

Change of Class room: See schedule on website

Where:

**Chemistry building
(attached to Physics building)
Room # 1402**

When:

October: 8, 13, 20, 27, and 29

Exam I – Will be graded and posted by Tuesday next week

Problem 1 - B and E transformation between moving frames

Quiz #3b, Hwk #35.5

Problem 2 - Biot-Savart law from current Arc

example done in class, Hwk problem 34.46, quiz #1a and #1d

Problem 3 - Solenoid - very similar to Hwk # 34.40

a) derivation in class and in book, one of the few examples of the utility of ampere's law

b) RHR for direction of B-field from current

c) E-field inside solenoid very similar to quiz #3a

d) E-field direction application of Lenz's law (lots of homework)

Problem 4 - E&M traveling wave

Derived in class and in book (eq 35.24), wave direction $\sim \mathbf{S} \sim \mathbf{E} \times \mathbf{B}$, $B=E/c$, S is intensity, meaning of "plane wave"

Problem 5 - RLC circuit -

Hwk problem 36.8, and strongly related to 36.7

Problem 6 - displacement current between parallel capacitors

Quiz #3d, Hwk problem 35.38

Light behaves as a wave

Maxwell's equations \rightarrow Wave equations \rightarrow Plane wave solutions

Superposition of wave solutions applies

Light diffracts and interferes like other well known wave phenomena.

Three limits for observing properties of light:

1. Physical optics – light as a wave

Energy of photon, $h\nu \ll$ resolution of detector

size of objects $\lesssim \lambda$

2. Ray optics --- light

Energy of photon, $h\nu \ll$ resolution of detector

size of objects $\gg \lambda$

3. quantum regime

Energy of photon, $h\nu \gtrsim$ resolution of detector

Assume monochromatic and coherent light

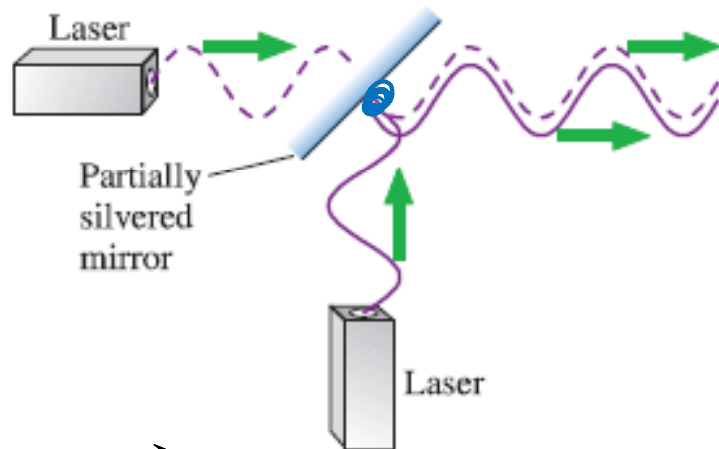
Light diffracts when traversing apertures like the water waves depicted below



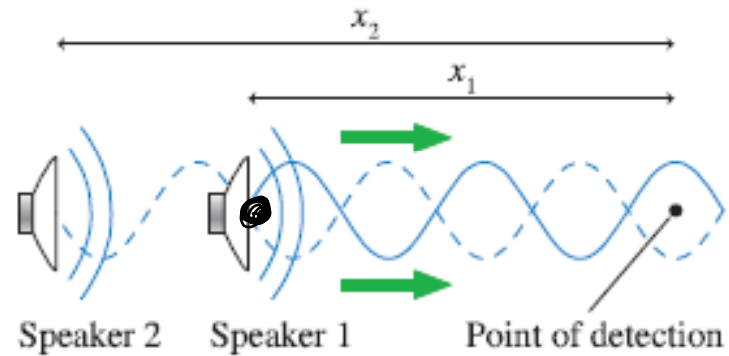
At a beach in Tel Aviv, Israel, plane water waves pass through two openings in a breakwall. Notice the diffraction effect—the waves exit the openings with circular wave fronts, as in Figure 37.1b. Notice also how the beach has been shaped by the circular wave fronts.

FIGURE 21.17 Two overlapped waves travel along the x -axis.

(a) Two overlapped light waves



(b) Two overlapped sound waves



$$E_1 = E_0 \cos(kx - \omega t + \phi_{10})$$

↑ Same ↑

$$E_2 = E_0 \cos(kx - \omega t + \phi_{20})$$

in pic $\pi/2$

• is "starting" point

$$\left. \begin{aligned} \phi_1 &= kx - \omega t + \phi_{10} \\ \phi_2 &= kx - \omega t + \phi_{20} \end{aligned} \right\} \Delta\phi = 0 \text{ Constructive}$$

at •:

$$D_1 = A_0 \cos(kx - \omega t - \pi/2)$$

$$D_2 = A_0 \cos(kx - \omega t + \pi/2)$$

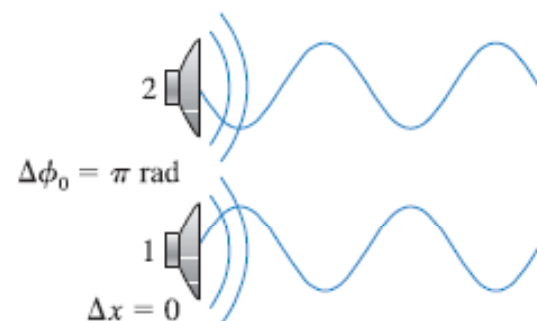
$$\phi_1 = kx - \omega t - \pi/2$$

$$\phi_2 = kx - \omega t + \pi/2$$

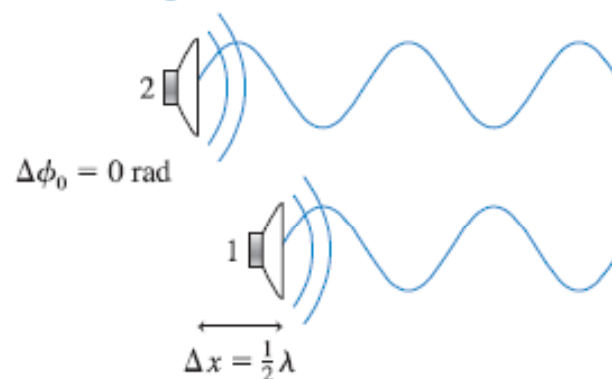
$$\Rightarrow \Delta\phi = \pi \text{ Destructive}$$

FIGURE 21.21 Destructive interference three ways.

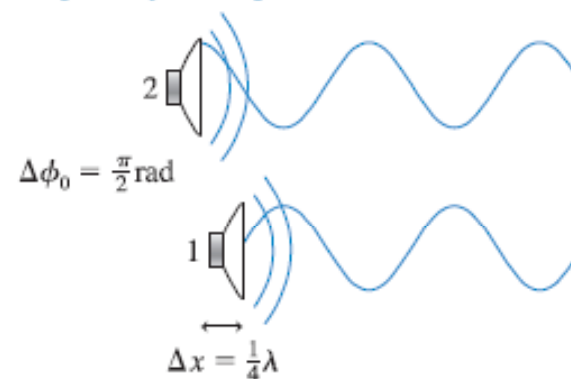
(a) The sources are out of phase.



(b) Identical sources are separated by half a wavelength.



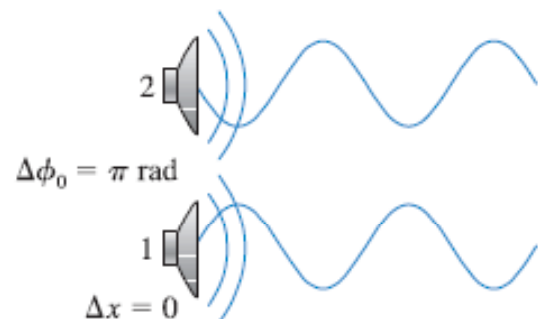
(c) The sources are both separated and partially out of phase.



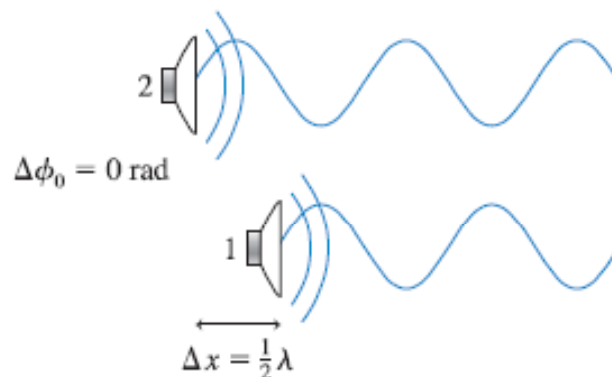
NOTE ► Don't confuse the phase difference of the waves ($\Delta\phi$) with the phase difference of the sources ($\Delta\phi_0$). It is $\Delta\phi$, the phase difference of the waves, that governs interference. ◀

FIGURE 21.21 Destructive interference three ways.

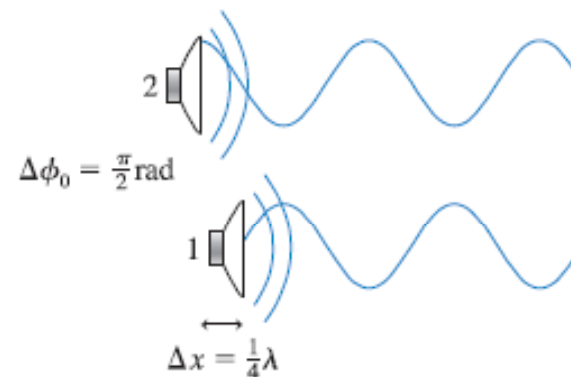
(a) The sources are out of phase.



(b) Identical sources are separated by half a wavelength.



(c) The sources are both separated and partially out of phase.



NOTE ▶ Don't confuse the phase difference of the waves ($\Delta\phi$) with the phase difference of the sources ($\Delta\phi_0$). It is $\Delta\phi$, the phase difference of the waves, that governs interference. ◀

$$a) \quad D_2 = A_0 \cos(\underbrace{kx - \omega t + \frac{\pi}{2}}_{\phi_2}) \quad \& \quad D_1 = A_0 \cos(\underbrace{kx - \omega t - \frac{\pi}{2}}_{\phi_1})$$

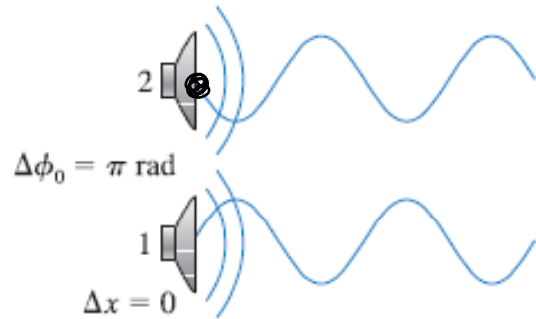
$$D_t = D_1 + D_2 = 0 \quad \text{when} \quad \Delta\phi = \pi \quad \text{Destructive interference}$$

$$\text{or when } \Delta\phi = \pm(2m+1)\pi$$

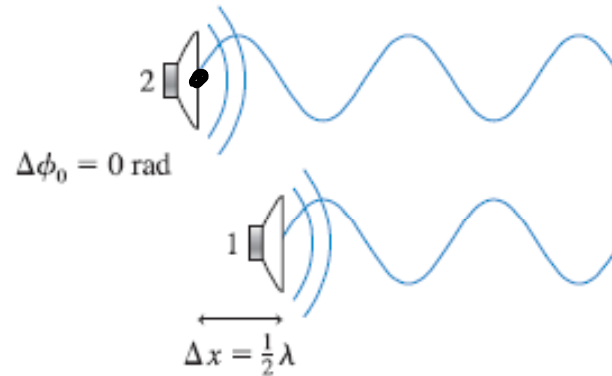
where $m = 0, 1, 2, 3, \dots$

FIGURE 21.21 Destructive interference three ways.

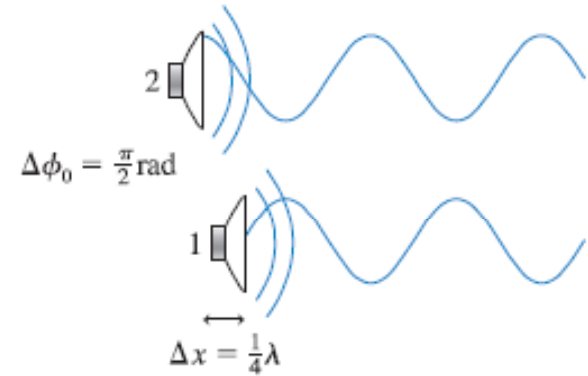
(a) The sources are out of phase.



(b) Identical sources are separated by half a wavelength.



(c) The sources are both separated and partially out of phase.



NOTE ▶ Don't confuse the phase difference of the waves ($\Delta\phi$) with the phase difference of the sources ($\Delta\phi_0$). It is $\Delta\phi$, the phase difference of the waves, that governs interference. ◀

$$b) D_2 = \cos\left(kx - \omega t - \frac{\pi}{2}\right), D_1 = \cos\left(k\left(x + \frac{1}{2}\lambda\right) - \omega t - \frac{\pi}{2}\right)$$

$$\Delta\phi = \phi_2 - \phi_1 = -\frac{k}{2}\lambda = -\frac{2\pi}{2\lambda}\lambda = -\pi$$

ΔOPL

or generally when $\Delta\phi = (2m+1)\pi$

or, when sources are in phase:

$$\Delta OPL = \frac{(2m+1)}{2}\lambda$$

Destructive interference

In general: (assuming $E_1 + E_2$ linearly polarized in same plane)

$$E_1 = E_0 \cos(kx_1 - \omega t + \phi_{10}), \quad E_2 = E_0 \cos(kx_2 - \omega t + \phi_{20})$$

$$= E_0 e^{i(kx_1 - \omega t + \phi_{10})} \quad = E_0 e^{i(kx_2 - \omega t + \phi_{20})}$$

Provided we take the real part after summation since:

$$\text{Re}(e^{i\theta}) = \text{Re}(\cos\theta + i\sin\theta) = \cos\theta$$

$$E_t = E_1 + E_2 = E_0 \left[e^{ikx_1} e^{-i\omega t} e^{i\phi_{10}} + e^{ikx_2} e^{-i\omega t} e^{i\phi_{20}} \right]$$

$$= E_0 e^{-i\omega t} \left[e^{ikx_1} e^{i\phi_{10}} + e^{ikx_2} e^{i\phi_{20}} \right]$$

$$= E_0 e^{-i\omega t} e^{i\frac{kx_1}{2}} e^{i\frac{kx_2}{2}} e^{i\frac{\phi_{10}}{2}} e^{i\frac{\phi_{20}}{2}} \left[e^{i\frac{kx_1}{2}} e^{-i\frac{kx_2}{2}} e^{i\frac{\phi_{10}}{2}} e^{-i\frac{\phi_{20}}{2}} + e^{i\frac{kx_2}{2}} e^{-i\frac{kx_1}{2}} e^{i\frac{\phi_{20}}{2}} e^{-i\frac{\phi_{10}}{2}} \right]$$

$$= E_0 e^{i(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})} \left[e^{-i(k\frac{\Delta x}{2} + \Delta\phi_0/2)} + e^{i(k\Delta x/2 + \Delta\phi_0/2)} \right]$$

Note: $e^{i\alpha} + e^{-i\alpha} =$
 $(\cos\alpha + i\sin\alpha) + (\cos\alpha - i\sin\alpha) = 2\cos\alpha$
 $2\cos\left(\frac{k\Delta x + \Delta\phi_0}{2}\right)$

Take Real part:

$$E_t = 2E_0 \cos\left(\frac{k\Delta x + \Delta\phi_0}{2}\right) \cos(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})$$

$$E_t = 2E_0 \cos\left(\underbrace{\frac{k\Delta x + \Delta\phi_0}{2}}_{\substack{\Delta OPL \\ \equiv \Delta\phi/2}}\right) \cos(kx_{avg} - \omega t + \phi_{avg})$$

Amplitude, A

$$A = \left| 2E_0 \cos \frac{\Delta\phi}{2} \right|$$

Constructive interference (amplitude @ maximum $\Rightarrow \pm 2E_0$)

$$\Rightarrow \cos \frac{\Delta\phi}{2} = \pm 1 \Rightarrow \frac{\Delta\phi}{2} = m\pi \Rightarrow \Delta\phi = 2m\pi, \quad m=0, \pm 1, \pm 2, \dots$$

$$\text{If sources are in phase } (\Delta\phi_0 = 0) \Rightarrow \frac{\Delta\phi}{2} = \frac{k\Delta x}{2} = \pi \frac{\Delta x}{\lambda} = m\pi$$

$$\text{or } \Delta x = \Delta OPL = m\lambda$$

Destructive interference (amplitude = 0)

$$\Rightarrow \cos \frac{\Delta\phi}{2} = 0 \Rightarrow \frac{\Delta\phi}{2} = \left(\frac{2m+1}{2}\right)\pi \Rightarrow \Delta\phi = (2m+1)\pi$$

$$\text{If sources are in phase } (\Delta\phi_0 = 0) \Rightarrow \Delta\phi = \frac{k\Delta x}{2} = \pi \frac{\Delta x}{\lambda} = \left(\frac{2m+1}{2}\right)\pi$$

$$\text{or } \Delta x = \Delta OPL = \left(\frac{2m+1}{2}\right)\lambda$$

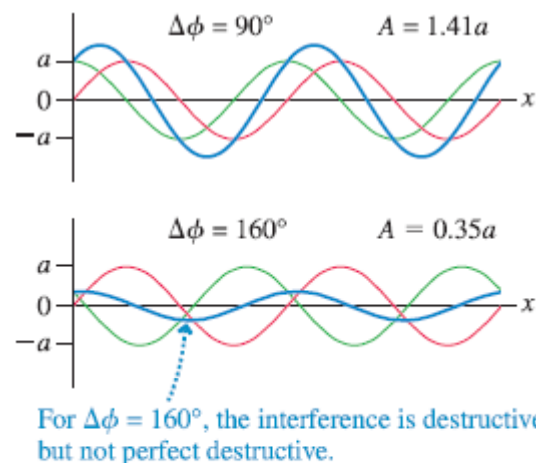


FIGURE 21.25 A circular or spherical wave.

The wave fronts are crests, separated by λ .

Troughs are halfway between wave fronts.

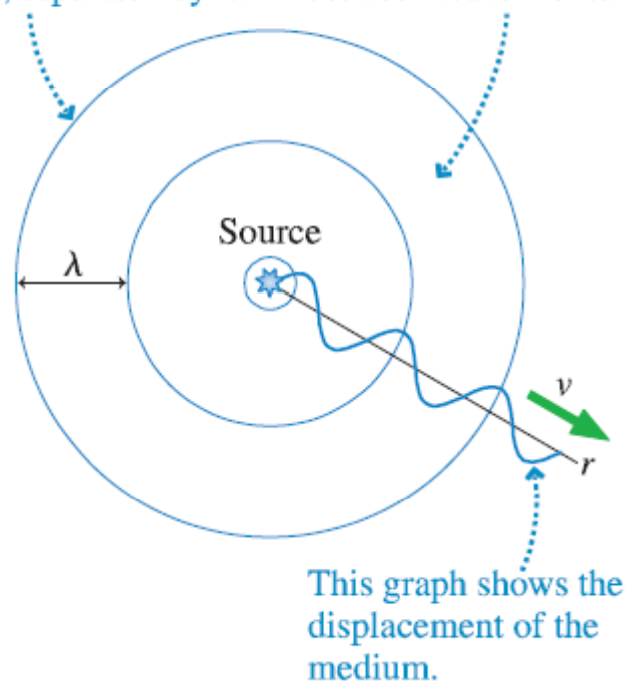
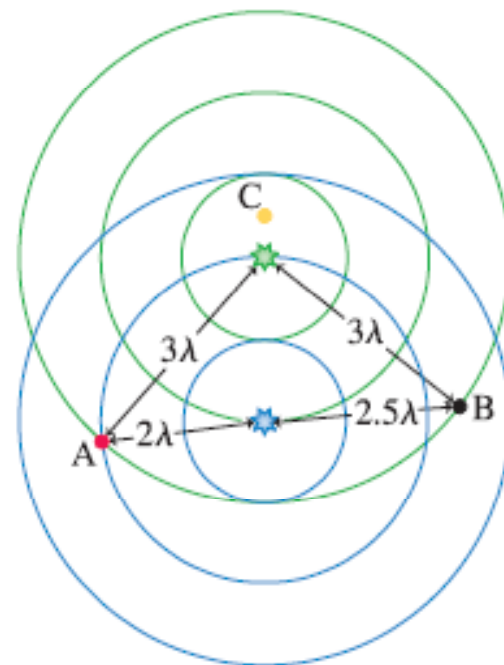


FIGURE 21.27 The path-length difference Δr determines whether the interference at a particular point is constructive or destructive.

- At A, $\Delta r_A = \lambda$, so this is a point of constructive interference.



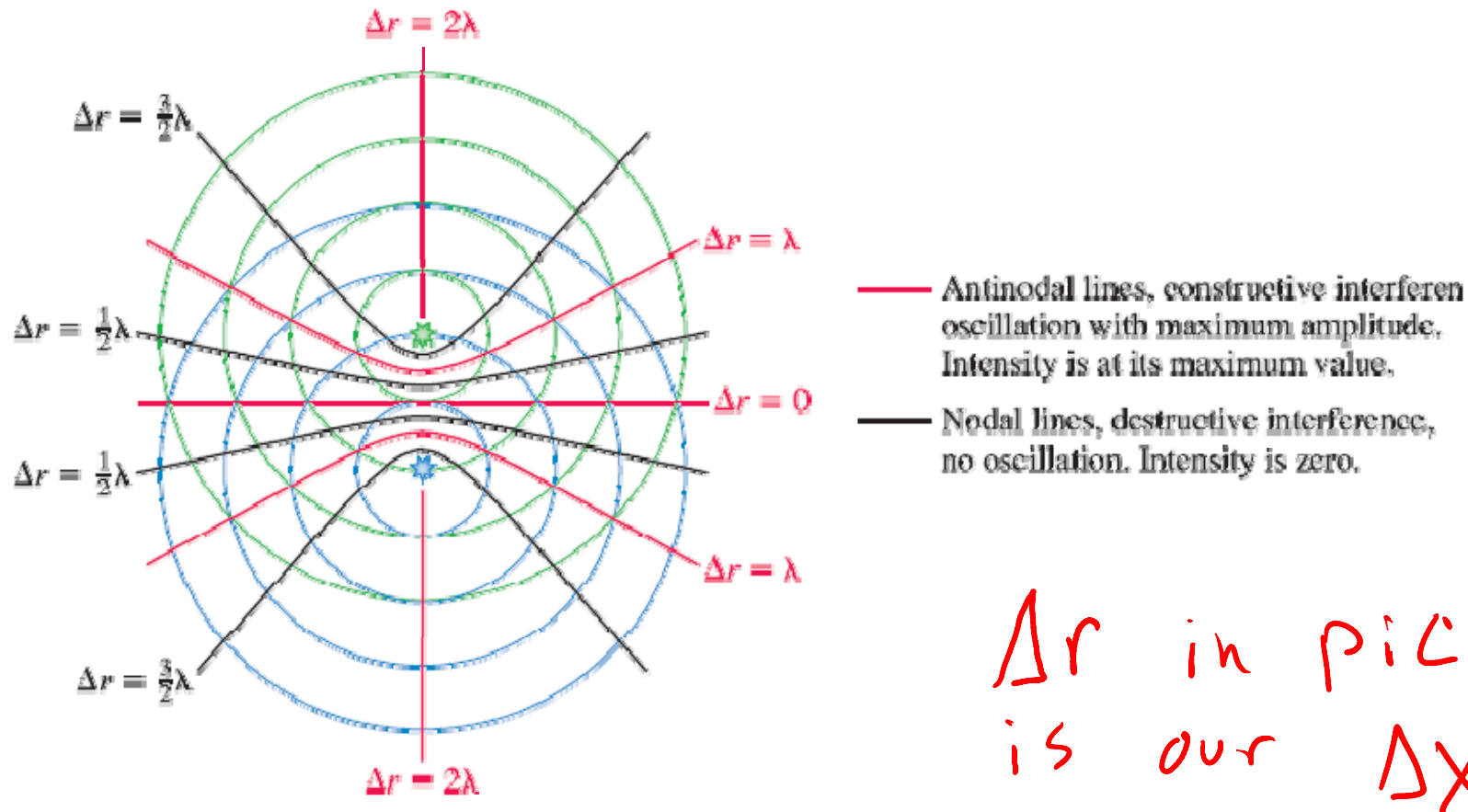
- At B, $\Delta r_B = \frac{1}{2}\lambda$, so this is a point of destructive interference.

$$E_t = A \cos(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})$$

$$A = |2 E_0 \cos \frac{\Delta \phi}{2}| \quad \text{Constructive} \quad \Delta \text{OPL} = m\lambda$$

$$\Delta \phi = k \Delta x + \Delta \phi_0 \quad \text{Destructive} \quad \Delta \text{OPL} = \left(\frac{2m+1}{2}\right) \lambda$$

FIGURE 21.28 The points of constructive and destructive interference fall along antinodal and nodal lines.



$$E_t = A \cos(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})$$

$$A = \left| 2 E_0 \cos \frac{\Delta \phi}{2} \right| \quad \text{Constructive } \Delta \text{OPL} = m\lambda$$

$$\Delta \phi = k \Delta x + \Delta \phi_0 \quad \text{Destructive } \Delta \text{OPL} = \left(\frac{2m+1}{2} \right) \lambda$$

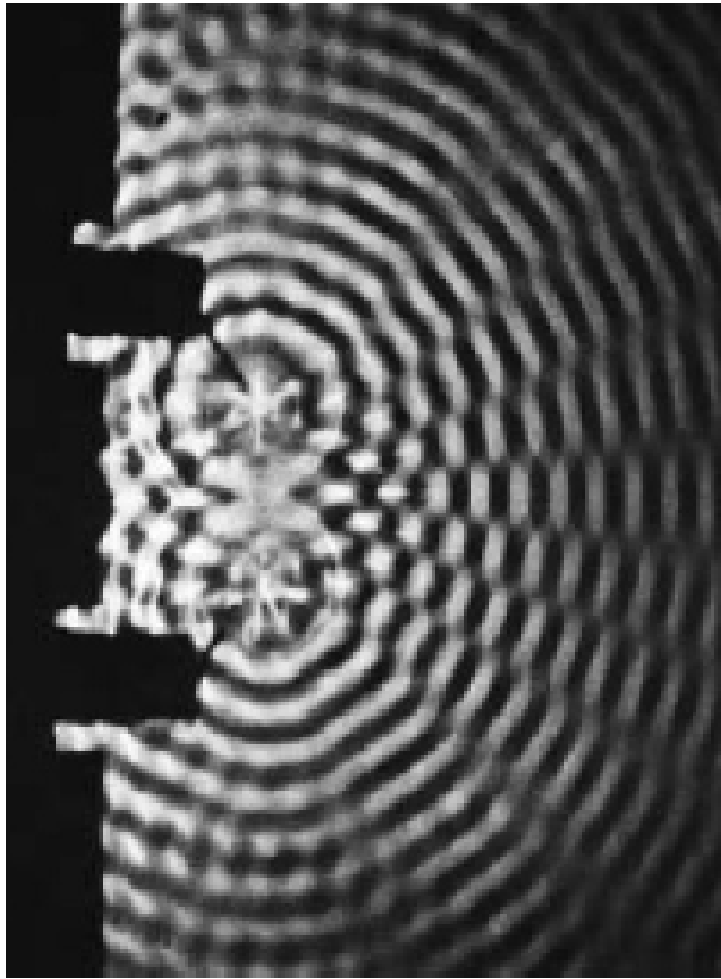
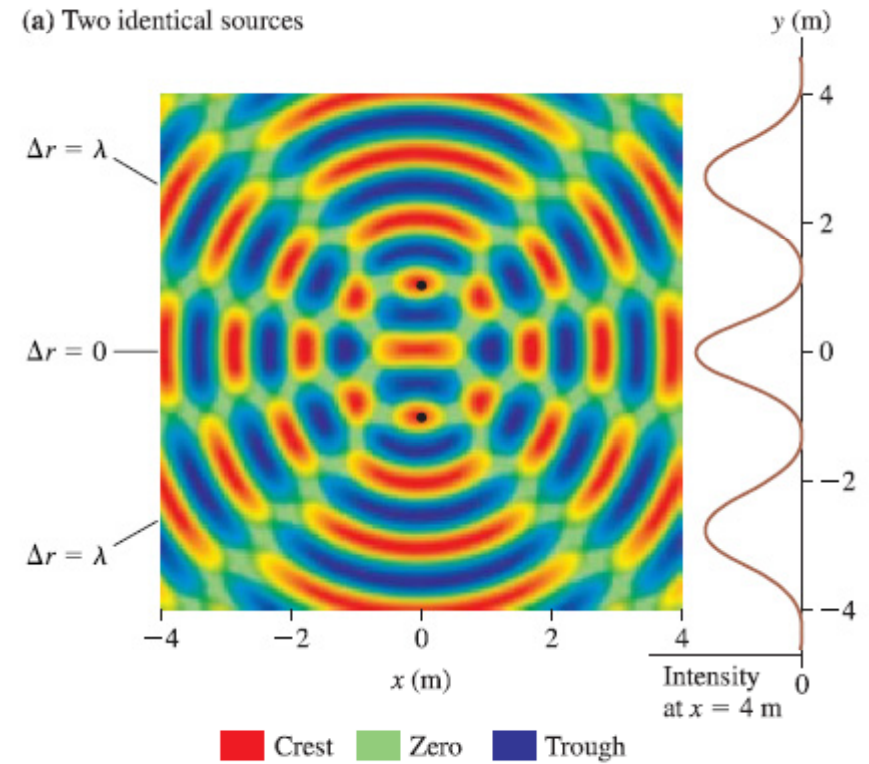


FIGURE 21.30 A contour map of the interference pattern of two sources. The right side of each figure shows the wave intensity along a vertical

(a) Two identical sources



$$E_t = A \cos(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})$$

$$A = \left| 2 E_0 \cos \frac{\Delta \phi}{2} \right| \quad \text{Constructive} \quad \Delta \text{OPL} = m\lambda$$

$$\Delta \phi = k \Delta x + \Delta \phi_0 \quad \text{Destructive} \quad \Delta \text{OPL} = \left(\frac{2m+1}{2} \right) \lambda$$

Huygen's principle

Each point on a wavefront acts like a source for an outgoing (half) spherical wave front, producing the new wavefront.

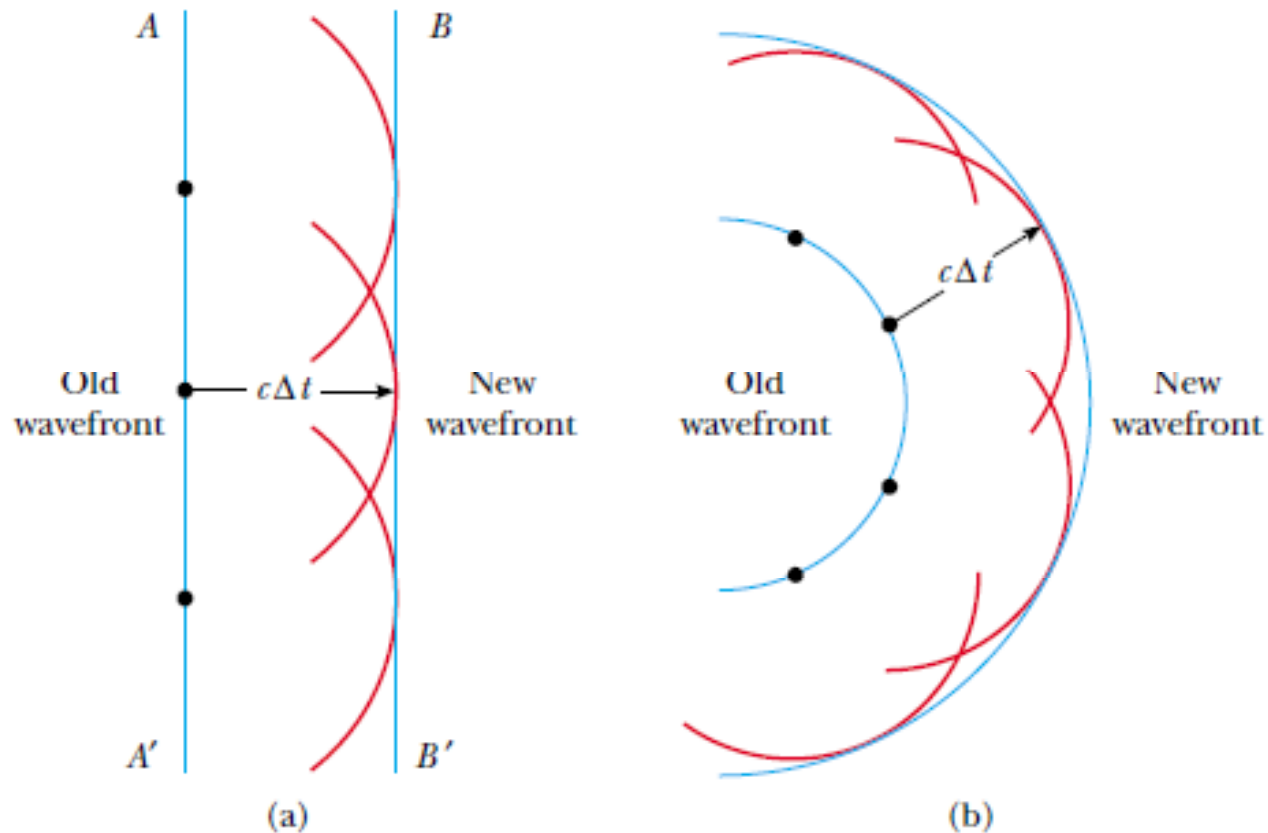


Figure 35.17 Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

Ideal double slit pattern

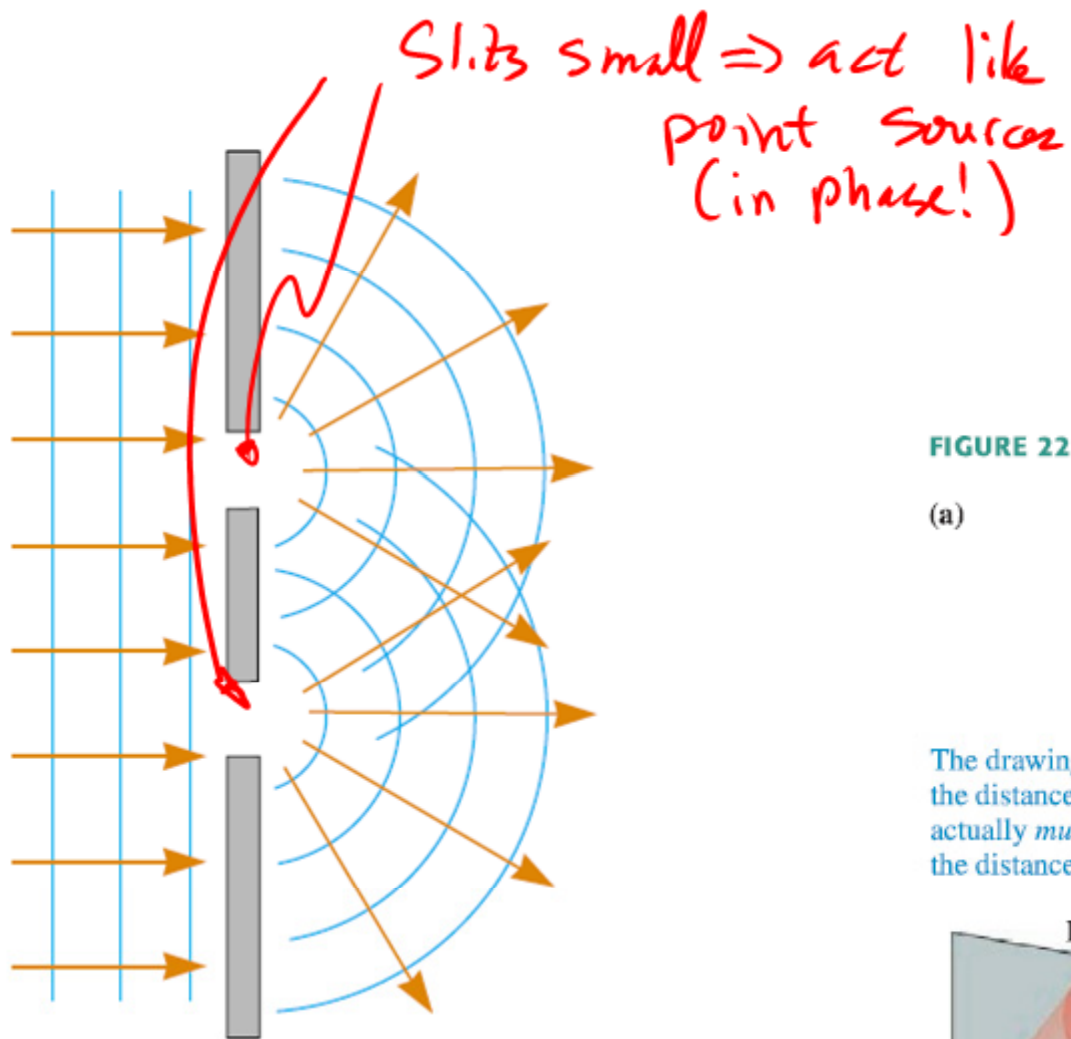
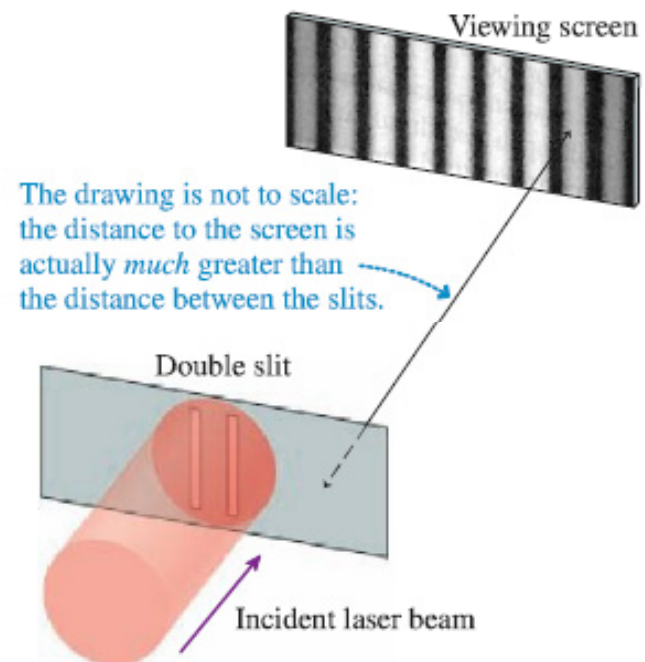
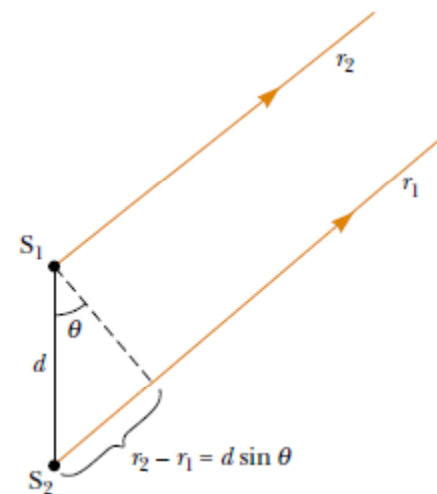
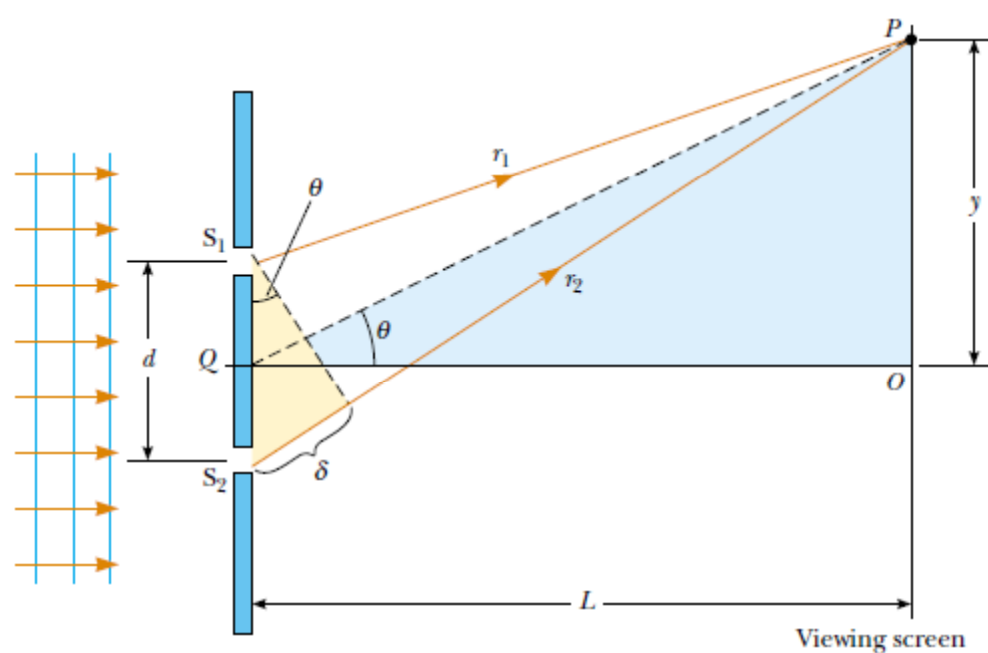


FIGURE 22.3 A double-slit interference experiment.

(a)



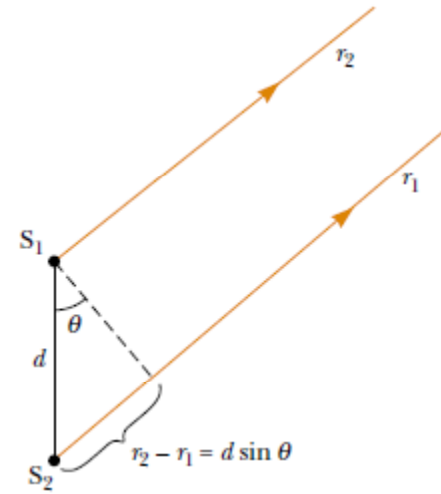
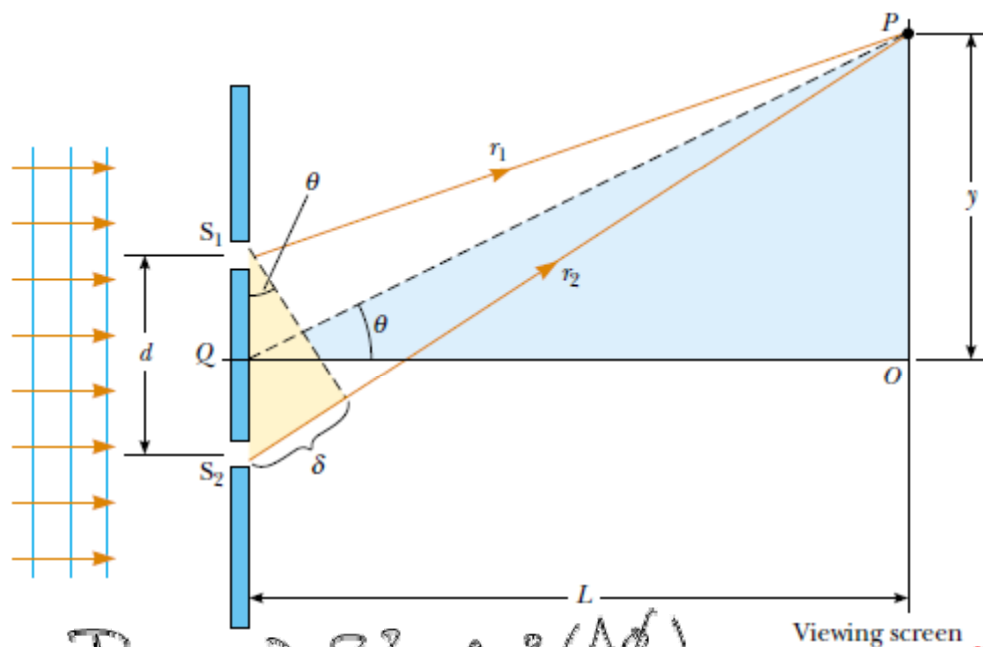


$$\delta = d \sin \theta = \Delta OPL \quad \Delta \phi = k\delta = \frac{2\pi}{\lambda} d \sin \theta$$

$$E = 2E_0 \cos\left(\frac{\Delta\phi}{2}\right) \cos(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})$$

$$I \propto |E|^2 \therefore I = \left| 2E_0 \cos\left(\frac{\Delta\phi}{2}\right) \right|^2 \cos^2(kx_{\text{avg}} - \omega t + \phi_{\text{avg}})$$

$$I_{\text{avg}} = 2E_0^2 \cos^2\left(\frac{\Delta\phi}{2}\right)$$



$$I_{avg} = \underbrace{2 E_0^2}_{I_0} \cos^2\left(\frac{\Delta\phi}{2}\right)$$

Constructive $\Delta OPL = m\lambda$

Destructive $\Delta OPL = \left(\frac{2m+1}{2}\right)\lambda$

$$\Delta\phi = k\delta = \frac{2\pi}{\lambda} d \sin\theta$$

for $L \gg d + y$, $\sin\theta \approx \theta = \frac{y}{L}$

$$\Rightarrow \frac{\Delta\phi}{2} \approx \frac{\pi d}{\lambda} \frac{y}{L}$$

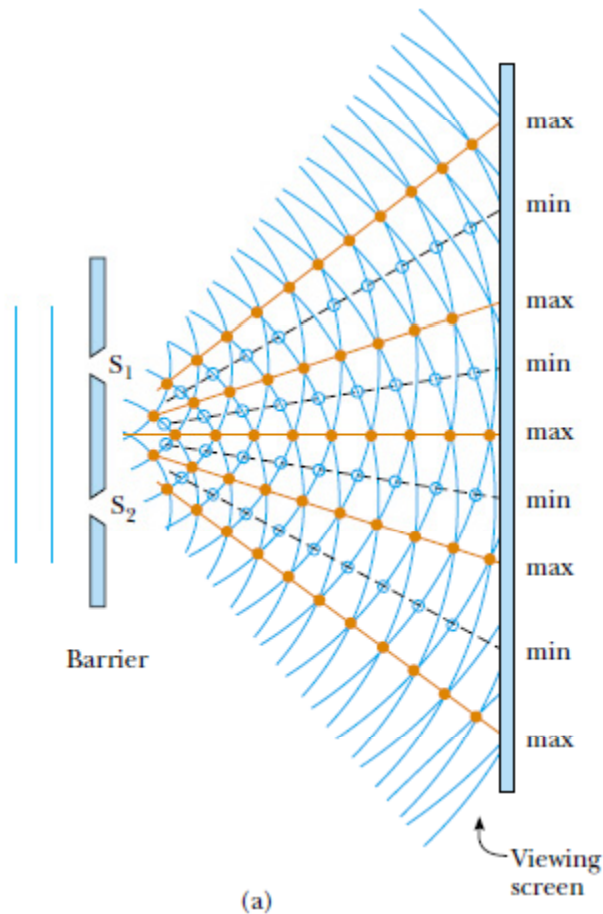
$$\therefore I_{avg} = I_0 \cos^2\left(\frac{\pi y d}{\lambda L}\right)$$

$$\begin{aligned} \Delta OPL &= \delta = d \sin\theta \\ &\approx d \frac{y}{L} \end{aligned}$$

Ideal double slit pattern

$$I_{avg} = I_0 \cos^2\left(\frac{\pi y d}{\lambda L}\right)$$

$$I_{avg} \quad yd/L = d \sin\theta$$



Active Figure 37.2 (a) Schematic diagram of Young's double-slit experiment. Slits S_1 and S_2 behave as coherent sources of light waves that produce an interference pattern on the viewing screen (drawing not to scale). (b) An enlargement of the center of a fringe pattern formed on the viewing screen.

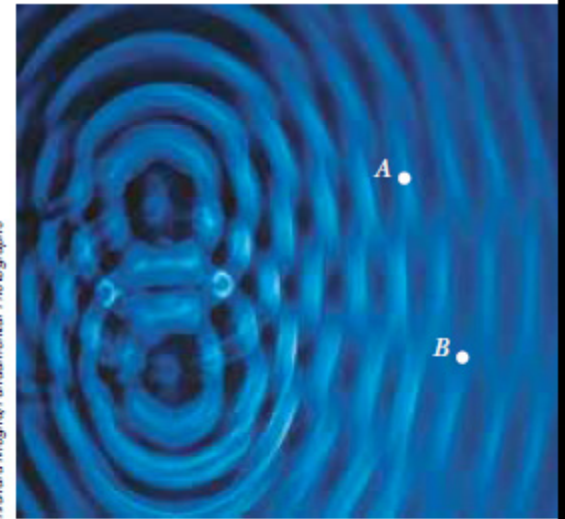
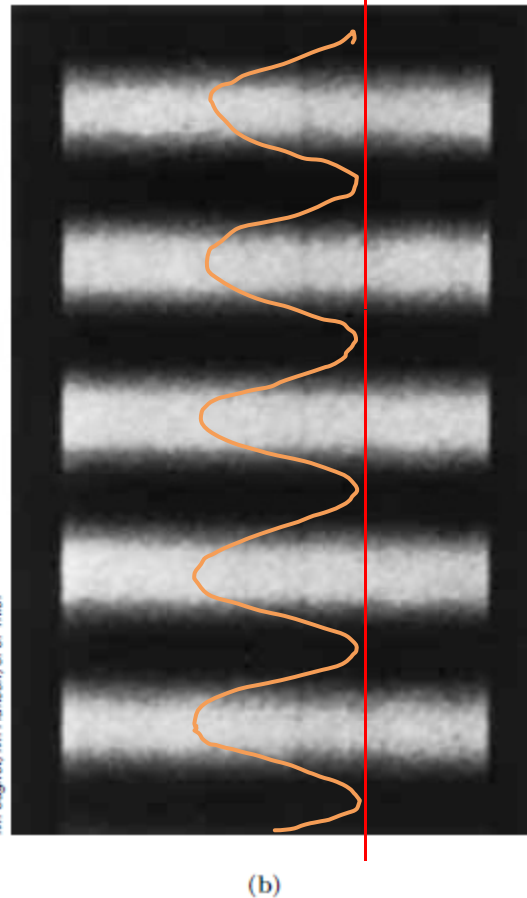
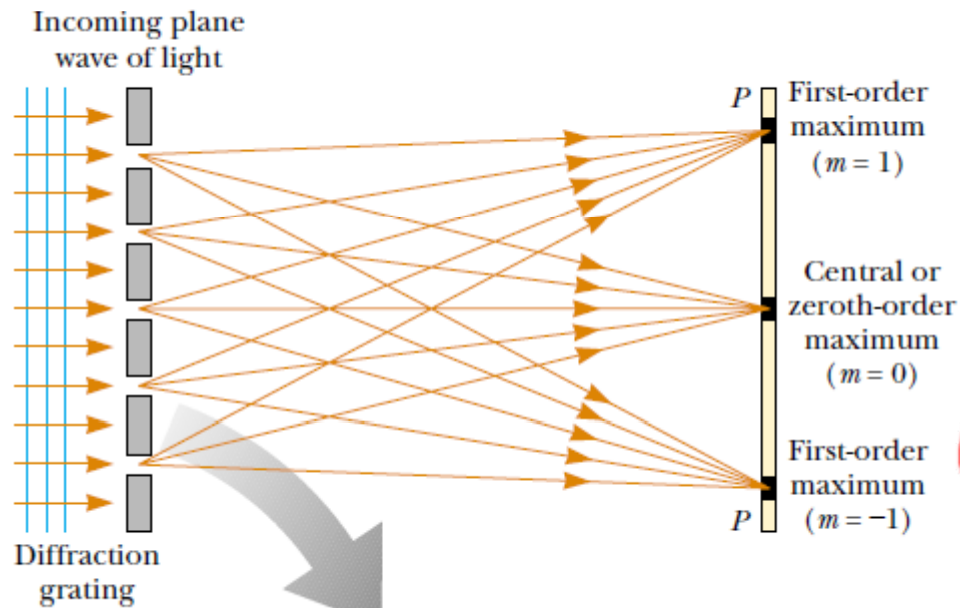


Figure 37.3 An interference pattern involving water waves is produced by two vibrating sources at the water's surface. The pattern is analogous to that observed in Young's double-slit experiment. Note the regions of constructive (A) and destructive (B) interference.



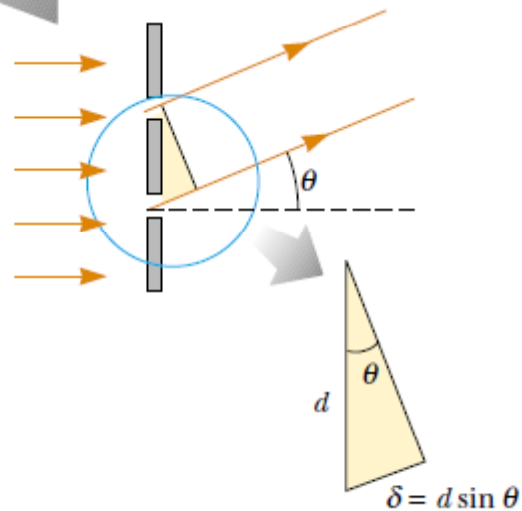
At the Active Figures link at <http://www.pse6.com>, you can adjust the slit separation and the wavelength of the light to see the effect on the interference pattern.

Multiple slit pattern – Diffraction grating



Constructive $\Delta OPL = m\lambda$

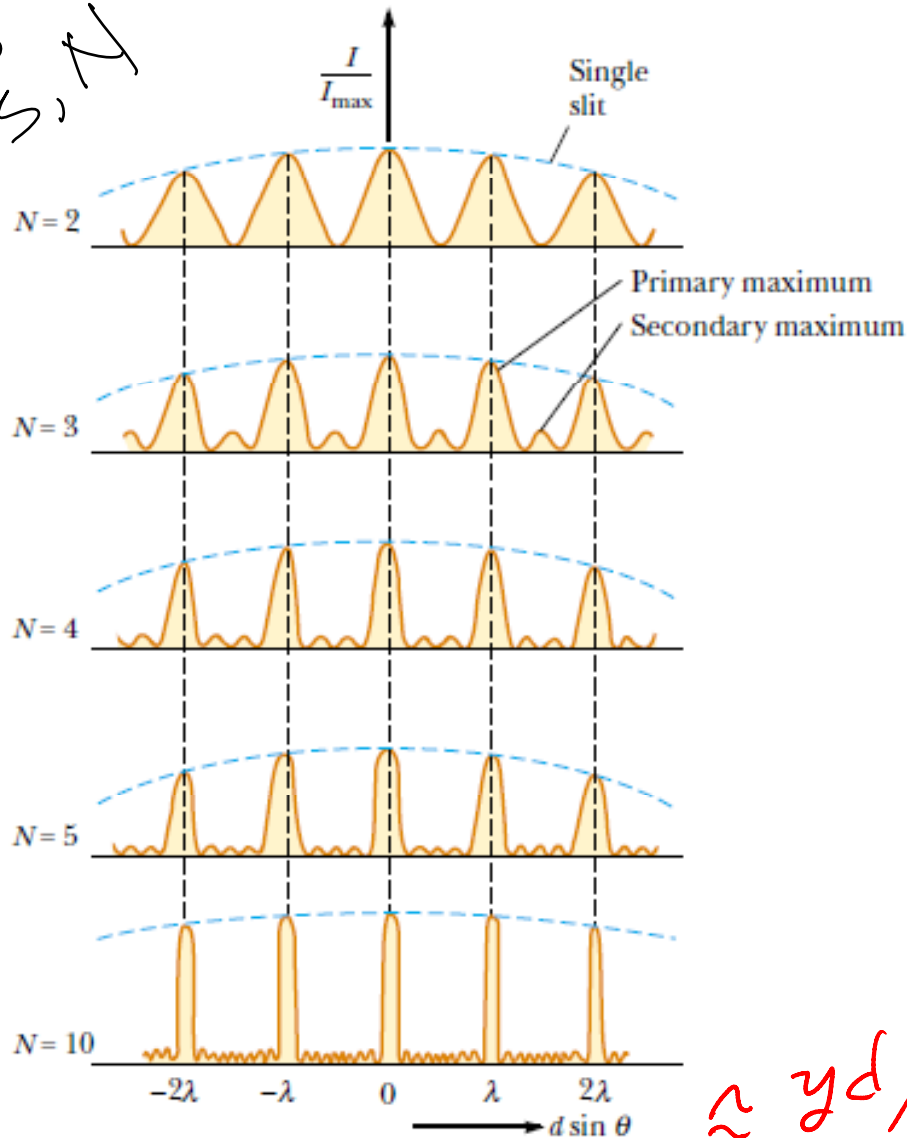
$$\Delta OPL = \delta = d \sin \theta \approx d \frac{y}{L}$$



Multiple slit pattern – Diffraction grating

of
slits, N

↓



Constructive $\Delta OPL = m\lambda$

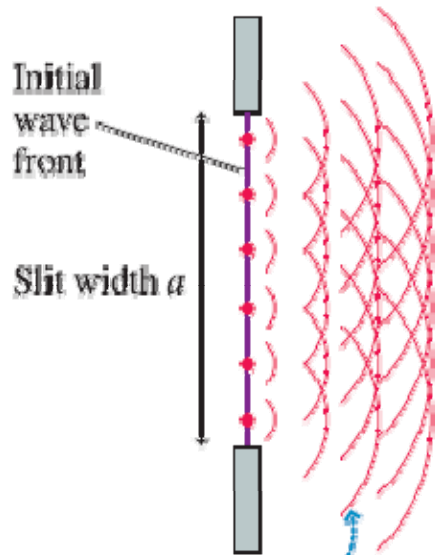
$$\Delta OPL = \delta = d \sin \theta$$

$$\approx \frac{d y}{L}$$

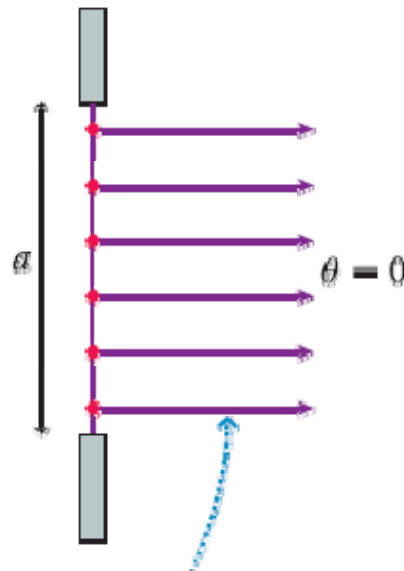
$$\approx yd/L$$

Single slit diffraction pattern

What happens if single slit is too large to be considered a "point" source?

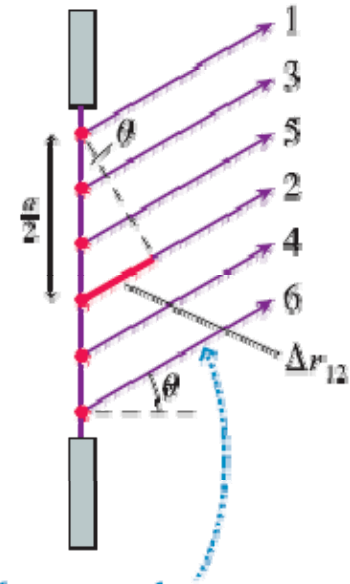


The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.



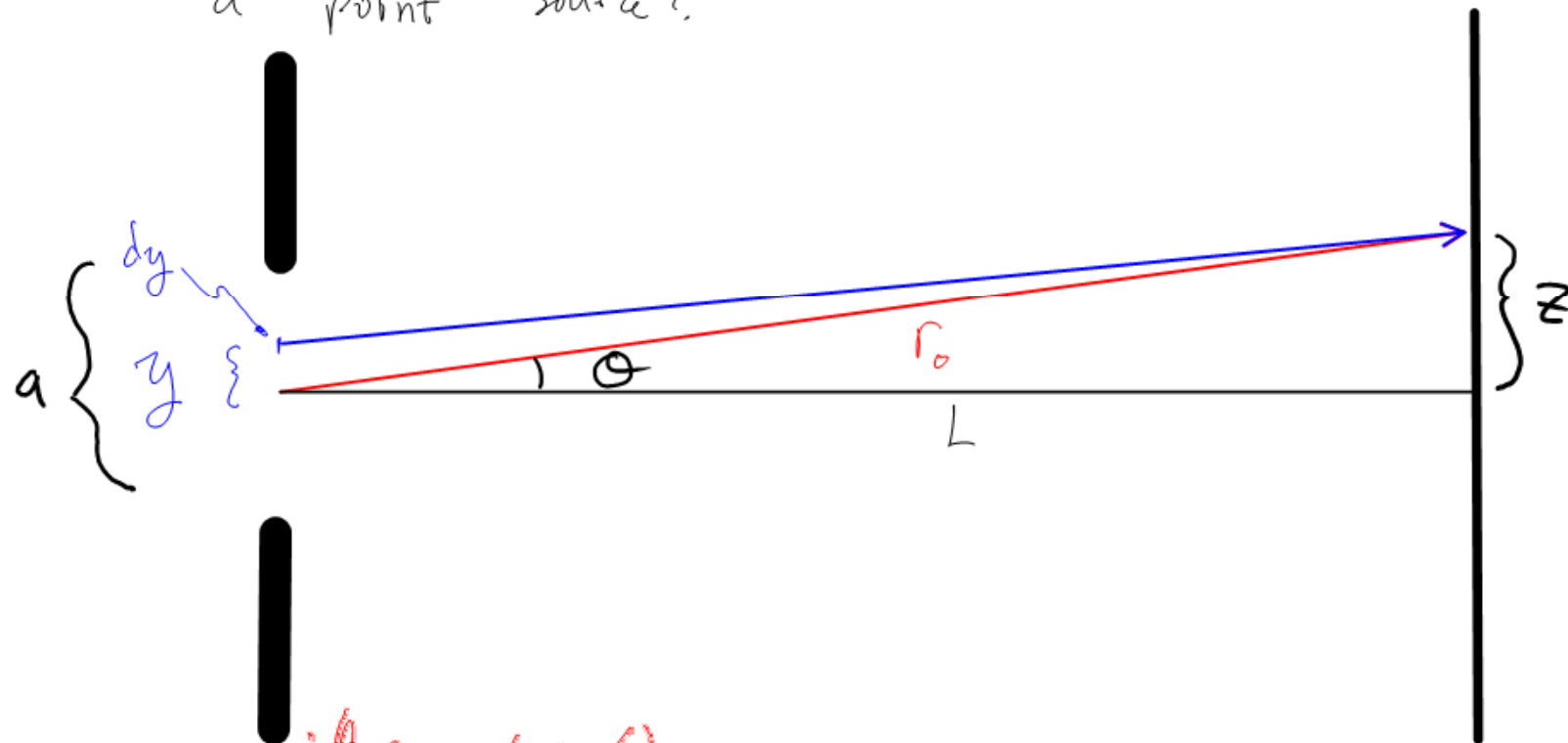
(c)

Each point on the wave front is paired with another point distance $a/2$ away.



These wavelets all meet on the screen at angle θ . Wavelet 2 travels distance $\Delta r_{12} = (a/2)\sin\theta$ farther than wavelet 1.

What happens if single slit is too large to be considered a "point" source?

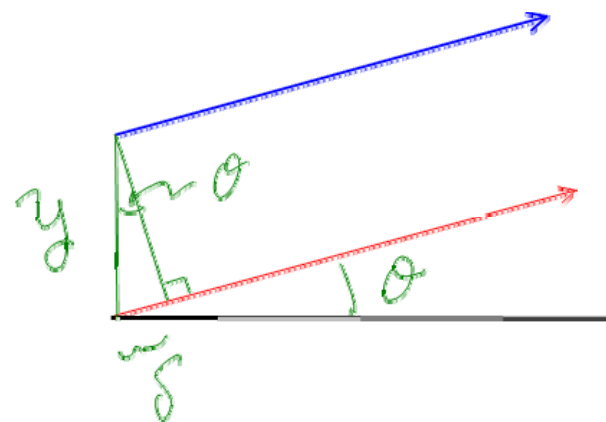


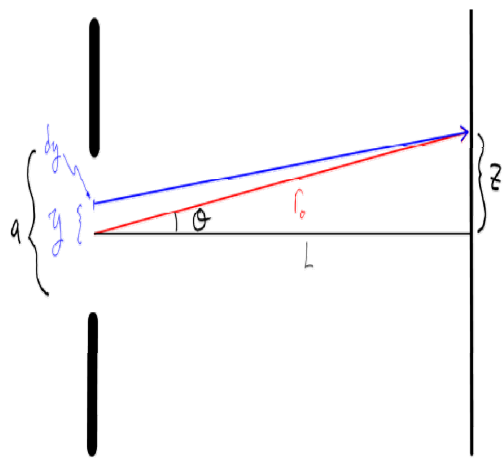
$$E_0 = E_A e^{i(kr_0 - \omega t + \phi_0)} \quad \text{@ center of slit, let } \phi_0 = 0$$

$$dE = E_A e^{i(kr - \omega t + \phi)} \frac{dy}{a} = E_0 e^{i\phi} \frac{dy}{a}$$

$$\phi = \cancel{kr} = k\delta = ky \sin\theta$$

$$E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a} \int_{-a/2}^{a/2} e^{i(k \sin\theta) y} dy$$





$$E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a} \int_{-a/2}^{a/2} e^{i(k \sin \theta) y} dy$$

Note: $\int_{-a/2}^{a/2} e^{i\alpha x} dx = \frac{1}{i\alpha} e^{i\alpha x} \Big|_{-a/2}^{a/2} = \frac{e^{i\alpha a/2} - e^{-i\alpha a/2}}{i\alpha} \cdot \frac{2 a/2}{2 a/2}$

$$= \frac{e^{i\alpha a/2} - e^{-i\alpha a/2}}{2i} \cdot \frac{a}{2 a/2} = a \frac{\sin \alpha \frac{a}{2}}{\alpha \frac{a}{2}}$$

$\underbrace{\hspace{10em}}_{\sin(\alpha \frac{a}{2})}$

$$\therefore E = E_0 \frac{\sin\left(\frac{a}{2} k \sin \theta\right)}{\left(\frac{a}{2} k \sin \theta\right)}, \quad k = \frac{2\pi}{\lambda}$$

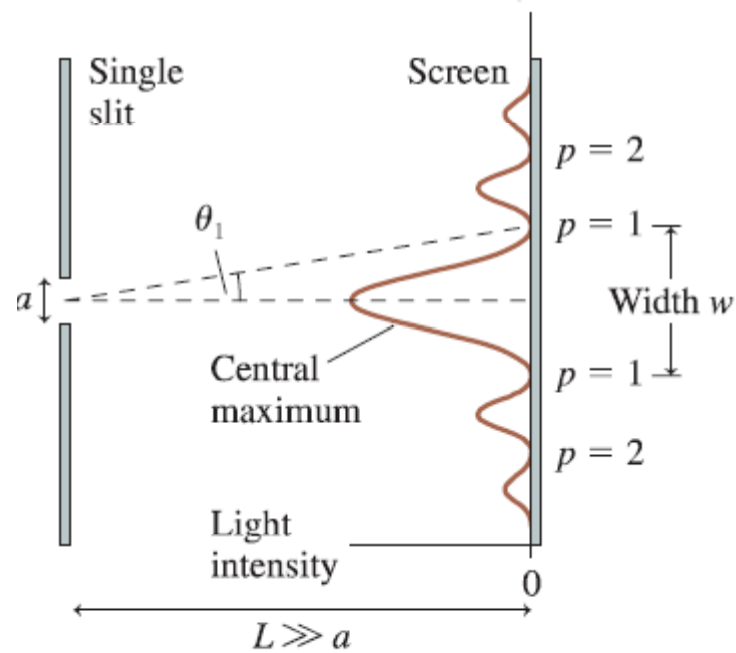
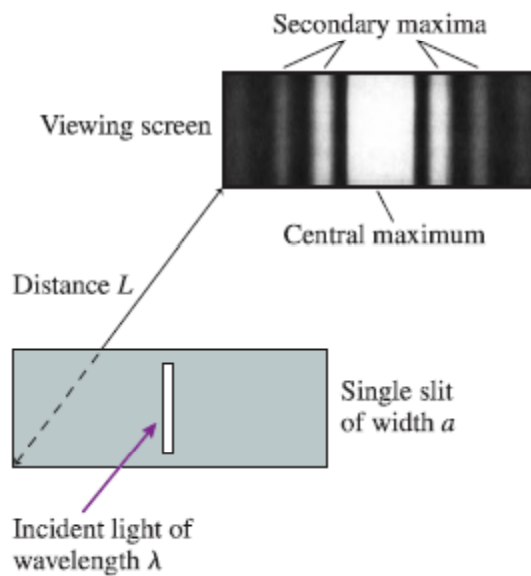
$$I_{\text{avg}} = \frac{1}{2} |E_{\text{max}}|^2 = I_{\text{max}} \left[\frac{\sin\left(\frac{\pi a \sin \theta}{\lambda}\right)}{\frac{\pi a \sin \theta}{\lambda}} \right]^2, \quad I_{\text{max}} = \frac{1}{2} E_A^2$$

$$\sin \theta \approx \frac{z}{L} \Rightarrow I_{\text{avg}} = I_{\text{max}} \left[\frac{\sin \pi \frac{a z}{\lambda L}}{\pi \frac{a z}{\lambda L}} \right]^2$$

Destructive $\pi \frac{a z}{\lambda L} = m\pi \Rightarrow z = m L \left(\frac{\lambda}{a} \right)$

Single slit diffraction pattern

FIGURE 22.10 A single-slit diffraction experiment.



$$\sin \theta \approx \frac{z}{L} \Rightarrow I_{avg} = I_{max} \left[\frac{\sin \pi \frac{az}{\lambda L}}{\pi \frac{az}{\lambda L}} \right]^2$$

Destructive $\pi \frac{az}{\lambda L} = m\pi \Rightarrow z = m L \left(\frac{\lambda}{a} \right)$

Consider ray 1 + 5:

$$\Delta \phi = k\delta, \delta = \frac{a}{2} \sin \theta$$

Destructive: $\Delta \phi = (2m+1)\pi = \frac{2\pi}{\lambda} \frac{a}{2} \sin \theta$

$$\Rightarrow \frac{a}{2} \sin \theta = \left(\frac{2m+1}{2}\right) \lambda = \frac{\lambda}{2} \text{ for } m=1$$

Will also be destructive for ray pairs 2 + 6, 3 + 7, 4 + 8, 5 + 9

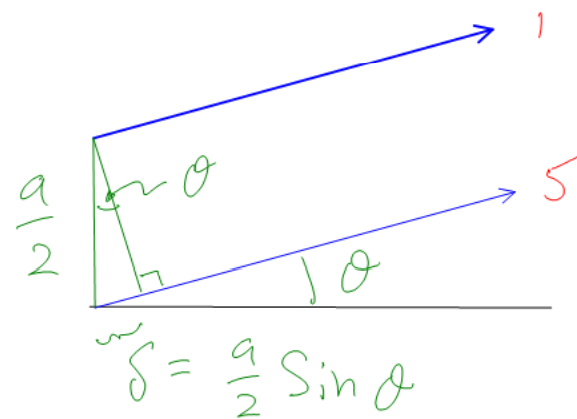
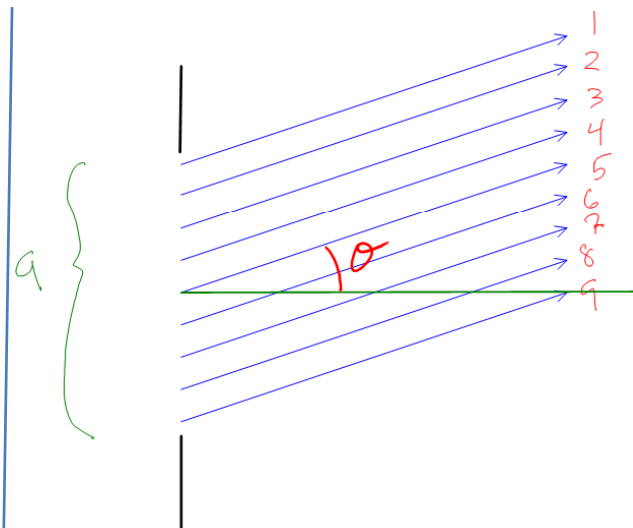
Consider ray ① + ③:

$$\frac{a}{4} \sin \theta = \left(\frac{2m+1}{2}\right) \lambda = \frac{\lambda}{2} \text{ for } m=1$$

Will also be destructive for ray pairs 2 + 4 etc

Dividing the slit up $2m$ times

$$\Rightarrow \frac{a}{2m} \sin \theta = \frac{\lambda}{2} \Rightarrow \frac{a}{m} \sin \theta = \lambda \text{ Destructive}$$



Dividing the slit up $2m$ times

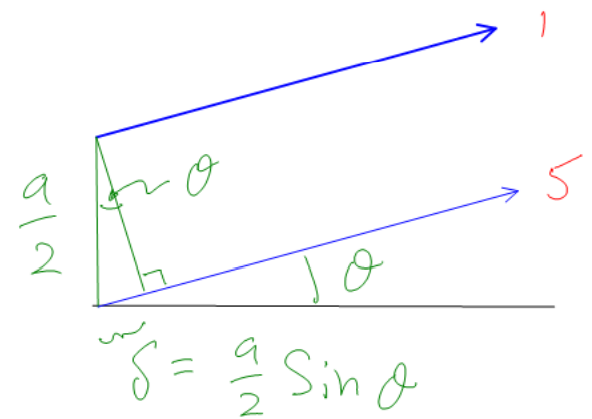
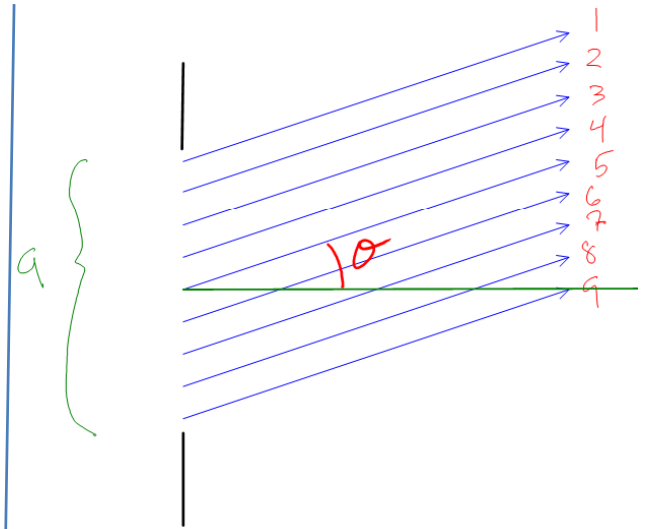
$$\Rightarrow \frac{a}{2m} \sin \theta = \frac{\lambda}{2}$$

$$\Rightarrow \frac{a}{m} \sin \theta = \lambda \quad \underline{\text{Destructive}}$$

Recall: $I_{\text{avg}} = I_{\text{max}} \left[\frac{\sin \left(\frac{\pi a \sin \theta}{\lambda} \right)}{\frac{\pi a \sin \theta}{\lambda}} \right]^2$

Destructive: $\Rightarrow \frac{\pi a \sin \theta}{\lambda} = m\pi$

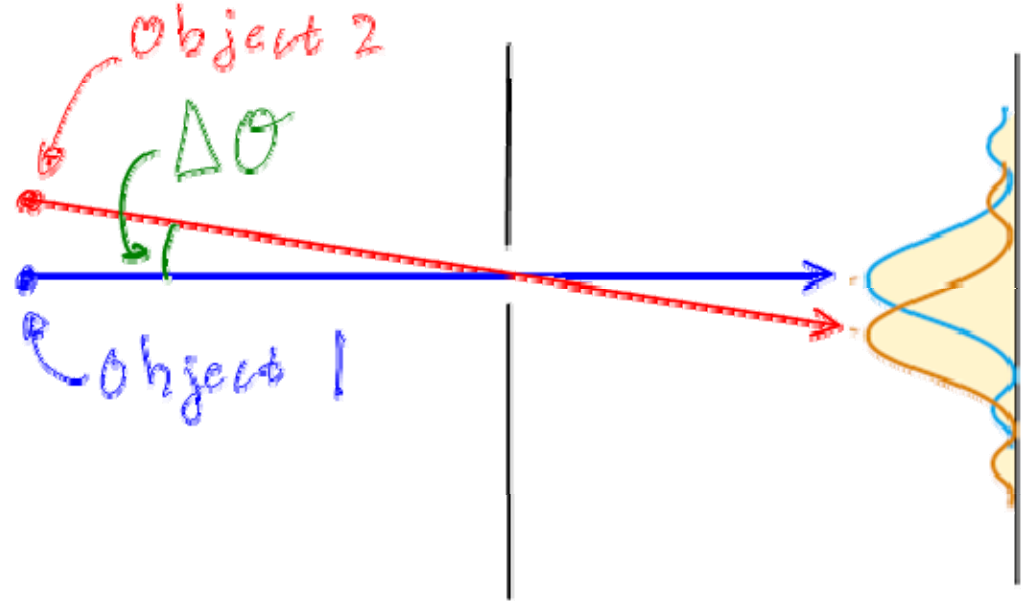
$$\Rightarrow \frac{a}{m} \sin \theta = \lambda$$



Resolution limit -- Rayleigh's criteria

Two separated objects, light goes through an aperture (or lens, or mirror, etc.). Can one resolve the two objects?

If the zeroth order diffraction pattern maximum from object two is at the first minimum from the diffraction pattern produced by object one, then they are resolvable:



$$I_{avg} = I_{avg} \left[\frac{\sin \left(\pi \frac{a}{\lambda} \sin \theta \right)}{\pi \frac{a}{\lambda} \sin \theta} \right]^2$$

Destructive $\pi \frac{a}{\lambda} \sin \theta = m \pi \Rightarrow \sin \theta \approx \Delta \theta = \frac{\lambda}{a}$

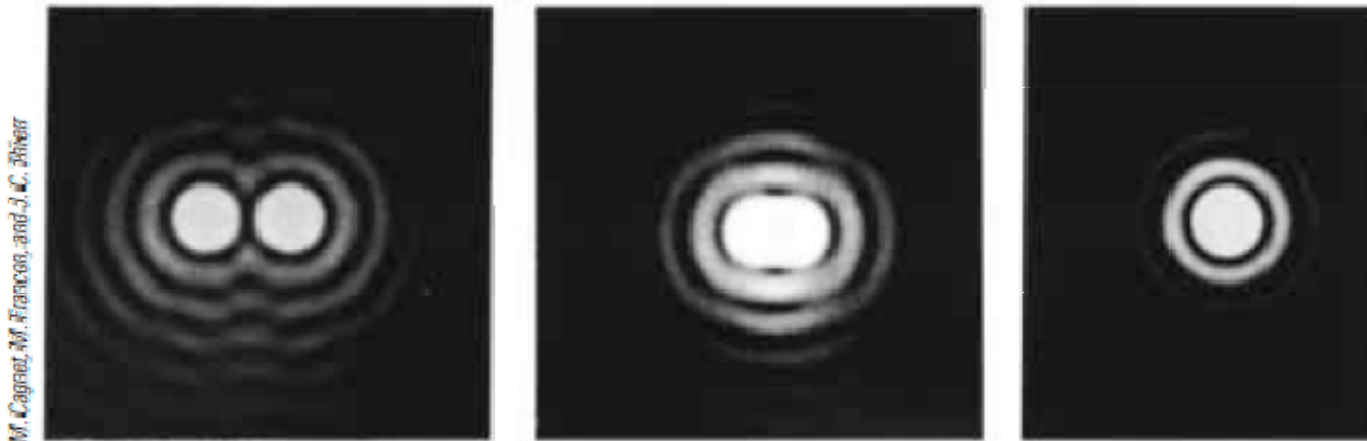
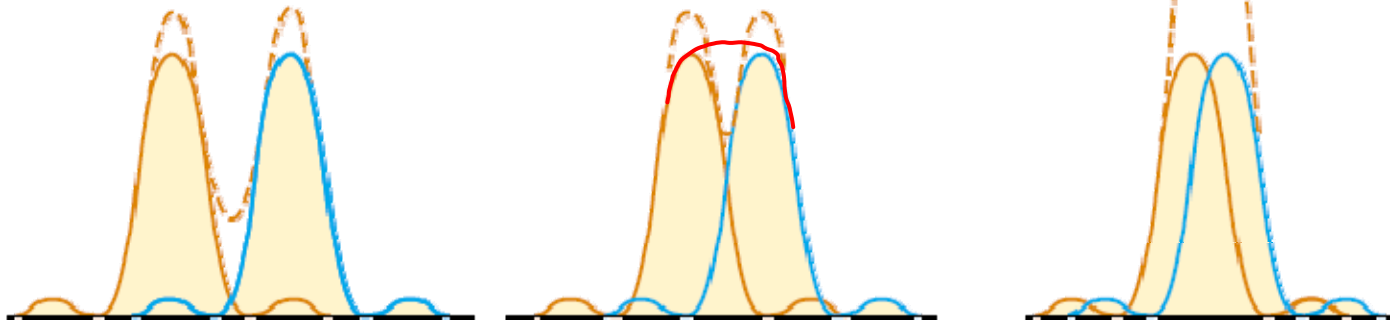
\uparrow
 $m=1$

Resolution limit -- Rayleigh's criteria

For circular aperture, slightly different:

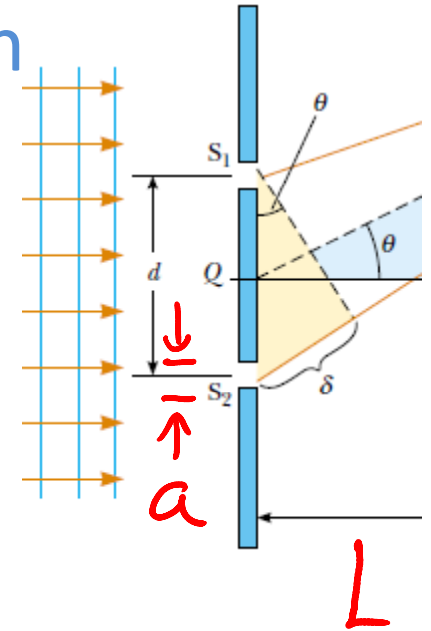
Instead of $\Delta\theta = \frac{\lambda}{a}$ for Slit,

$$\Delta\theta = 1.22 \frac{\lambda}{a} \quad \text{where } a = \text{diameter of opening}$$



Real double slit diffraction pattern

We could integrate across both slits to find total E-field. Answer is that the single slit intensity is modulated by the double slit pattern (for slit opening $a < d$, the slit spacing)



ideal double slit

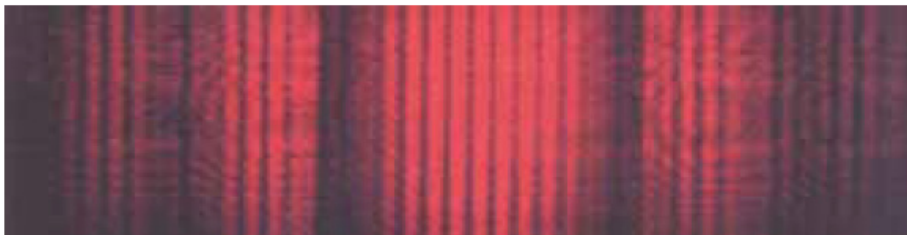
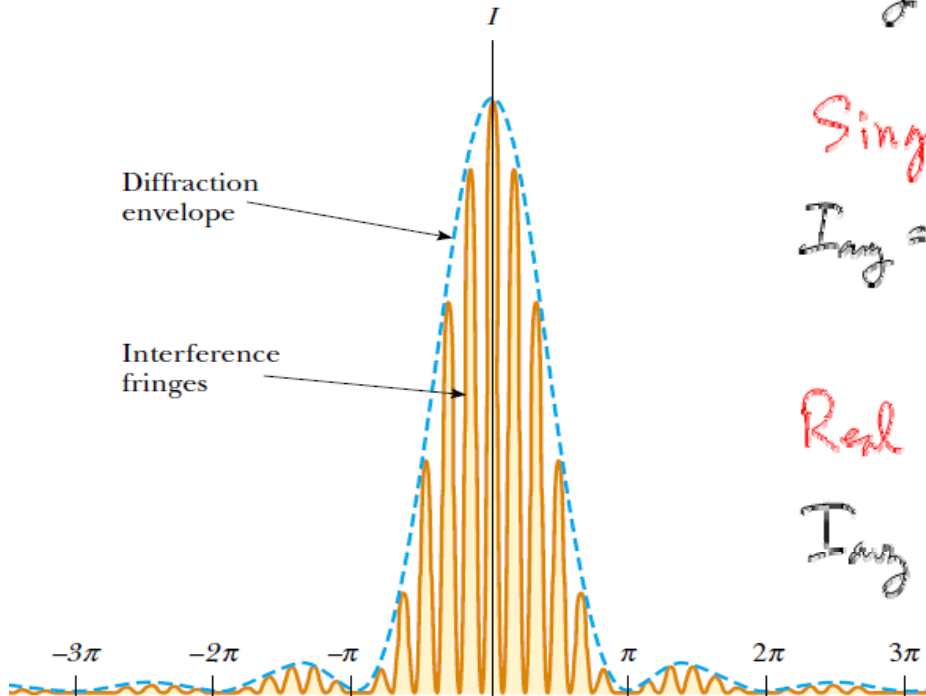
$$I_{avg} = I_{max} \cos^2\left(\frac{\pi y d}{\lambda L}\right)$$

Single Slit

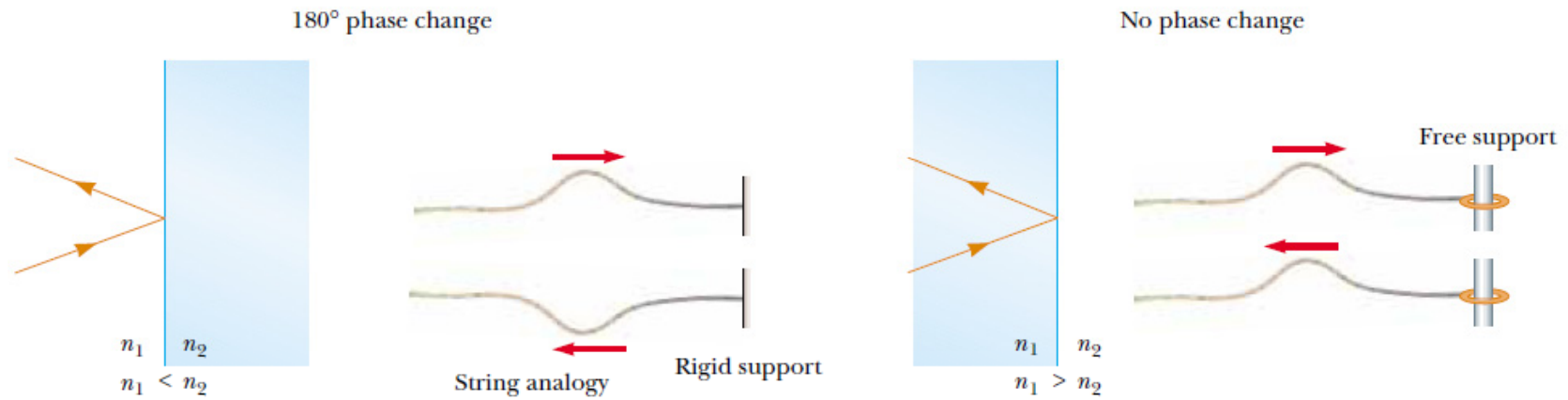
$$I_{avg} = I_{max} \left[\frac{\sin \pi \frac{a y}{\lambda L}}{\pi \frac{a y}{\lambda L}} \right]^2$$

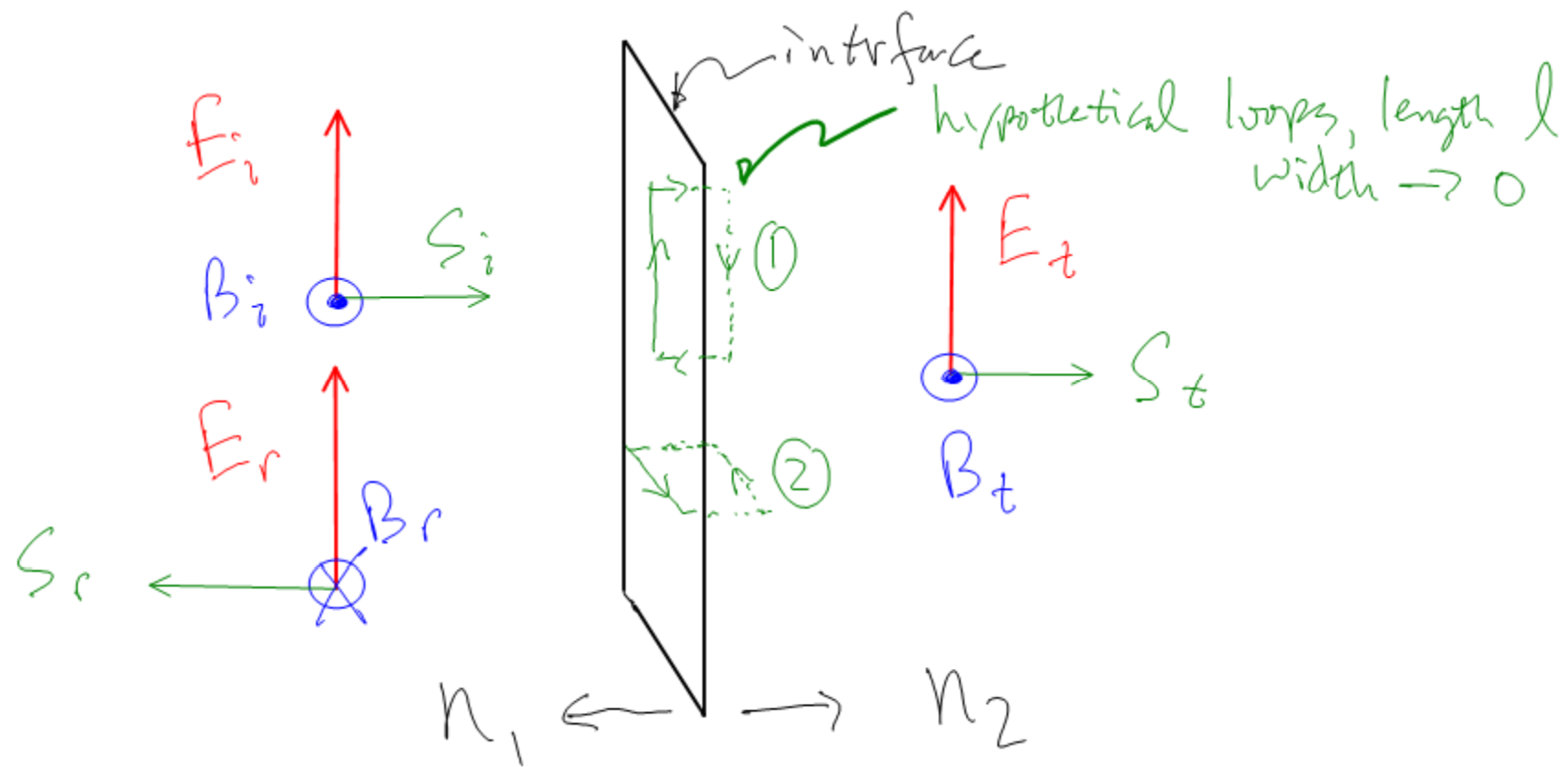
Real double slit

$$I_{avg} = I_{max} \cos^2\left(\pi \frac{y d}{\lambda L}\right) \left[\frac{\sin \pi \frac{a y}{\lambda L}}{\pi \frac{a y}{\lambda L}} \right]^2$$



Interference in parallel dielectrics



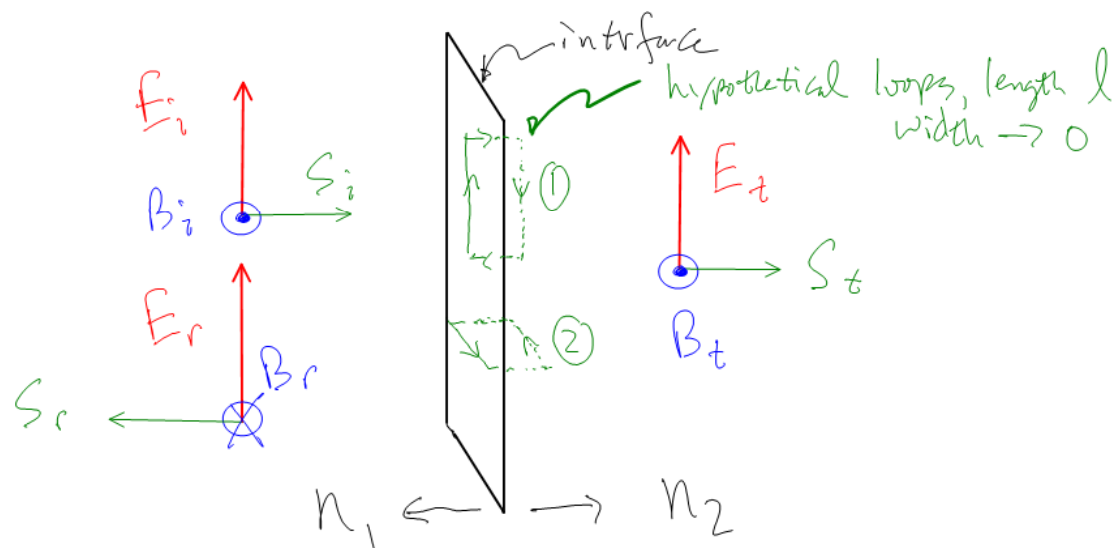


Maxwell's Equations:

$$\textcircled{1} \oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt} \rightarrow 0 \text{ as width} \rightarrow 0$$

$$\Rightarrow (E_i + E_r)l - E_t l = 0$$

$$\Rightarrow E_i + E_r = E_t \quad \textcircled{1}$$



$$E_i + E_r = E_t \quad (1)$$

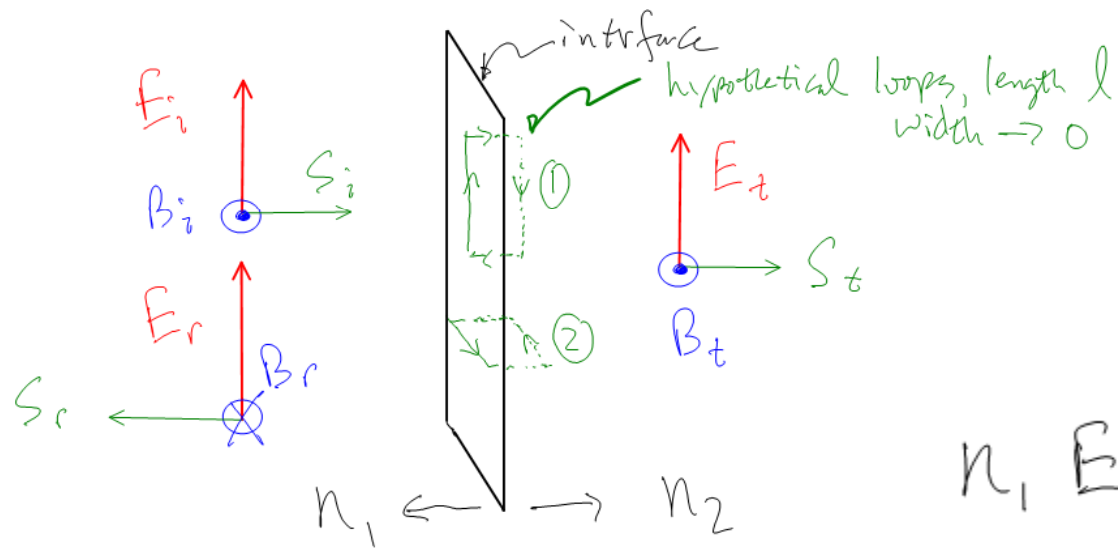
$$(2) \quad \int \vec{B} \cdot d\vec{S} = \mu_0 \left(\underbrace{I_{enc}}_{L_0} + \epsilon_0 \frac{d\Phi_E}{dt} \right) \rightarrow 0 \text{ as width} \rightarrow 0$$

Also know $B_i = \frac{E_i}{v_1}$, $B_r = \frac{E_r}{v_1}$, $B_t = \frac{E_t}{v_2}$

where $n_1 \equiv \frac{c}{v_1}$, $n_2 \equiv \frac{c}{v_2}$

$$(B_i - B_r)l - B_t l = 0$$

$$\Rightarrow n_1 E_i - n_1 E_r = n_2 E_t \quad (2)$$



$$E_i + E_r = E_t \quad (1)$$

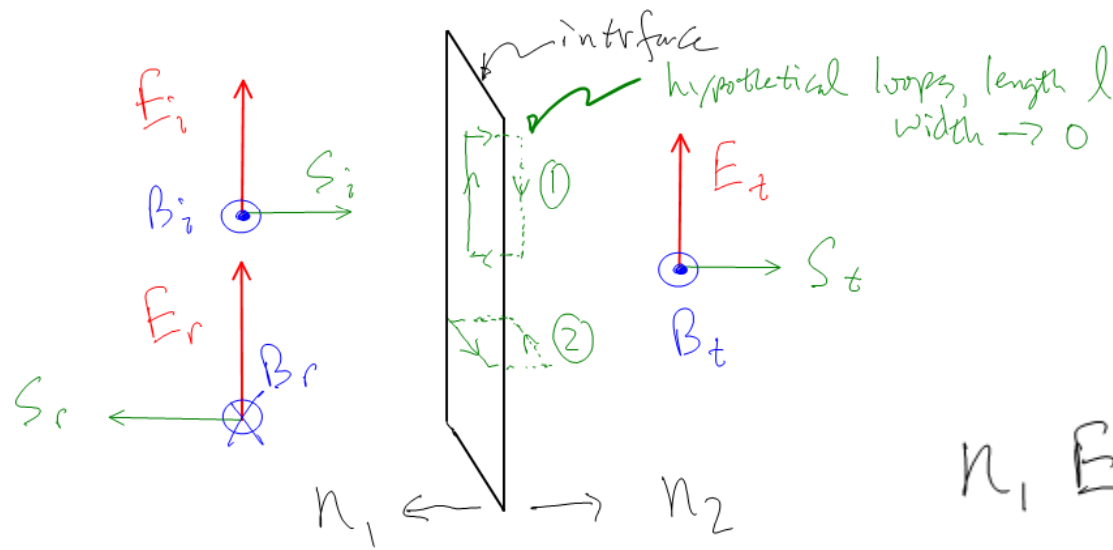
$$n_1 E_i - n_1 E_r = n_2 E_t \quad (2)$$

$$(eq 2) - n_2 (eq 1) \Rightarrow (n_1 - n_2) E_i + (-n_1 - n_2) E_r = 0$$

$$\Rightarrow r \equiv \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$(eq 2) + n_1 (eq 1) \Rightarrow 2 n_1 E_i = (n_1 + n_2) E_t$$

$$\Rightarrow t \equiv \frac{E_t}{E_i} = \frac{2 n_1}{n_1 + n_2}$$



$$E_i + E_r = E_t \quad (1)$$

$$n_1 E_i - n_1 E_r = n_2 E_t \quad (2)$$

Fresnel Coefficients:

$$r \equiv \frac{E_r}{E_i} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$t \equiv \frac{E_t}{E_i} = \frac{2n_1}{n_1 + n_2}$$

$r < 0$ when $n_2 > n_1 \Rightarrow E_r$ pts in opposite direction from E_i ! like π phase shift!
 $r > 0$ when $n_1 > n_2 \Rightarrow E_r$ & E_i point in same direction

Dielectrics Antireflection Coatings

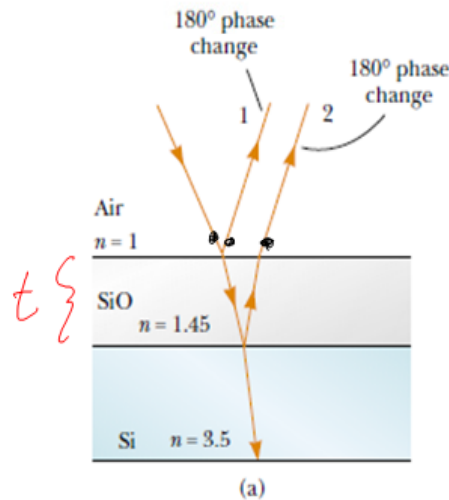


Figure 37.20 (Example 37.4) (a) Reflective losses from a silicon solar cell are minimized by coating the surface of the cell with a thin film of silicon monoxide. (b) The reflected light from a coated camera lens often has a reddish-violet appearance.

Lens AR Coating: (assume normal incidence)

Can adjust path length in SiO to minimize reflection via destructive interference:

incoming wave, just outside SiO surface, has phase ϕ_i

ray ①: $\phi_i + \pi$ ← from "hard" reflection

ray ②: $\phi_i + \pi + k\delta$, $\delta = 2 \cdot t$, $k = \frac{2\pi}{(\lambda/n)}$
 ← "hard" reflection

Note: Since $v = \frac{c}{n}$ & $v = \lambda f$

& freq on both sides of interface must be same

$$\Rightarrow c = \lambda_0 f, v = \lambda_n f = \frac{c}{n} \Rightarrow \lambda_n = \frac{\lambda_0}{n}$$

Destructive: $\Delta\phi = (2m+1)\pi \Rightarrow k\delta = 2\pi(m + \frac{1}{2})$

$$\Rightarrow \frac{2t}{(\lambda/n)} = (m + \frac{1}{2}) \Rightarrow t = \frac{1}{2} \frac{\lambda}{n} (m + \frac{1}{2})$$

Interference in thin films dielectrics

For given thickness, find max reflection:

ray ①: $\phi_i + \pi$ from "hard" reflection

ray ②: $\phi_i + 0 + k\delta$, $\delta = 2 \cdot t$, $k = \frac{2\pi}{(\lambda/n)}$
"soft" reflection

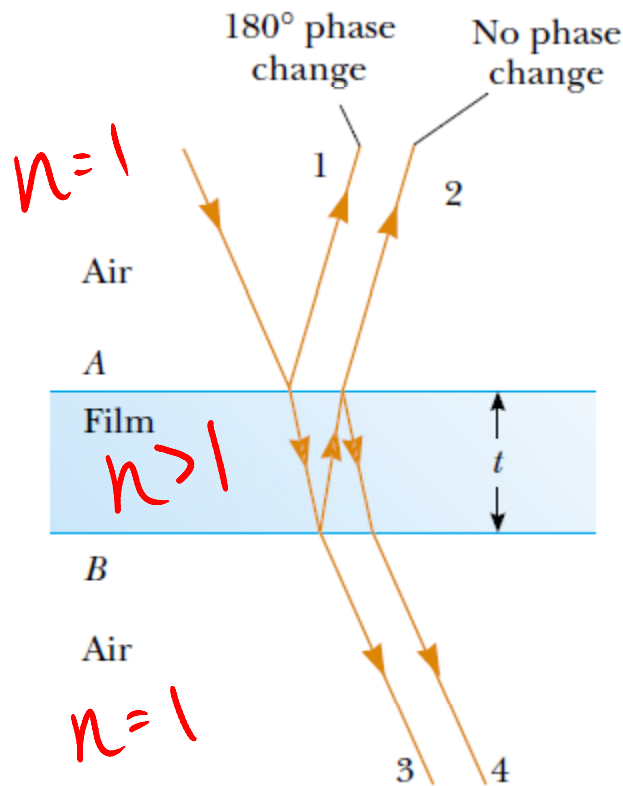
Constructive: $\Delta\phi = 2m\pi \Rightarrow k\delta - \pi = 2m\pi$

$$\Rightarrow k\delta = (2m+1)\pi \Rightarrow \frac{2t}{(\lambda/n)} = \frac{(2m+1)}{2}$$

$$\Rightarrow t = \frac{1}{4} \frac{\lambda_0}{n} (2m+1)$$

or, since $\lambda f = c$

$$\Rightarrow f_{\max} = \frac{1}{4} \frac{c}{nt} (2m+1)$$



Interference in thin films dielectrics

Dr. Jeremy Burgess/Science Photo Library

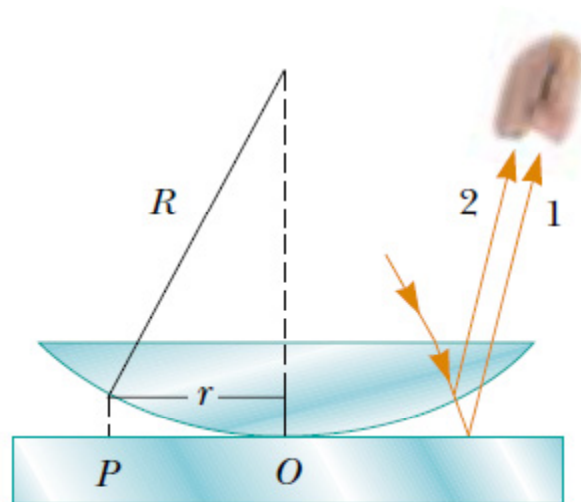


Peter Aprahamian/Science Photo Library/Photo Researchers, Inc.



$$f_{\text{max}} = \frac{1}{4} \frac{c}{n t} (2m+1)$$

Newton's rings



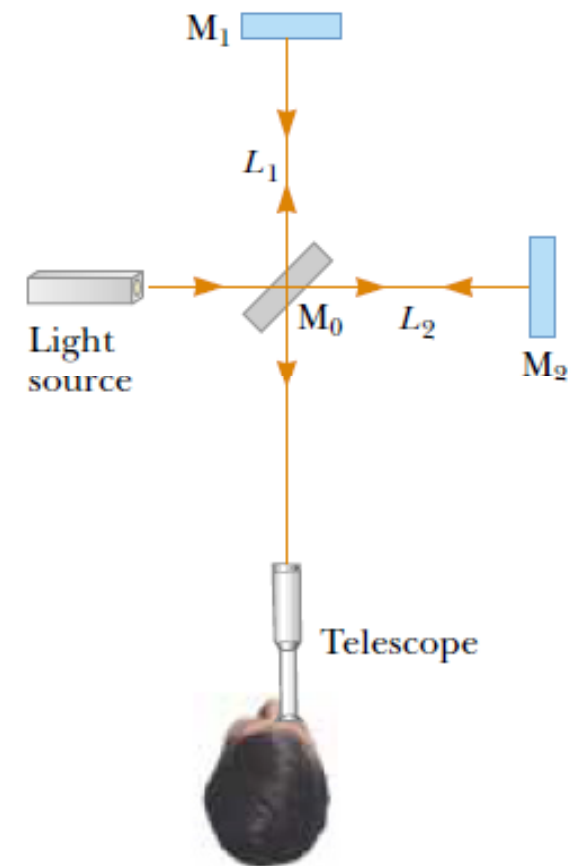
(a)



(b)

Figure 37.18 (a) The combination of rays reflected from the flat plate and the curved lens surface gives rise to an interference pattern known as Newton's rings. (b) Photograph of Newton's rings.

Michelson Interferometer



$$\Delta OPL = 2(L_2 - L_1) \equiv \delta$$

$$\Delta \phi = \frac{2\pi}{\lambda} \delta = \frac{2\pi}{\lambda} 2(L_2 - L_1) = \frac{4\pi}{c} (L_2 - L_1) f$$

$2f = c$

$$E = 2 E_0 \cos\left(\frac{\Delta \phi}{2}\right) \cos(kx_{avg} - \omega t + \phi_{avg})$$

$$I_{avg} = I_0 \cos^2\left(\frac{\Delta \phi}{2}\right) = I_0 \cos^2\left(\frac{2\pi}{c} (L_2 - L_1) f\right), \quad 2\pi f = \omega$$

FTIR Spectrometer $\Rightarrow L_2(t) = v_0 t$

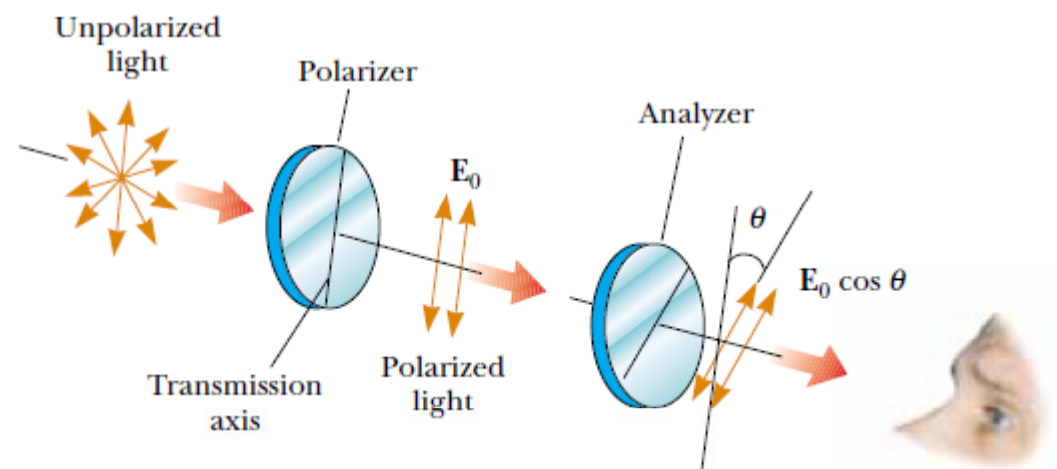
Mirror moves @ constant velocity

$$I_{avg}(t) = I_0 \cos^2\left(\frac{v_0}{c} \omega t\right) \quad \text{for } L_2 = L_1 \text{ @ } t = 0$$

Laser Interferometer -- LIGO



Figure 37.23 The Laser Interferometer Gravitational-Wave Observatory (LIGO) near Richland, Washington. Note the two perpendicular arms of the Michelson interferometer.



Active Figure 38.30 Two polarizing sheets whose transmission axes make an angle θ with each other. Only a fraction of the polarized light incident on the analyzer is transmitted through it.

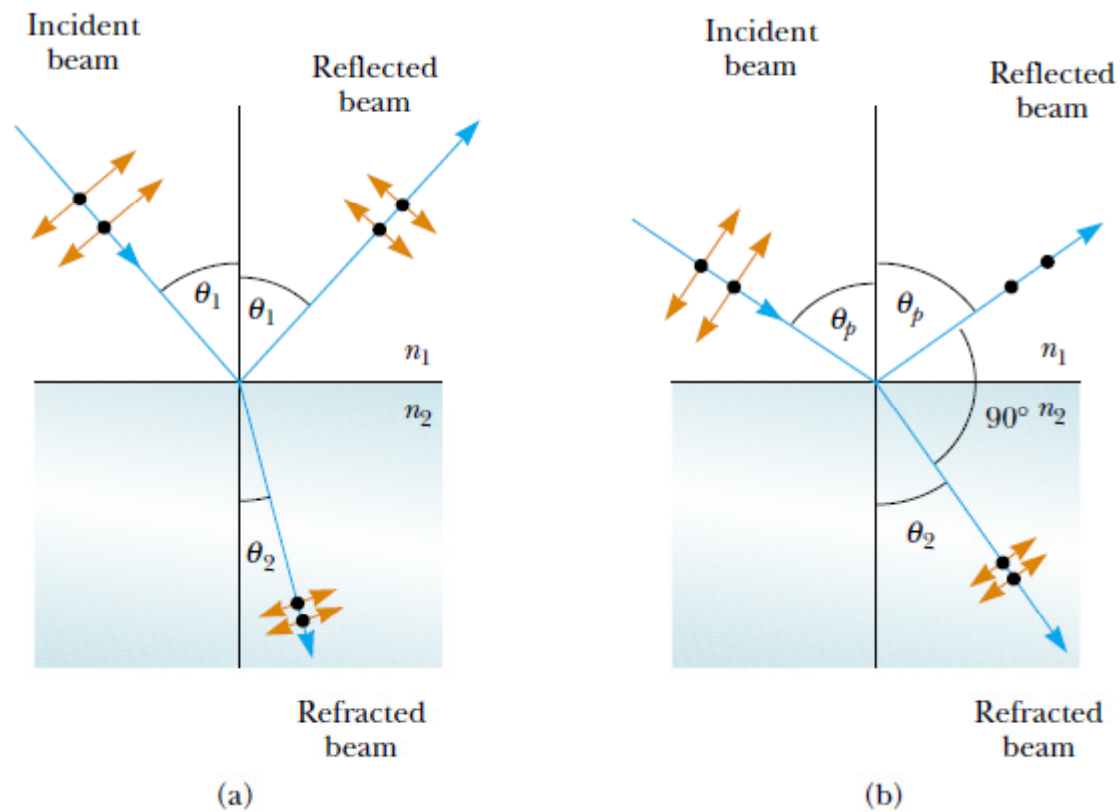


Figure 38.32 (a) When unpolarized light is incident on a reflecting surface, the reflected and refracted beams are partially polarized. (b) The reflected beam is completely polarized when the angle of incidence equals the polarizing angle θ_p , which satisfies the equation $n = \tan\theta_p$. At this incident angle, the reflected and refracted rays are perpendicular to each other.

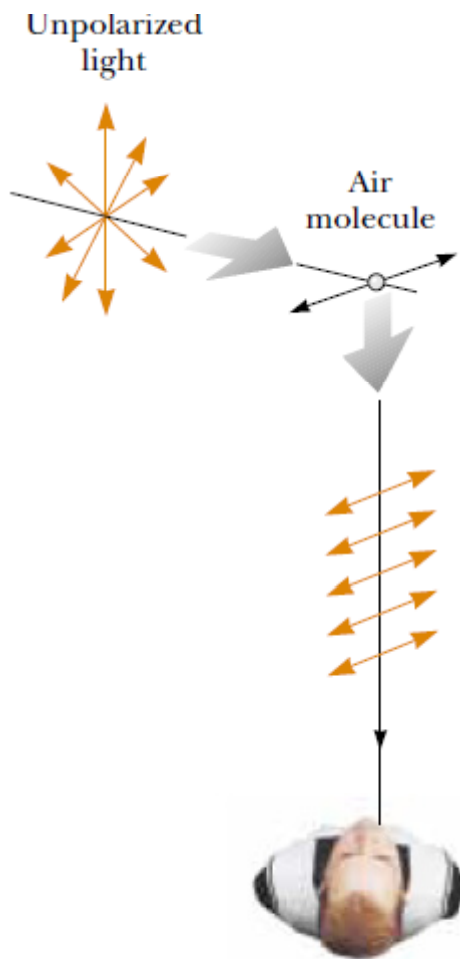


Figure 38.37 The scattering of unpolarized sunlight by air molecules. The scattered light traveling perpendicular to the incident light is plane-polarized because the vertical vibrations of the charges in the air molecule send no light in this direction.