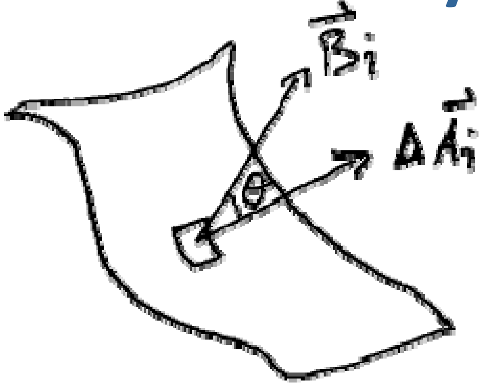
$$\Phi_m = A_{\mathrm{eff}} B = AB\cos\theta$$

The magnetic flux measures the amount of magnetic field (proportional to the net number of field lines) passing through a loop of area *A* if the loop is tilted at an angle ϑ from the field, *B*. As a dot-product, the equation becomes:

$$\Phi_m = \vec{A} \cdot \vec{B}$$

Note that contributions can be positive and negative!

Magnetic flux for an arbitrary magnetic field and some arbitrary surface.

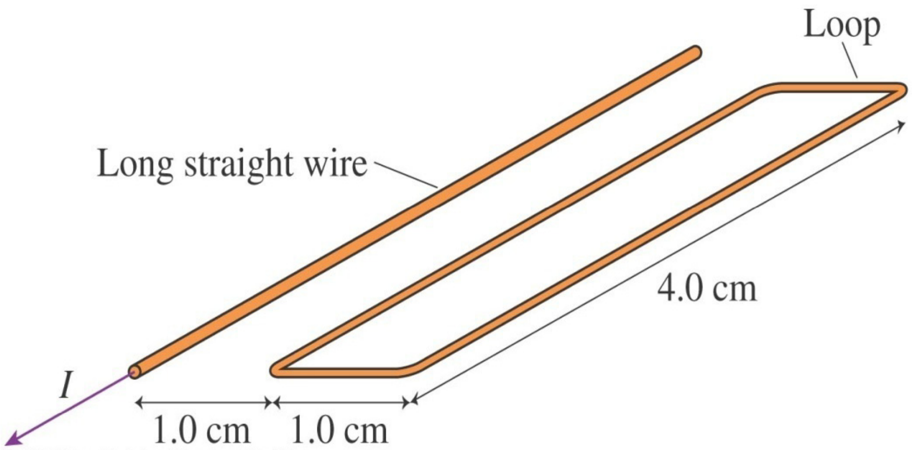


$$\Delta \Phi_{B_i} = \vec{B}_i \cdot \Delta \vec{A}_i \Rightarrow$$

$$\Phi_B \approx \sum_i \vec{B}_i \cdot \Delta \vec{A}_i$$

$$\Phi_B = \int_{\text{Surface}} \vec{B} \cdot d\vec{A}$$

Magnetic flux from the current in a long straight wire



Magnetic field from long straight wire:

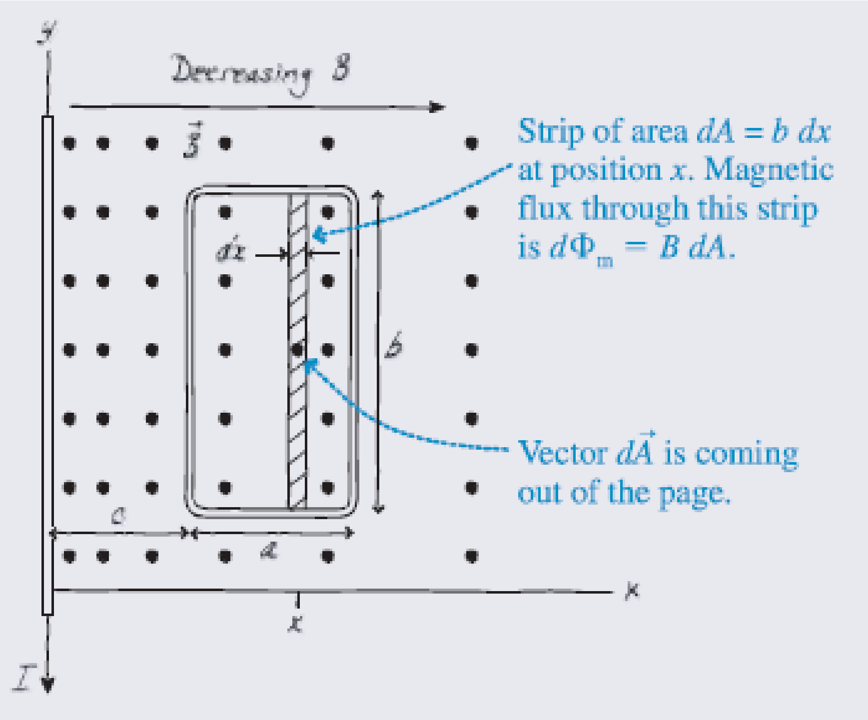
$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} \Rightarrow \int_0^{2\pi} B r\,d\theta = \mu_0 I$

Amperian loop

Direction from RHR.

$\vec{B} \parallel d\vec{s}$, $ds = r\,d\theta$

$\therefore \vec{B} = \frac{\mu_0}{2\pi}\frac{I}{r}\hat{\theta}$

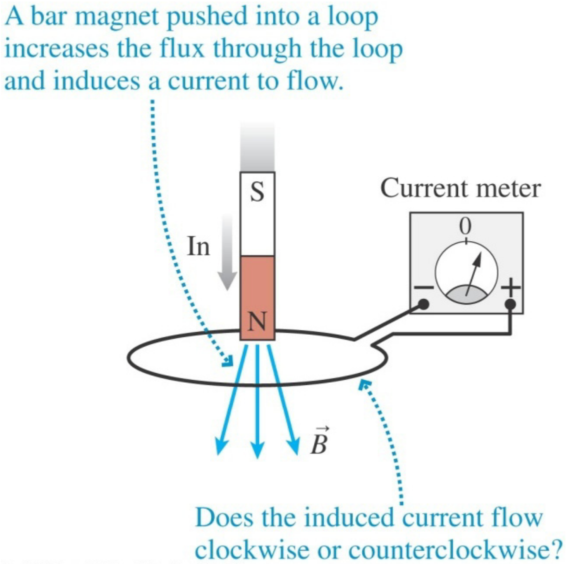


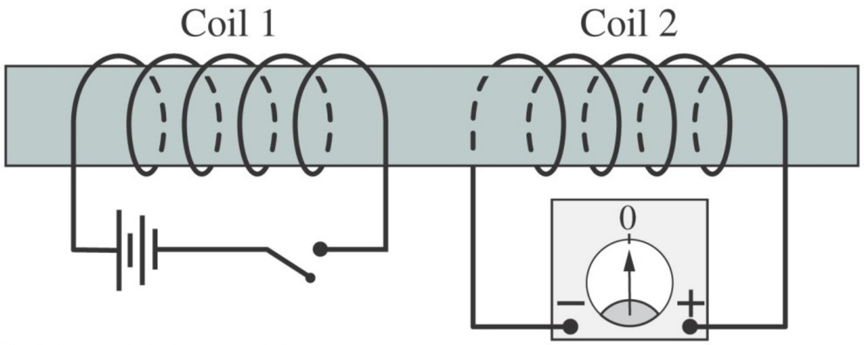
$$d\Phi_m = \vec{B}\cdot d\vec{A} = B\,dA = Bb\,dx = \frac{\mu_0 I b}{2\pi x}\,dx$$

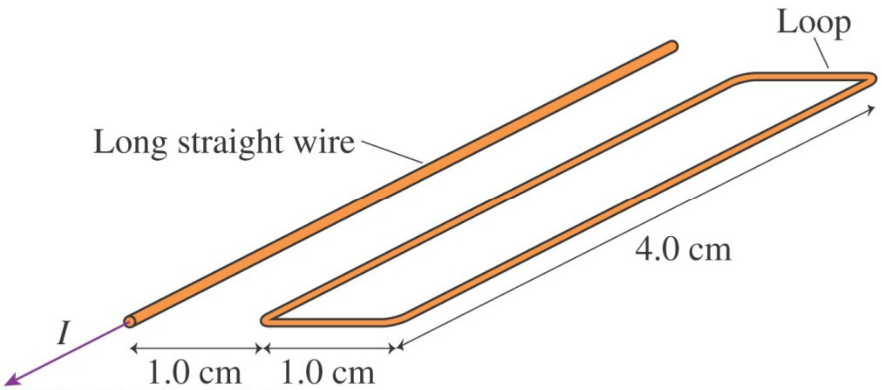
$$\Phi_m = \frac{\mu_0 I b}{2\pi}\int_c^{c+a}\frac{dx}{x} = \frac{\mu_0 I b}{2\pi}\ln x\Big|_c^{c+a} = \frac{\mu_0 I b}{2\pi}\ln\left(\frac{c+a}{c}\right)$$

Lenz's Law:

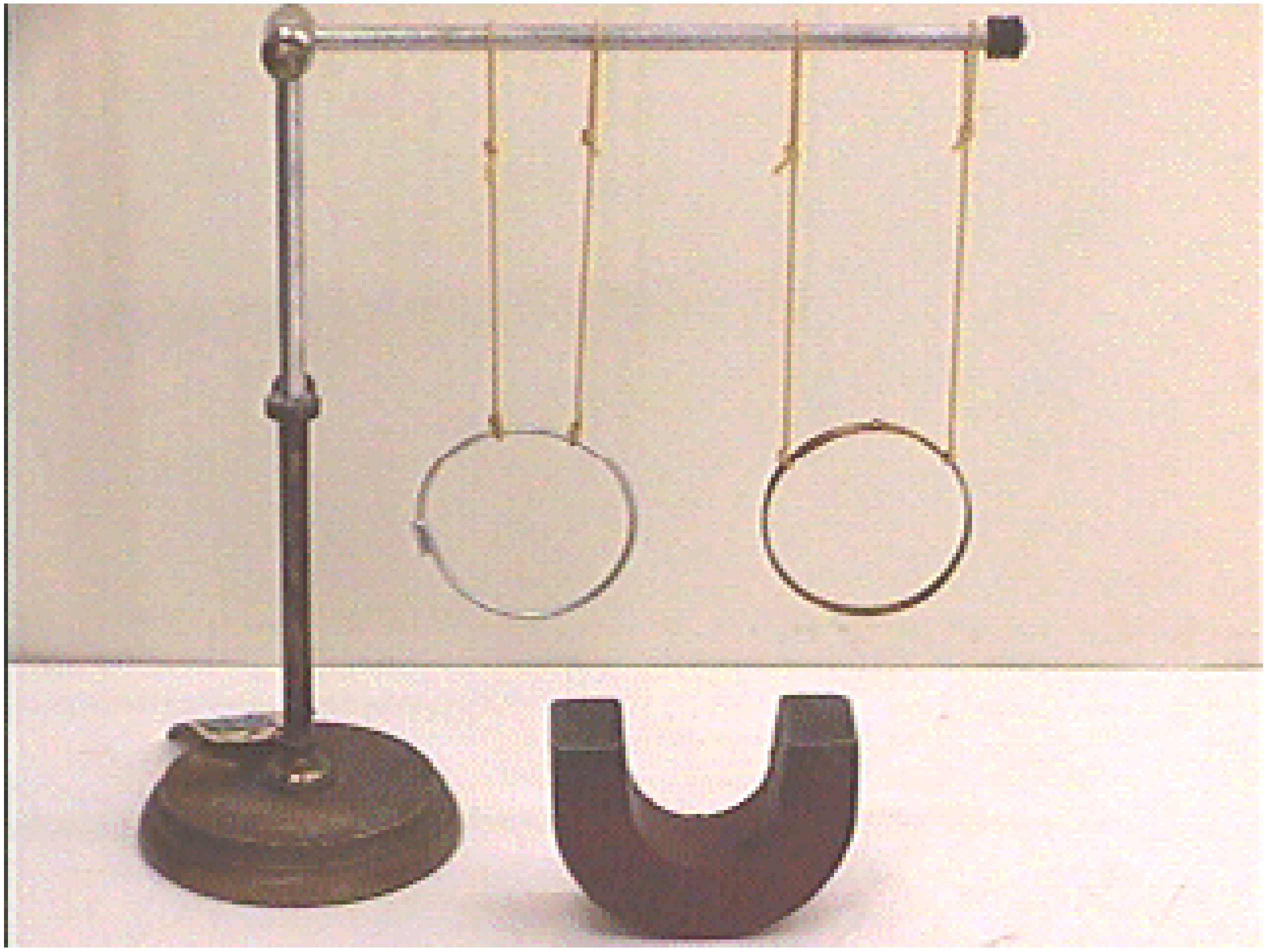
Lenz's law There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.







K2-43: LENZ'S LAW - PERMANENT MAGNET AND COILS



Faraday's Law:

Faraday's law An emf $\mathcal{E}$ is induced around a closed loop if the magnetic flux through the loop changes. The magnitude of the emf is

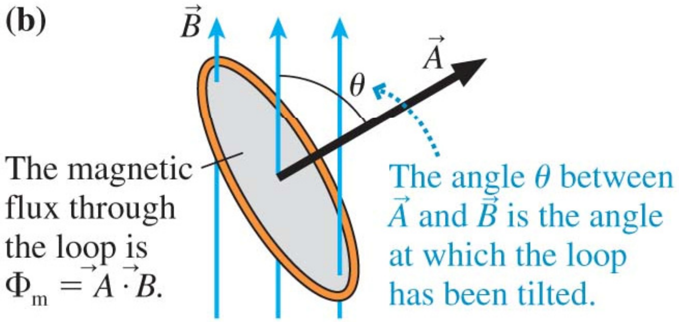
$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| \qquad (34.14)$$

and the direction of the emf is such as to drive an induced current in the direction given by Lenz's law.

emf is the same thing as voltage.

Can change the flux through a loop three ways:

1. Change the size of the loop
2. Change the strength of magnetic field
3. Change the orientation of the loop



$$\Phi_m = A_{\text{eff}}B = AB\cos\theta$$

Direction of the induced current (same as the direction of the emf) will be 1 of two directions in the loop. "Lenz's Law" gives the direction.

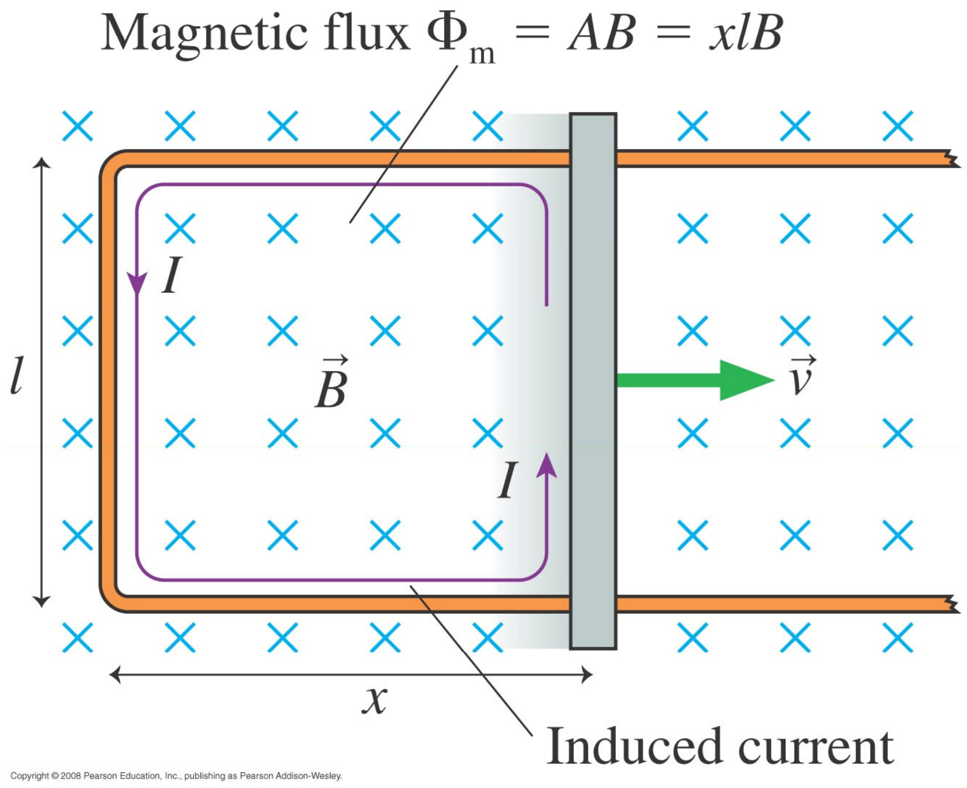
Faraday’s Law: Multiple turns (N-loops)

$$\mathcal{E}_{\text{coil}} = N\left|\frac{d\Phi_{\text{per coil}}}{dt}\right| \quad \text{(Faraday's law for an } N\text{-turn coil)}$$

Since each coil is ‘wired up’ serially, it is exactly like wiring up series batteries.

If you wire up N batteries of voltage V, the total voltage is N x V.

Faraday's Law example – Changing area of loop

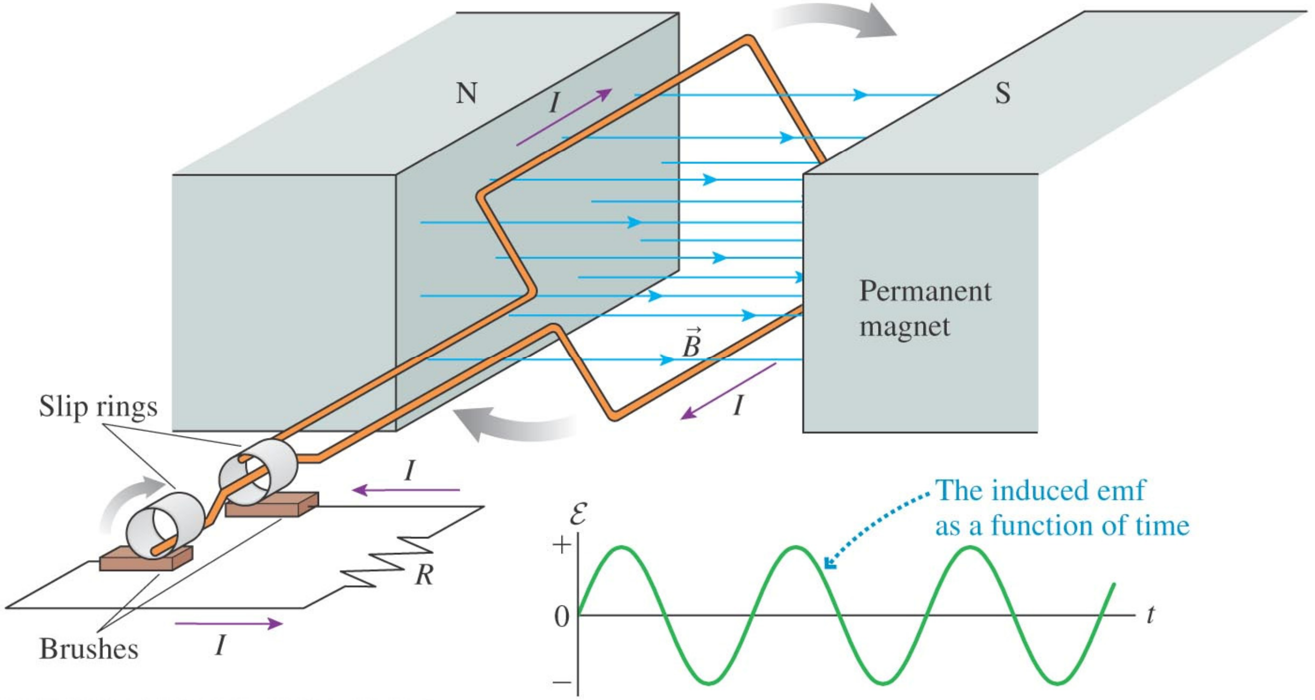


$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| = \frac{d}{dt}(xlB) = \frac{dx}{dt}lB = vlB$$

$$I = \frac{\mathcal{E}}{R} = \frac{vlB}{R}$$

Faraday's Law example – Changing Orientation of Loop: Generators

Assume loop rotates at constant angular frequency:



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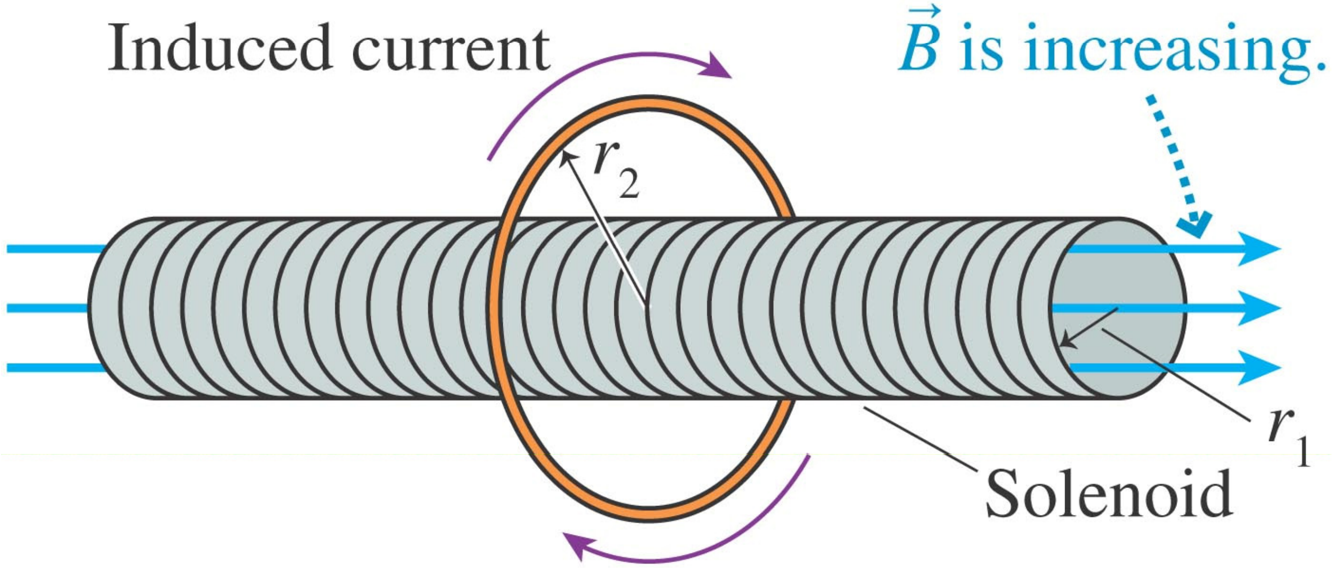
$$\Phi_{\mathrm{m}} = \vec{A} \cdot \vec{B} = AB\cos\theta = AB\cos\omega t$$

$$\mathcal{E}_{\mathrm{coil}} = -N\frac{d\Phi_{\mathrm{m}}}{dt} = -ABN\frac{d}{dt}(\cos\omega t) = \omega ABN\sin\omega t$$

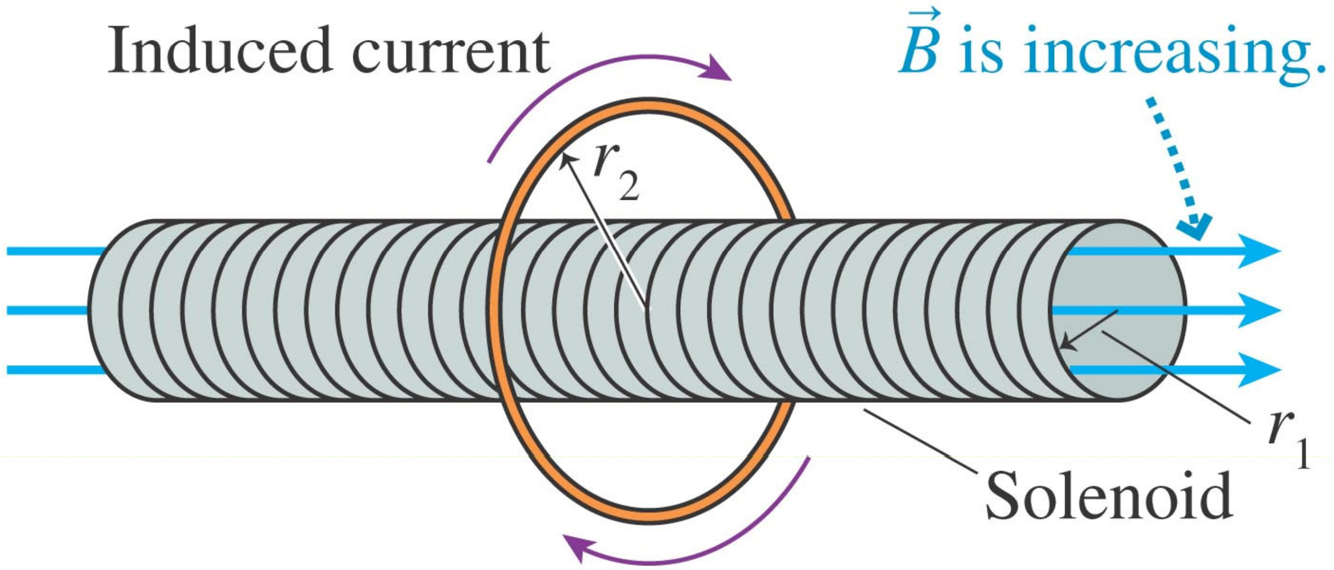
Faraday’s Law – New Physics



B=0 outside the infinite solenoid. Faraday’s law states that a current will flow in the hoop if the B-field is changing in solenoid. How do the charges in the hoop ‘know’ the flux is changing? There MUST be something causing the charges to move, and it is NOT directly related to the B-field like motional emf since the charges are initially stationary in the loop.

Wherever there is voltage (emf), there is an E-field. A time varying B-field evidently causes an E-field (even outside the solenoid) which push the charge in the hoop. However, the E-field still exists even outside the hoop!

E-field driving the current in hoop – induced E-field

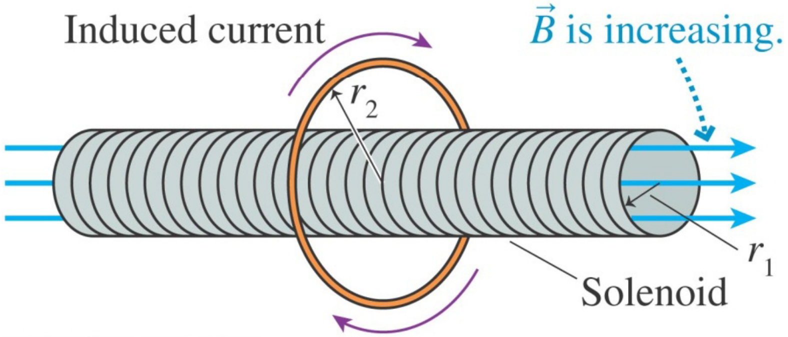


Radial E has to be zero everywhere:

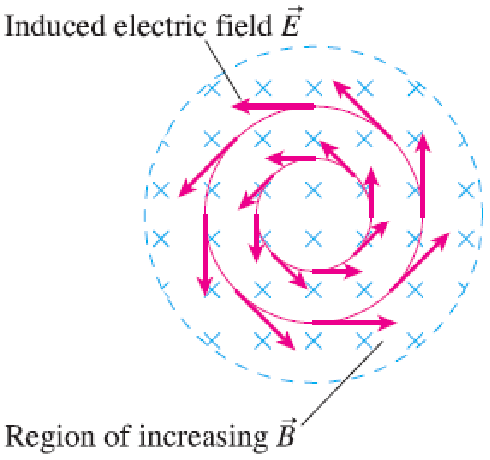
> If we reverse current, we expect E radial to change direction. However, we must end up with the above picture if we reverse current AND flip the solenoid over 180 degrees. Therefore, E radial must be zero.

Since charge flows circumferentially, there must be a tangential component of E-field pushing the charge around the ring. **This E-field exists even without the ring.**

Induced E-field



To push a test charge q around the ring (or along the same path without the ring!) requires work.



$$W = q \underbrace{\Delta V}_{\equiv \mathcal{E}\text{, Voltage to go fully around ring}} = q \oint \vec{E} \cdot d\vec{s}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = \underset{\substack{\uparrow \\ \text{Lenz's Law}}}{-} \frac{d\Phi_B}{dt}$$

K2-62: CAN SMASHER - ELECTROMAGNETIC

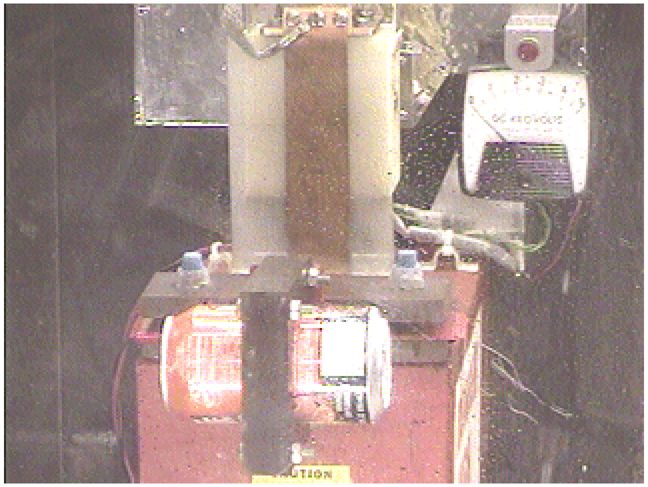


FIGURE 34.35 Maxwell hypothesized the existence of induced magnetic fields.

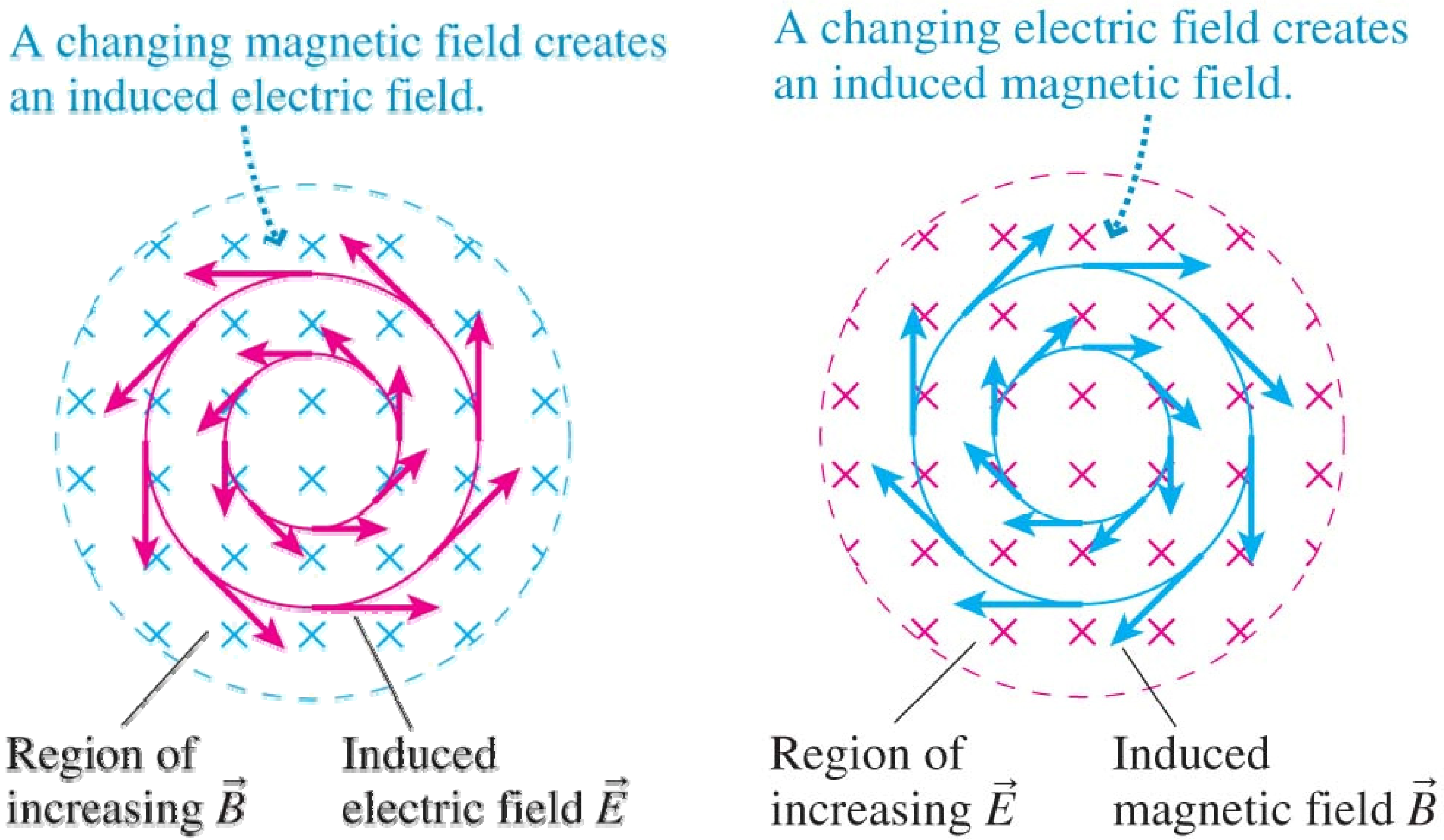
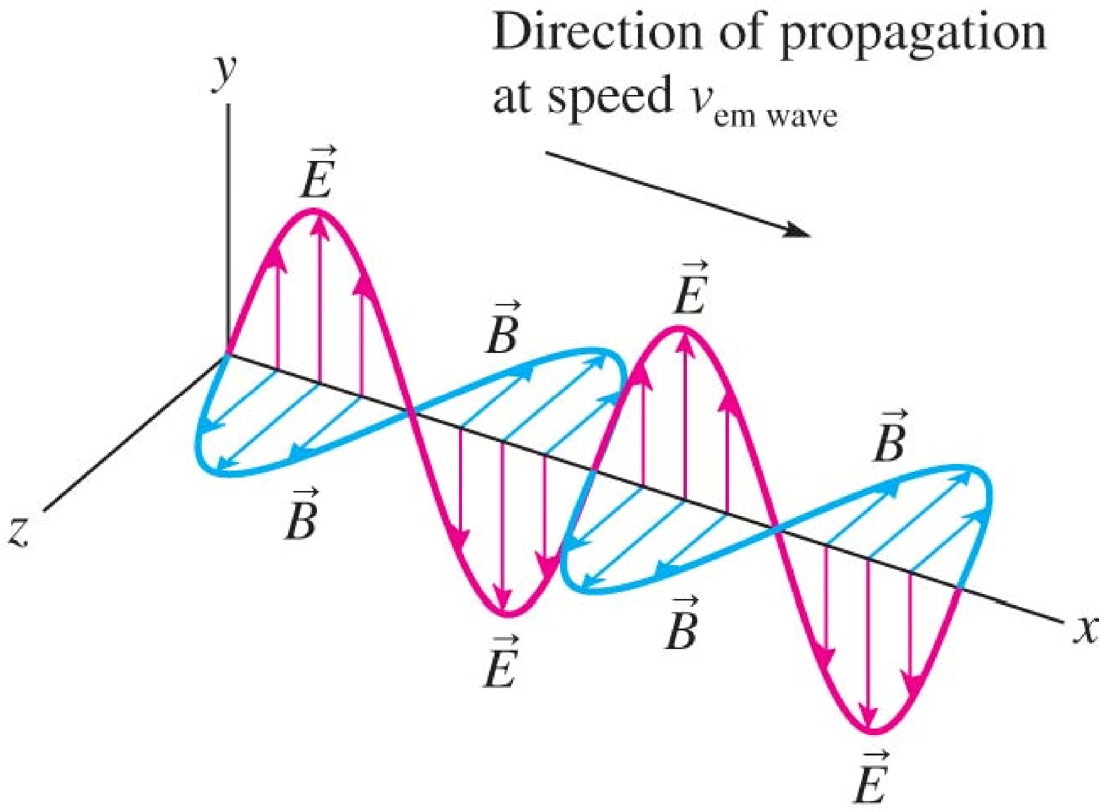
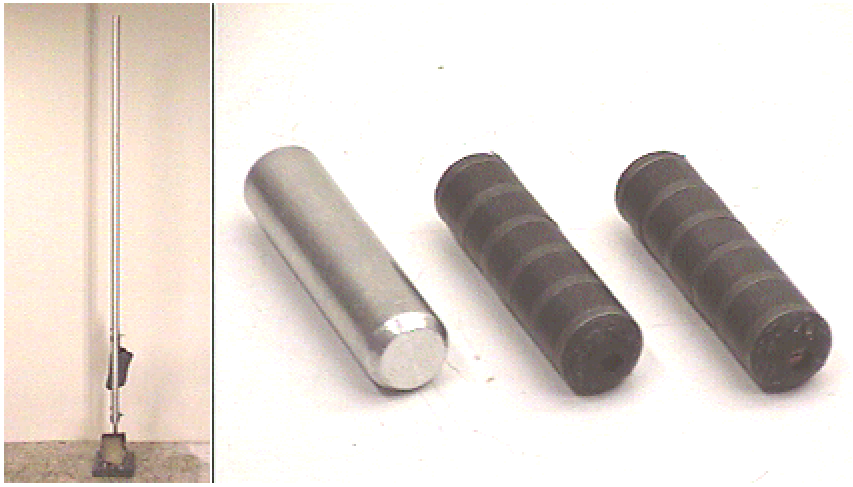


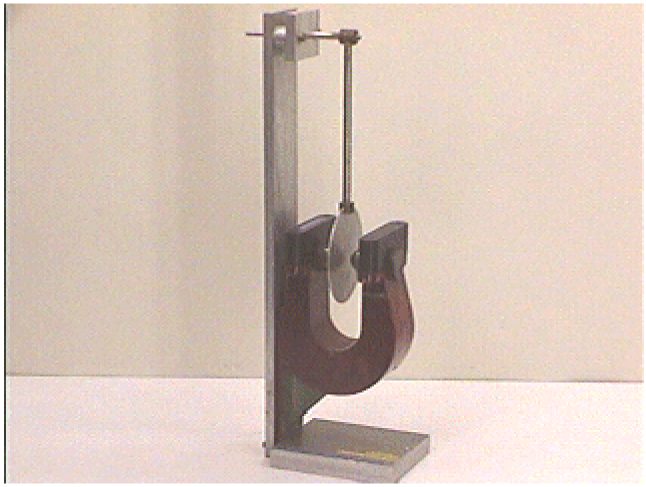
FIGURE 34.36 A self-sustaining electromagnetic wave.



Eddy Currents

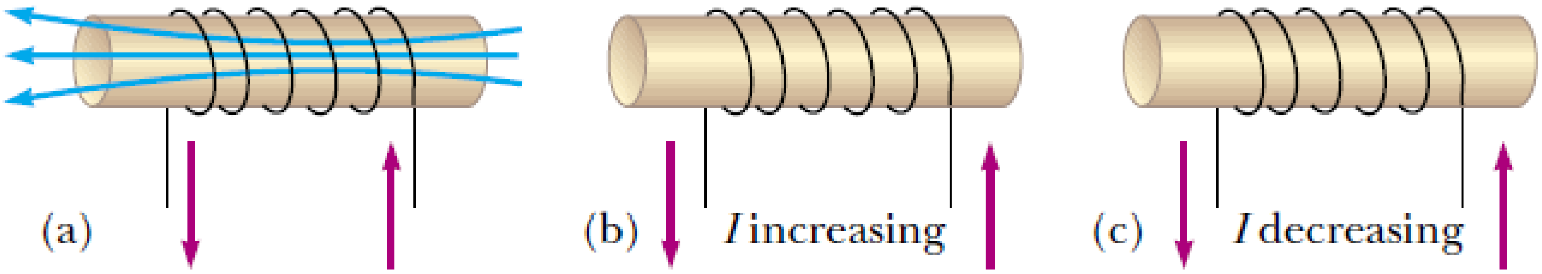


K2-42: LENZ'S LAW - MAGNET IN ALUMINUM TUBE



K2-44: EDDY CURRENT PENDULUM

Back emf and inductors

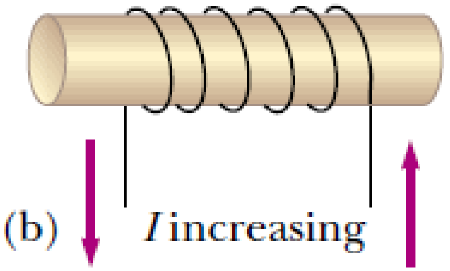


(a) Steady current, magnetic field is to the left
(b) Current increasing, magnetic field increasing to the left. Lenz's law states that an emf is set-up to oppose this increasing flux, thus creating a voltage that opposes the increasing current.

Considering case (b) for the case of argument, the back emf from Faraday's Law (we do not assume an infinite solenoid here!)

Back emf and inductors

Considering case (b) for the case of argument, the back emf from Faraday's Law (we do not assume an infinite solenoid here!)



$$|\mathcal{E}_{back}| = -N\frac{d\Phi}{dt} \; ; \quad \Phi = \int \vec{B}\cdot d\vec{A} \propto B$$

(That is, if you double B everywhere, you double Φ)

& $B \propto I$ (Biot-Savart Law: $B = \frac{\mu_0}{4\pi} I \int \frac{d\vec{s}\times\hat{r}}{r^2}$)

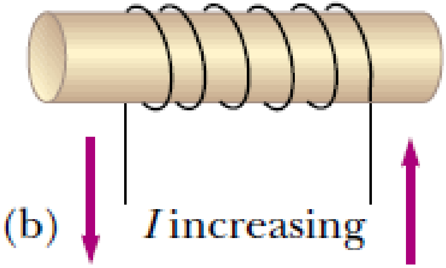
$$\therefore \; |\mathcal{E}_{back}| \propto \frac{dI}{dt}$$

Define the self inductance L as the proportionality constant:

$$\Delta V_L = -L\frac{dI}{dt}$$

Back emf and inductors

Considering case (b) for the case of argument, the back emf from Faraday's Law (we do not assume an infinite solenoid here!)



For infinite solenoid:

$$B = \mu_0 \frac{N}{\ell} I \quad \& \quad \mathcal{E}_{back} = -N \frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

$$\frac{d\Phi}{dt} = \frac{d}{dt}(\vec{B}\cdot\vec{A}) = A \mu_0 \frac{N}{\ell} \frac{dI}{dt}$$

$$\therefore L = A \mu_0 \frac{N^2}{\ell}$$

Energy stored in B-field

$$|\mathcal{E}| = L\frac{dI}{dt}$$

Work dW required to push a little more charge dq through inductor in time dt

$$\frac{dW}{dt} = \underbrace{\frac{dq}{dt}\,\mathcal{E}}_{dW = dq\,\mathcal{E}} = L\,I\,\frac{dI}{dt}$$

or $P = I\mathcal{E}$

Work Required to ramp Current up to "I"

$$\Rightarrow dW = L\,I\,dI \quad \text{or} \quad W = \int_0^I L\,I'\,dI' = \frac{1}{2}L I^2$$

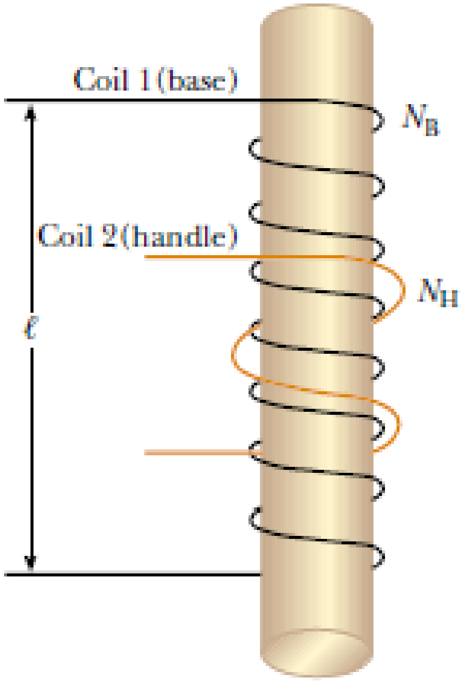
For infinite Solenoid: $L = A\mu_0 N^2/\ell$, $B = \mu_0 \frac{N}{\ell} I$

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\frac{A\mu_0 N^2}{\ell}\left(\frac{\ell}{\mu_0 N}\right)^2 B^2 = \frac{1}{2}\frac{A\ell}{\mu_0} B^2$$

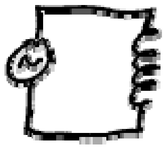
$u_B = \frac{PE}{\text{Volume}} = \frac{1}{2\mu_0}B^2$ where $A\ell$ is volume of Solenoid

Recall for ||-plate capacitor: $u_E = \frac{\varepsilon_0}{2}E^2$

Transformer



Drive coil ①: $\mathcal{E}_1(t) = N_1 \frac{d\Phi_1}{dt}$; Φ_1 is flux per coil

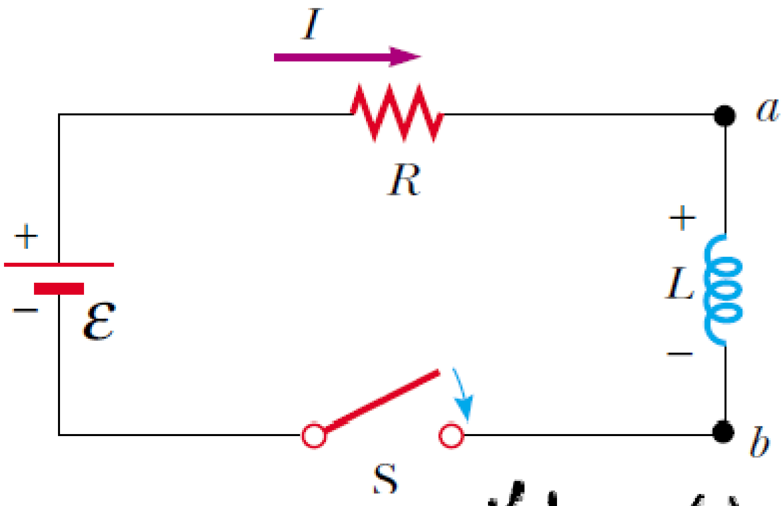
(Note: $\mathcal{E}_{Applied} = \mathcal{E}_{back}$ )

$\mathcal{E}_2 = N_2 \frac{d\Phi_2}{dt}$, Φ_2 is flux per coil

$\Phi_1 = \Phi_2 \Rightarrow \frac{d\Phi_1}{dt} = \frac{d\Phi_2}{dt}$

$\therefore \quad \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \Rightarrow \mathcal{E}_2 = \frac{N_2}{N_1}\mathcal{E}_1$

RL Circuits



Charging:

1. Throw switch at t=0, current is zero since it must ramp up gradually (back emf)
2. As t-> infinity, dI/dt->0, and all voltage across resistor

Kirchoff's loop rule: $\mathcal{E} - IR - L\frac{dI}{dt} = 0$

Let $x(t) = \frac{\mathcal{E}}{R} - I(t) \Rightarrow \frac{dx}{dt} = -\frac{dI}{dt}$

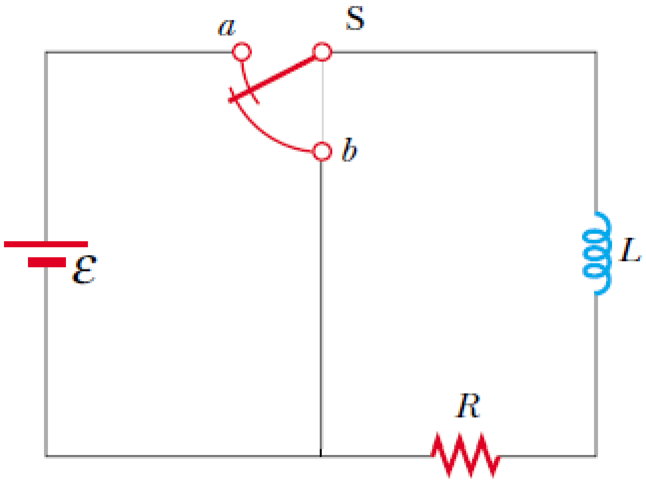
$\therefore \; x + \frac{L}{R}\frac{dx}{dt} = 0 \Rightarrow x(t) = x_0 e^{-t/(L/R)}$

Let $\tau \equiv L/R$, & $I(t=0)=0 \Rightarrow \underbrace{x(t=0)}_{\equiv x_0} = \frac{\mathcal{E}}{R}$

$\frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R} e^{-t/\tau}$

$\Rightarrow I = \frac{\mathcal{E}}{R}\left(1-e^{-t/\tau}\right)$

RL Circuits



Discharging:

1. Throw switch at t=0, current is maximum and given by I=ε/R
2. As t-> infinity, I->0 as it is dissipated by resistor

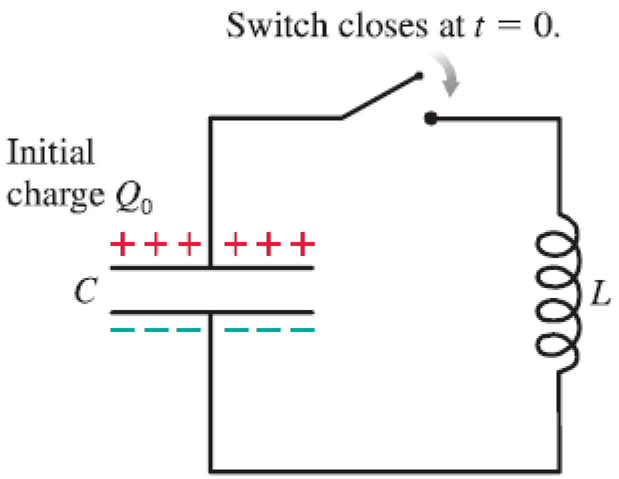
Kirchoff's loop rule: $IR + L\frac{dI}{dt} = 0$

$$\Rightarrow \frac{dI}{I} = -\frac{R}{L}\,dt \Rightarrow \ln\frac{I}{I_0} = -t/\tau$$

where $\tau \equiv L/R$

$$\therefore I(t) = \frac{\varepsilon}{R}e^{-t/\tau}$$

LC Circuits



Discharging:

1. Throw switch at t=0, current is zero (back emf only allows current to ramp up)

Kirchoff's loop rule: $$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

$$I = \frac{dQ}{dt} \Rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{LC} Q$$

This is a simple Harmonic oscillator!

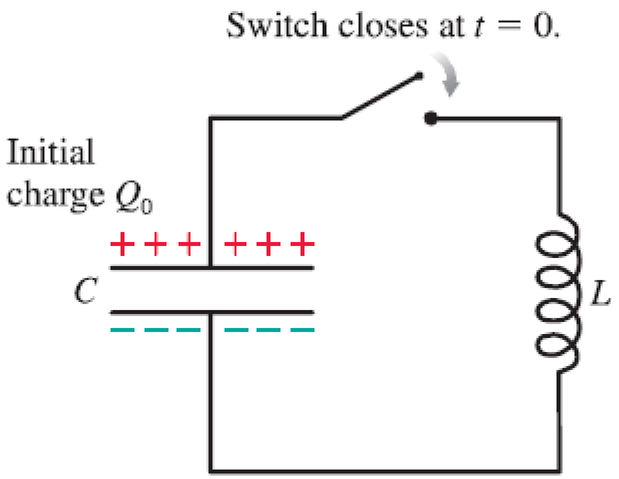
Recall, mass on a spring:

$$m\frac{d^2x}{dt^2} = -kx \Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m} x$$

where $x = A\cos(\omega t + \phi_0)$

$$\Rightarrow \omega = \sqrt{k/m}$$

LC Circuits



Discharging:

1. Throw switch at t=0, current is zero (back emf only allows current to ramp up)

Kirchoff's loop rule: $\frac{Q}{C} - L\frac{dI}{dt} = 0$

$$I = \frac{dQ}{dt}^{\checkmark} \Rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{LC} Q$$

$$\therefore \; Q = Q_0 \, Cos(\omega t + \phi) \qquad \omega = \sqrt{\frac{1}{LC}}$$

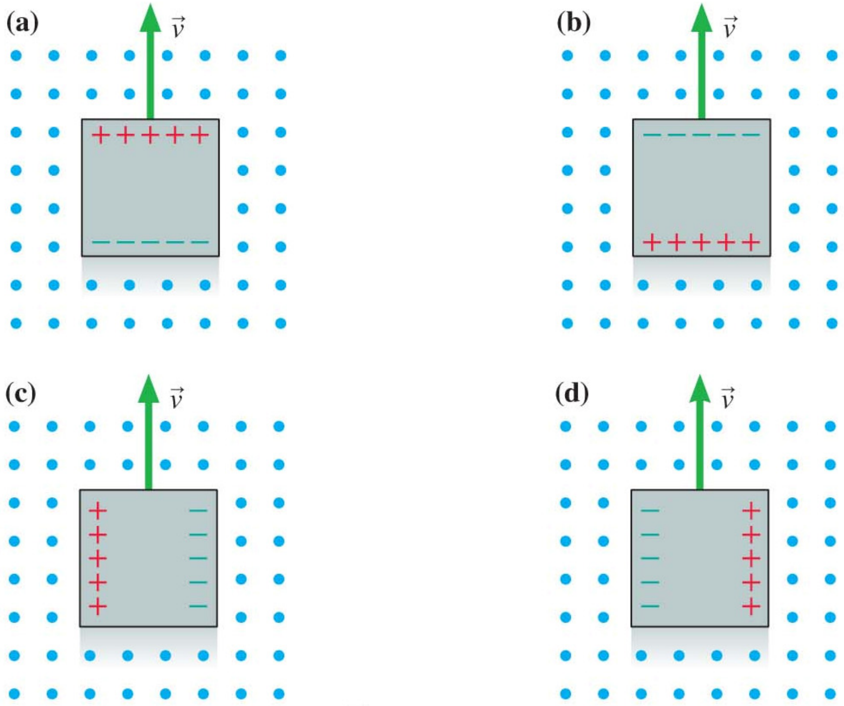
@ $t=0$, $Q = Q_0$ on Capacitor (fully charged)

$\Rightarrow \phi = 0$

$$\therefore \; Q = Q_0 \, Cos(\omega t)$$

Chapter 34. Clicker Questions

A square conductor moves through a uniform magnetic field. Which of the figures shows the correct charge distribution on the conductor?



$\vec{B}$ out of page

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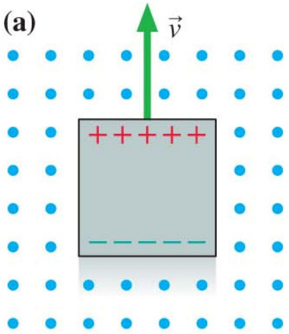
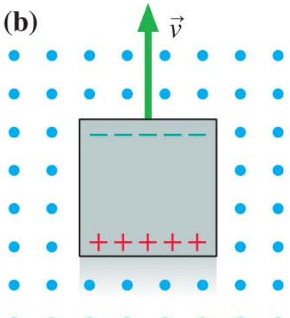
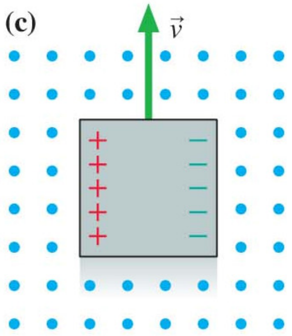
(a)



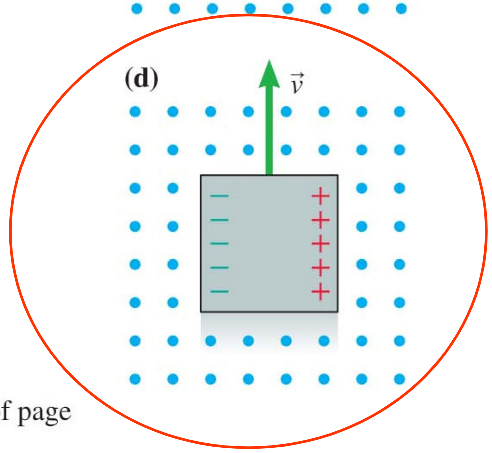
(b)



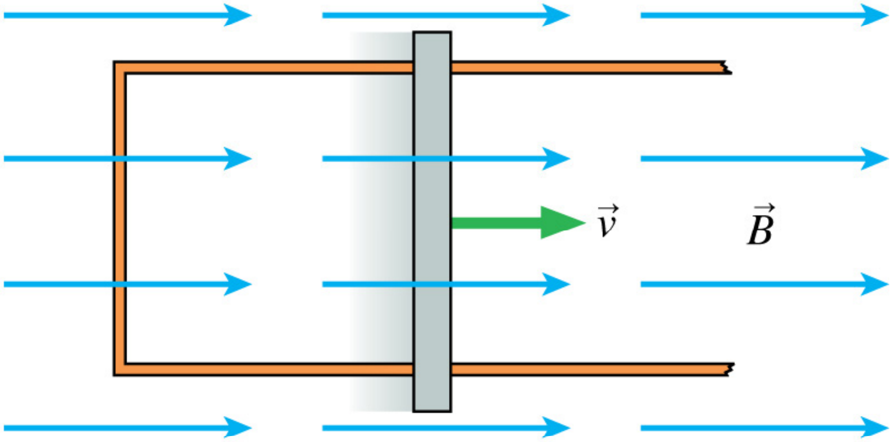
(c)



(d)

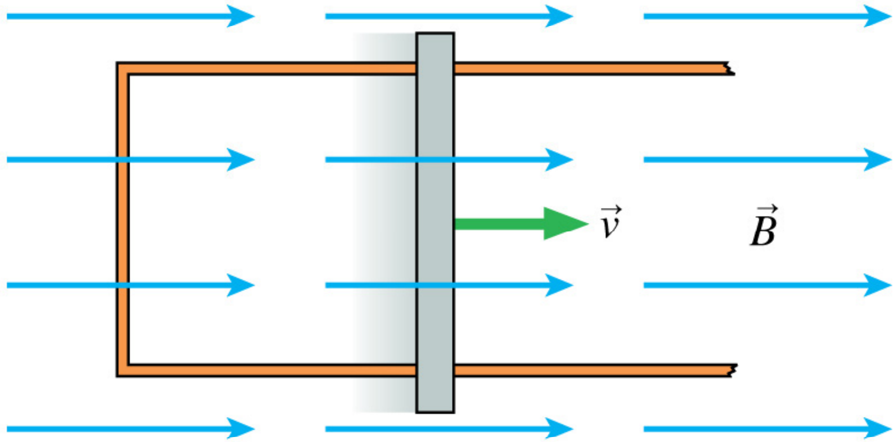


$\vec{B}$ out of page



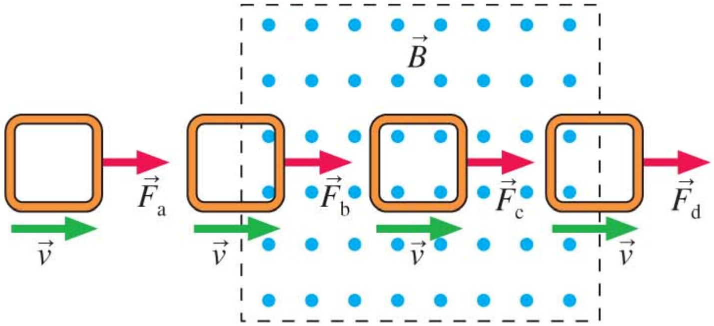
Is there an induced current in this circuit? If so, what is its direction?

A. No
B. Yes, clockwise
C. Yes, counterclockwise



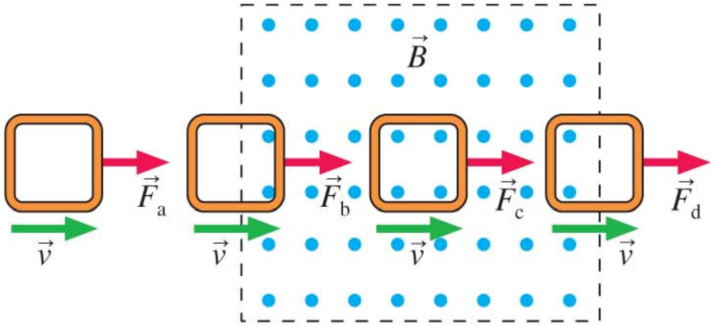
Is there an induced current in this circuit? If so, what is its direction?

[✓ **A. No**]
B. Yes, clockwise
C. Yes, counterclockwise



A square loop of copper wire is pulled through a region of magnetic field. Rank in order, from strongest to weakest, the pulling forces F_a, F_b, F_c and F_d that must be applied to keep the loop moving at constant speed.

A. $F_b = F_d > F_a = F_c$
B. $F_c > F_b = F_d > F_a$
C. $F_c > F_d > F_b > F_a$
D. $F_d > F_b > F_a = F_c$
E. $F_d > F_c > F_b > F_a$



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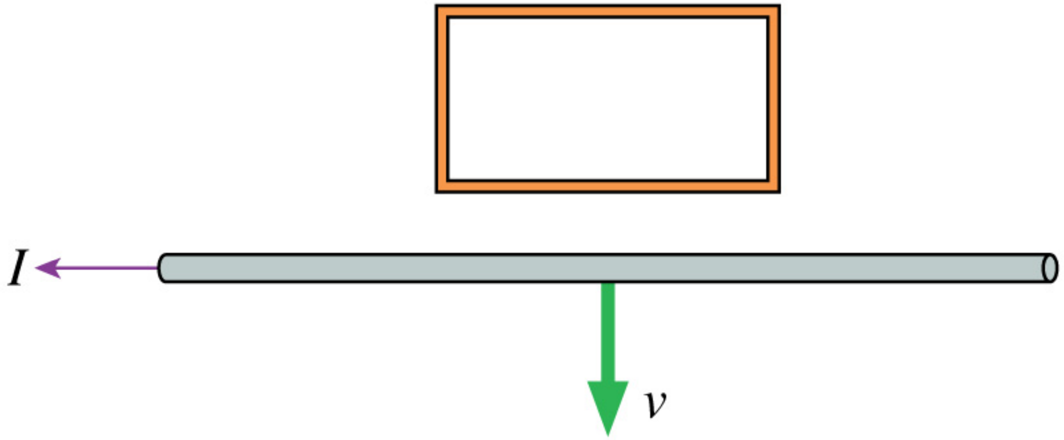
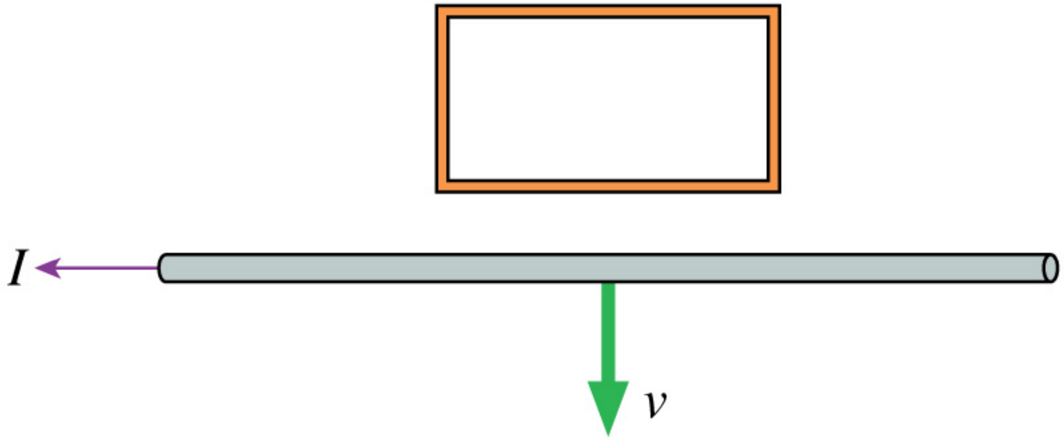
✓ **A.** $\boldsymbol{F_b = F_d > F_a = F_c}$
B. $F_c > F_b = F_d > F_a$
C. $F_c > F_d > F_b > F_a$
D. $F_d > F_b > F_a = F_c$
E. $F_d > F_c > F_b > F_a$



A current-carrying wire is pulled away from a conducting loop in the direction shown. As the wire is moving, is there a cw current around the loop, a ccw current or no current?

A. There is no current around the loop.
B. There is a clockwise current around the loop.
C. There is a counterclockwise current around the loop.



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[✓ **B. There is a clockwise current around the loop.**]
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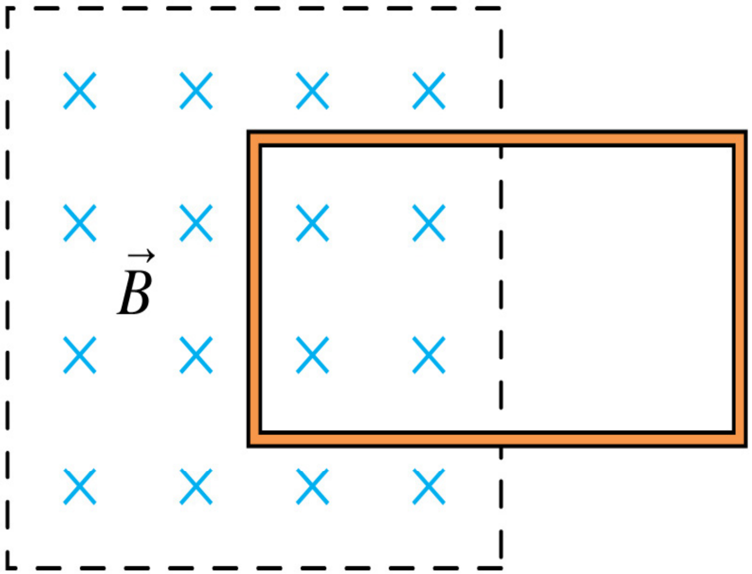
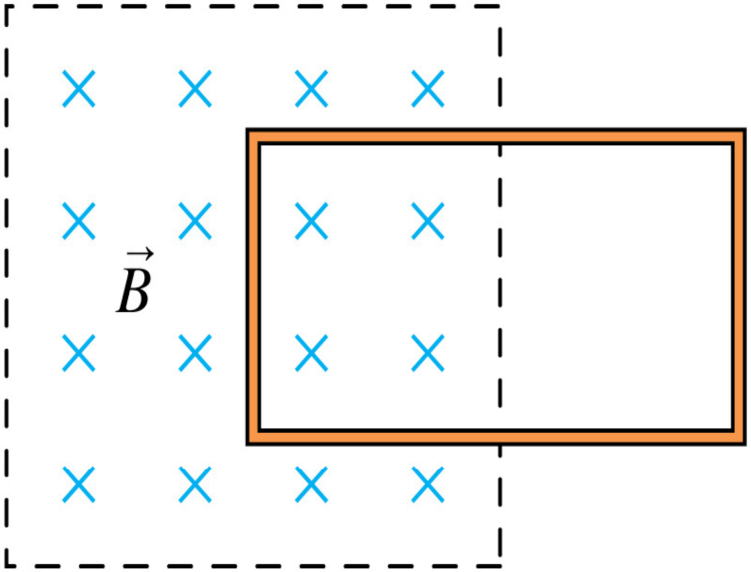
A conducting loop is halfway into a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



A. The loop is pulled to the left, into the magnetic field.
B. The loop is pushed to the right, out of the magnetic field.
C. The loop is pushed upward, toward the top of the page.
D. The loop is pushed downward, toward the bottom of the page.
E. The tension is the wires increases but the loop does not move.

A conducting loop is halfway into a magnetic field. Suppose the magnetic field begins to increase rapidly in strength. What happens to the loop?



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[✓ **B. The loop is pushed to the right, out of the magnetic field.**]
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