HW # 9

Ch – 37: RELATIVITY

CONCEPTUAL QUESTIONS

37.3. Event 1 occurs after event 2. The flash of light from event 2 has to travel twice as far as the flash of light from event 1 and will take twice as long to travel that longer distance.

37.4. Your lab partner is in the same reference frame as you are and so, with appropriate calculations and allowances for light travel time, will conclude, as you do, that the two events are simultaneous.

37.5. **(a)** Yes, they are simultaneous in Peggy's reference frame because she and the firecrackers are at rest relative to one another and she is halfway between them and saw the explosions at the same time.
(b) No, the left one occurred first because it had a farther distance for its light to reach Peggy since she was moving toward the right one.

37.6. **(a)** No. During the time the flashes of light are traveling, the rocket is moving to the right. So the flash from the right lightning strike has less distance to travel to get to the rocket and therefore reaches the rocket pilot first.
(b) No. Two events which are simultaneous in reference frame S are not simultaneous in any reference frame moving relative to S. The student sees the tree on the left hit first. He is moving to the left relative to the frame of the rocket where the strikes were simultaneous. So he moves toward the wave front on the left and away from the one on the right.

37.7. **(a)** Event 1 is your friend leaving Los Angeles; event 2 is your friend arriving in New York.
(b) Your friend.
(c) Your friend.

37.8. **(a)** No, you measured the left end first.
(b) Yes, experimenters in S' are at rest relative to the meter stick so they are measuring the proper length and the proper time.

37.9. Yes, the experimenters on the ground will measure the train as length contracted, and if it is going fast enough $L < 80$ m.

EXERCISES AND PROBLEMS

37.2. Model: S and S′ are inertial frames. S′ moves relative to S with speed v.
Solve: **(a)** Using the Galilean transformations of position,

$$x_1' = x_1 - vt_1 \Rightarrow 4.0\text{ m} = x_1 - v\,(1.0\text{ s}) \Rightarrow x_1 = 4.0\text{ m} + v\,(1.0\text{ s})$$

$$x_2' = x_2 - vt_2 \Rightarrow -4.0\text{ m} = x_2 - v\,(3.0\text{ s}) \Rightarrow x_2 = -4.0\text{ m} + v\,(3.0\text{ s})$$

Because $x_1 = x_2$,

$$4.0\text{ m} + v\,(1.0\text{ s}) = -4.0\text{ m} + v\,(3.0\text{ s}) \Rightarrow v = 4.0\text{ m/s}$$

(b) The positions of the two explosions in the S frame are

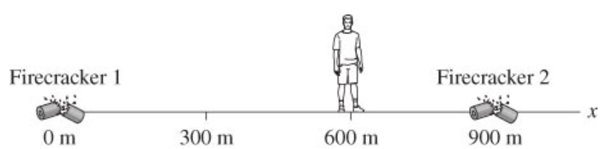
$$x_1 = 4.0\text{ m} + (4.0\text{ m/s})(1.0\text{ s}) = 8.0\text{ m} \qquad x_2 = -4.0\text{ m} + (4.0\text{ m/s})(3.0\text{ s}) = 8.0\text{ m}$$

37.6. Model: Assume the spacecraft is an inertial reference frame.
Solve: Light travels at speed c in all inertial reference frames, regardless of how the reference frames are moving with respect to the light source. Relative to the spacecraft, the starlight is approaching at the speed of light $c = 3.00 \times 10^8$ m/s.

37.7. Model: Assume the starship and the earth are inertial reference frames.
Solve: It has been found that light travels at 3.00×10^8 m/s in every inertial frame, regardless of how the reference frames are moving with respect to each other. An observer on the earth will measure the laser beam's speed as 3.00×10^8 m/s.

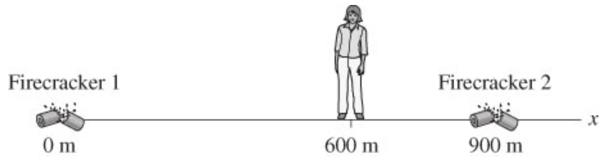
37.8. Model: Assume the earth is an inertial reference frame.
Solve: Light travels at speed c in all inertial reference frames, regardless of their motion with respect to the light source. The speed of each photon will be c in any such inertial reference frame.

37.10. Model: Bjorn and firecrackers 1 and 2 are in the same reference frame. Light from both firecrackers travels towards Bjorn at 300 m/μs.
Visualize:



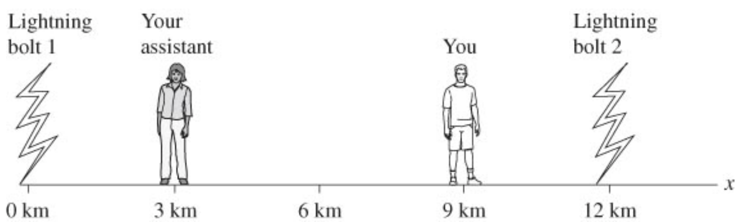
Solve: Bjorn is 600 m from the origin. Light with a speed of 300 m/μs takes 2.0 μs to reach Bjorn. Since this flash reaches Bjorn at $t = 3.0$ μs, it left firecracker 1 at $t_1 = 1.0$ μs. The flash from firecracker 2 takes 1.0 μs to reach Bjorn. So, the light left firecracker 2 at $t_2 = 2.0$ μs. Note that the two events are *not* simultaneous although Bjorn *sees* the events as occurring at the same time.

37.11. Model: Bianca and firecrackers 1 and 2 are in the same reference frame. Light from both firecrackers travels toward Bianca at 300 m/μs.
Visualize:



Solve: The flash from firecracker 1 takes 2.0 μs to reach Bianca $(600 \text{ m} \div 300 \text{ m}/\mu\text{s})$. The firecracker exploded at t_1 = 1.0 μs because it reached Bianca's eye at 3.0 μs. The flash from the firecracker 2 takes 1.0 μs to reach Bianca. Since firecrackers 1 and 2 exploded simultaneously, the explosion occurs at t_2 = 1.0 μs. So, the light from firecracker 2 reaches Bianca's eye at 2.0 μs. Although the events are simultaneous, Bianca *sees* them occurring at different times.

37.12. Model: You and your assistant are in the same reference frame. Light from the two lightning bolts travels toward you and your assistant at 300 m/μs. You and your assistant have synchronized clocks.
Visualize:



Solve: Bolt 1 is 9.0 km away, so it takes 30 μs for the light to reach you $(9000 \text{ m} \div 300 \text{ m}/\mu\text{s})$. Bolt 2 is 3.0 km away from you, so it takes 10 μs to reach you. Since both flashes reach your eye at the same time, event 1 happened 20 μs before event 2. If event 1 happened at time $t_1 = 0$ then event 2 happened at time $t_2 = 20$ μs. For your assistant, it takes light from bolt 1 10 μs to reach her and light from bolt 2 30 μs to reach her. She sees the flash from bolt 1 at $t = 10$ μs and the flash from bolt 2 at $t = 50$ μs. That is, your assistant sees flash 2 40 μs after she sees flash 1.

37.15. Model: Your personal rocket craft is an inertial frame moving at 0.9*c* relative to stars A and B.
Solve: In your frame, star A is moving away from you and star B is moving toward you. When you are exactly halfway between them, both the stars explode simultaneously. The flashes from the two stars travel toward you with speed *c*. Because (i) you are at rest in your frame, (ii) the explosions are equally distant, and (iii) the light speed is *c*, independent of the fact that the stars are moving in your frame, the light will arrive simultaneously.

37.16. Model: The earth is in reference frame S and the cosmic ray is in reference frame S′. Frame S′ travels with velocity v relative to frame S.
Solve: Two events are "cosmic ray enters the atmosphere" and "cosmic ray hits the ground." These can both be measured with a single clock in the cosmic ray's frame, frame S′, so the time interval between them in S′ is the proper time interval: $\Delta t' = \Delta\tau$. The time interval measured in the earth's frame, frame S, is $\Delta t = 400$ μs. The time-dilation result is

$$\Delta\tau = \sqrt{1-\beta^2}\ \Delta t$$

The cosmic ray's speed in frame S is simply

$$v = \frac{\Delta L}{\Delta t} = \frac{60{,}000 \text{ m}}{400 \times 10^{-6} \text{ s}} = 1.5 \times 10^8 \text{ m/s} \Rightarrow \beta = \frac{v}{c} = 0.50$$

Thus the time interval measured by the cosmic ray is

$$\Delta\tau = \sqrt{1-(0.50)^2}\ (400\ \mu\text{s}) = 346\ \mu\text{s}$$

37.17. Model: Let the moving clock be in frame S′ and an identical at-rest clock be in frame S.
Solve: The ticks being measured are those of the moving clock. The interval between 2 ticks is measured by the same clock in S′—namely, the clock that is ticking—so this is the proper time: $\Delta t' = \Delta\tau$. The rest clock measures a longer interval Δt between two ticks of the moving clock. These are related by

$$\Delta\tau = \sqrt{1-\beta^2}\ \Delta t$$

The moving clock ticks at half the rate of the rest clock when $\Delta\tau = \frac{1}{2}\Delta t$. Thus
$\sqrt{1-\beta^2} = \sqrt{1-v^2/c^2} = 1/2 \Rightarrow v = c\sqrt{1-(1/2)^2} = 0.866c$

37.18. Solve: **(a)** The starting event is the astronaut leaving earth. The finishing event is the astronaut arriving at the star system. The time between these events as measured on earth is

$$\Delta t = \frac{4.5\text{ ly}}{0.9c} = \frac{4.5\text{ ly}}{0.9\text{ ly/y}} = 5.0\text{ y}$$

(b) For the astronaut, the two events occur at the same position and can be measured with just one clock. Thus, the time interval in the astronaut's frame is the proper time interval.

$$\Delta\tau = \sqrt{1-\frac{v^2}{c^2}}\Delta t = \sqrt{1-\left(\frac{0.9c}{c}\right)^2}\,5.0\text{ y} = \sqrt{0.19}\ 5.0\text{ y} = 2.2\text{ y}$$

(c) The total elapsed time is the time for the astronaut to reach the star system plus the time for light to travel from the star system to the earth. The time is

37.19. Model: Let S be the earth's reference frame and S′ the rocket's reference frame.
Solve: **(a)** The astronauts measure proper time i $\Delta t' = \Delta\tau$. Thus

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-(v/c)^2}} \Rightarrow 120\text{ y} = \frac{10\text{ y}}{\sqrt{1-(v/c)^2}} \Rightarrow v = 0.9965c$$

(b) In frame S, the distance of the distant star is

$$\Delta x = v\Delta t = (0.9965c)(60\text{ y}) = (0.9965\text{ ly/y})(60\text{ y}) = 59.8\text{ ly}$$

$$\Delta t + 4.5\text{ y} = 5.0\text{ y} + 4.5\text{ y} = 9.5\text{ y}$$

37.20. Model: The earth's frame is S and the airliner's frame is S′. S′ moves relative to S with velocity v. Also, assume zero acceleration/ deceleration times.

The first event is when the airliner takes off and almost instantly attains a speed of v = 250 m/s. The second event is when the airliner returns to its original position after 2 days. It is clear that the two events occur at the *same position* in frame S′ and can be measured with just one clock. This is however not the case for an observer in frame S.
Solve: **(a)** You have aged less because your proper time is less than the time in the earth frame.
(b) In the S frame (earth),

$$\Delta t = \frac{2\times5\times10^6\text{ m}}{250\text{ m/s}} = 4.0\times10^4\text{ s}$$

In the S′ frame (airliner),

$$\Delta\tau = \Delta t\left(1-v^2/c^2\right)^{\frac{1}{2}} \approx \Delta t\left(1-\frac{v^2}{2c^2}\right)$$

$$\Rightarrow \Delta t - \Delta\tau \approx \Delta t\frac{v^2}{2c^2} = \left(4.0\times10^4\text{ s}\right)\frac{1}{2}\left(\frac{250\text{ m/s}}{3.0\times10^8\text{ m}}\right)^2 = 1.4\times10^{-8}\text{ s} = 14\text{ ns}$$

You age 14 ns less than your stay-at-home friends.

37.22. Model: The ground is frame S and the moving rod is frame S′. The length of the rod in S′ is the proper length ℓ because the rod is at rest in S′. The rod is length contracted in S to

$$L = \sqrt{1-\beta^2}1$$

The length is contracted to 60% when $L = 0.60\,\ell$. Thus

$$\sqrt{1-\beta^2} = \sqrt{1-v^2/c^2} = 0.60 \Rightarrow v = c\sqrt{1-(0.60)^2} = 0.80c$$

37.24. Model: S′ is the muon's frame and S is the ground's frame. S′ moves relative to S with a speed of 0.9997c.

Solve: For an experimenter in the ground's frame, a distance of 60 km (or L) is always there for measurements. That is, L is the atmosphere's proper length ℓ. The muon measures the thickness of the atmosphere to be length contracted to

$$L' = \sqrt{1-\beta^2}\,\ell = \sqrt{1-(0.9997)^2}\,(60 \text{ km}) = 1.47 \text{ km}$$

37.26. Model: S is the galaxy's reference frame and S′ is the spaceship's reference frame. S′ moves relative to S with a speed v.

Solve: **(a)** For an experimenter in the galaxy's reference frame, the diameter (or length) of the galaxy is 10^5 ly. This is the proper length $\ell = L$ because it is at rest and is always there for measurements. However, in the spaceship's reference frame S′, the galaxy moves toward him/her with speed v. S′ measures the galaxy to be length contracted to L' = 1.0 ly. Thus

$$L' = \ell\sqrt{1-\beta^2} \Rightarrow 1.0 \text{ ly} = (10^5 \text{ ly})\sqrt{1-\beta^2}$$

$$\Rightarrow \beta = \sqrt{1-10^{-10}} \approx 1 - \tfrac{1}{2}\times 10^{-10} \Rightarrow v = (1-5.0\times 10^{-11})c = 0.99999999995c$$

(b) In S, the spaceship travels 100,000 ly at speed v = 0.99999999995c, taking

$$\Delta t = \frac{L}{v} \approx 100{,}000 \text{ y}$$