Publisher's Solution for HW # 8

CONCEPTUAL QUESTIONS

25.1. θ_1 decreases. As the crystal is compressed, the spacing d between the planes of atoms decreases. The Bragg condition is $m\lambda = 2d\cos\theta_m$ so as d decreases, $\cos\theta_m$ must increase. But $\cos\theta$ increases as θ decreases.

25.2.(a) $E_a > E_b > E_c$ because the energy per photon depends only on the frequency so $E = hf = hc/\lambda$. The smaller wavelengths correspond to higher frequencies. **(b)** $N_c > N_b > N_a$ because the powers are equal, there must be more photons when the energy per photon is less.

25.3. $\frac{E_2}{E_1} = \frac{hc/\lambda_2}{hc/\lambda_1} = \frac{\lambda_1}{2\lambda_1} = \frac{1}{2}$

25.5. Fast electrons will have a shorter wavelength leading to less diffraction spreading and better resolution.

25.7. Because $E_n = n^2\frac{h^2}{8mL^2}$ we see that for a given n, E_n is inversely proportional to L^2. If L is doubled then E_n is decreased by a factor of 4. So the new $E_1 = 1\times 10^{-19}$ J.

25.8. It is the same, or 1.0×10^{-20} J. $E_{H_1} = \frac{h^2}{8m_0L_0^2}$ $\quad E_{He_1} = \frac{h^2}{8(4m_0)\left(\frac{L_0}{2}\right)^2} = E_{H_1}$

EXERCISES AND PROBLEMS

25.4. Model: The angles of incidence for which diffraction from parallel planes occurs satisfy the Bragg condition.
Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where $m = 1, 2, 3, \ldots$ For first and second order diffraction,

$$2d\cos\theta_1 = (1)\lambda \quad , \quad 2d\cos\theta_2 = (2)\lambda$$

Dividing these two equations,

$$\frac{\cos\theta_2}{\cos\theta_1} = 2 \Rightarrow \theta_2 = \cos^{-1}(2\cos\theta_1) = \cos^{-1}(2\cos 68^\circ) = 41^\circ$$

25.7.Model: The angles corresponding to the various diffraction orders satisfy the Bragg condition.

Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where $m = 1, 2, 3, \ldots$ The maximum possible value of m is the number of possible diffraction orders. The maximum value of $\cos\theta_m$ is 1. Thus,

$$2d = m\lambda \Rightarrow m = \frac{2d}{\lambda} = \frac{2(0.180 \text{ nm})}{(0.085 \text{ nm})} = 4.2$$

We can observe up to the fourth diffraction order.

25.10.Model: Use the photon model of light.

Solve: The energy of a photon with wavelength λ_1 is $E_1 = hf_1 = hc/\lambda_1$. Similarly, $E_2 = hc/\lambda_2$. Since E_2 is equal to $2E_1$,

$$\frac{hc}{\lambda_2} = 2\frac{hc}{\lambda_1} \Rightarrow \lambda_2 = \frac{\lambda_1}{2} = \frac{600 \text{ nm}}{2} = 300 \text{ nm}$$

Assess: A photon with $\lambda = 300$ nm has twice the energy of a photon with $\lambda = 600$ nm. This is an expected result, because energy is inversely proportional to the wavelength.

25.12.Solve: Your mass is, say, $m \approx 70$ kg and your velocity is 1 m/s. Thus, your momentum is $p = mv \approx (70 \text{ kg})(1 \text{ m/s}) = 70$ kg m/s. Your de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34} \text{ Js}}{70 \text{ kg m/s}} \approx 9\times10^{-36} \text{ m}$$

25.14.Visualize: We'll employ Equations 25.8 $(\lambda = h/p)$ and 25.9 $(E = p^2/2m)$ to express the wavelength in terms of kinetic energy.

Solve: First solve Equation 25.9 for p: $p = \sqrt{2mE}$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34} \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31} \text{ kg})(2.4\times10^{-19} \text{J})}} = 1.0 \text{ nm}$$

Assess: The energy given is about 1.5 eV, which is a reasonable amount of energy. The resulting wavelength is a few to a few dozen times the size of an atom.

25.18.Model: Model the 5.0-fm-diameter nucleus as a box of length $L = 5.0$ fm.

Solve: The proton's energy is restricted to the discrete values

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{(6.63\times10^{-34} \text{ Js})^2 n^2}{8(1.67\times10^{-27} \text{ kg})(5.0\times10^{-15} \text{ m})^2} = (1.316\times10^{-12} \text{ J})n^2$$

where $n = 1, 2, 3, \ldots$ For $n = 1$, $E_1 = 1.3\times10^{-12}$ J, for $n = 2$, $E_2 = (1.316\times10^{-12} \text{ J})4 = 5.3\times10^{-12}$ J, and for $n = 3$, $E_3 = 9E_1 = 1.2\times10^{-11}$ J.

25.21.Model: Use the photon model of light.

Solve:(a) The wavelength is calculated as follows:

$$E_{\text{gamma}} = hf = h\left(\frac{c}{\lambda}\right) \Rightarrow \lambda = \frac{(6.63\times 10^{-34}\ \text{Js})(3.0\times 10^{8}\ \text{m/s})}{1.0\times 10^{-13}\ \text{J}} = 2.0\times 10^{-12}\ \text{m}$$

(b) The energy of a visible-light photon of wavelength 500 nm is

$$E_{\text{visible}} = h\left(\frac{c}{\lambda}\right) = \frac{(6.63\times 10^{-34}\ \text{Js})(3.0\times 10^{8}\ \text{m/s})}{500\times 10^{-9}\ \text{m}} = 3.978\times 10^{-19}\ \text{J}$$

The number of photons n such that $E_{\text{gamma}} = nE_{\text{visible}}$ is

$$n = \frac{E_{\text{gamma}}}{E_{\text{visible}}} = \frac{1.0\times 10^{-13}\ \text{J}}{3.978\times 10^{-19}\ \text{J}} = 2.5\times 10^{5}$$

25.22.Model: Use the photon model.

Solve: The energy of a 1000 kHz photon is

$$E_{\text{photon}} = hf = (6.63\times 10^{-34}\ \text{Js})(1000\times 10^{3}\ \text{Hz}) = 6.63\times 10^{-28}\ \text{J}$$

The energy transmitted each second is 20×10^{3} J. The number of photons transmitted each second is $20\times 10^{3}\ \text{J}/6.63\times 10^{-28}\ \text{J} = 3.0\times 10^{31}$.

25.25.Model: Use the photon model of light and the Bragg condition for diffraction.

Solve: The Bragg condition for the reflection of x-rays from a crystal is $2d\cos\theta_m = m\lambda$. To determine the angles of incidence θ_m, we need to first calculate λ. The wavelength is related to the photon's energy as $E = hc/\lambda$. Thus,

$$\lambda = \frac{hc}{E} = \frac{(6.63\times 10^{-34}\ \text{Js})(3.0\times 10^{8}\ \text{m/s})}{1.50\times 10^{-15}\ \text{J}} = 1.326\times 10^{-10}\ \text{m}$$

From the Bragg condition,

$$\theta_m = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left[\frac{(1.326\times 10^{-10}\ \text{m})m}{2(0.21\times 10^{-9}\ \text{m})}\right] = \cos^{-1}(0.3157m) \Rightarrow \theta_1 = \cos^{-1}(0.3157) = 71.6^\circ$$

Likewise, $\theta_2 = \cos^{-1}(0.3157\times 2) = 50.8^\circ$ and $\theta_3 = 18.7^\circ$. Note that $\theta_4 = \cos^{-1}(0.3157\times 4)$ is not allowed because the $\cos\theta$ cannot be larger than 1. Thus, the x-rays will be diffracted at angles of incidence equal to 18.7°, 50.8°, and 71.6°.

25.29.Model: Particles have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the particles causes an interference pattern in a double-slit apparatus.

Solve:(a) Since the speed of the neutron and electron are the same, the neutron's momentum is

$$p_n = m_n v_n = \frac{m_n}{m_e} m_e v_n = \frac{m_n}{m_e} m_e v_e = \frac{m_n}{m_e} p_e$$

where m_n and m_e are the neutron's and electron's masses. The de Broglie wavelength for the neutron is

$$\lambda_n = \frac{h}{p_n} = \frac{h}{p_e}\frac{p_e}{p_n} = \lambda_e \frac{m_e}{m_n}$$

From Section 22.2 on double-slit interference, the fringe spacing is $\Delta y = \lambda L/d$. Thus, the fringe spacing for the electron and neutron are related by

$$\Delta y_n = \frac{\lambda_n}{\lambda_e}\Delta y_e = \frac{m_e}{m_n}\Delta y_e = \left(\frac{9.11\times 10^{-31}\text{ kg}}{1.67\times 10^{-27}\text{ kg}}\right)\left(1.5\times 10^{-3}\text{ m}\right) = 8.18\times 10^{-7}\text{ m} = 0.818\ \mu\text{m}$$

(b) If the fringe spacing has to be the same for the neutrons and the electrons,

$$\Delta y_e = \Delta y_n \Rightarrow \lambda_e = \lambda_n \Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_n v_n} \Rightarrow v_n = v_e \frac{m_e}{m_n} = \left(2.0\times 10^6\text{ m/s}\right)\left(\frac{9.11\times 10^{-31}\text{ kg}}{1.67\times 10^{-27}\text{ kg}}\right) = 1.1\times 10^3\text{ m/s}$$

25.30.Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the electrons causes a diffraction pattern.

Solve: The width of the central maximum of a single-slit diffraction pattern is given by Equation 22.22:

$$w = \frac{2\lambda L}{a} = \frac{2Lh}{ap} = \frac{2Lh}{amv} = \frac{2(1.0\text{ m})\left(6.63\times 10^{-34}\text{ Js}\right)}{\left(1.0\times 10^{-6}\text{ m}\right)\left(9.11\times 10^{-31}\text{ kg}\right)\left(1.5\times 10^{6}\text{ m/s}\right)} = 9.7\times 10^{-4}\text{ m} = 0.97\text{ mm}$$

25.32.Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$.

Visualize: Please refer to Figure 25.11. [REFER TO THE SUPPLEMENTAL FOR THE IMAGE AND ANALYSIS THEREOF] Notice that a scattering angle $\phi = 60^\circ$ corresponds to an angle of incidence $\theta = 30^\circ$.

Solve: Equation 25.6 describes the Davisson-Germer experiment: $D\sin(2\theta_m) = m\lambda$. Assuming $m = 1$, this equation simplifies to $D\sin 2\theta = \lambda$. Using $\lambda = h/mv$, we have

$$D = \frac{h}{mv\sin 2\theta} = \frac{6.63\times 10^{-34}\text{ Js}}{\left(9.11\times 10^{-31}\text{ kg}\right)\left(4.30\times 10^{6}\text{ m/s}\right)\sin\left(60^\circ\right)} = 1.95\times 10^{-10}\text{ m} = 0.195\text{ nm}$$

25.33. Model: A confined particle can have only discrete values of energy.
Solve: (a) Equation 25.14 simplifies to

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{\left(6.63\times 10^{-34}\ \text{Js}\right)^2}{8\left(9.11\times 10^{-31}\ \text{kg}\right)\left(0.70\times 10^{-9}\ \text{m}\right)^2} = \left(1.231\times 10^{-19}\ \text{J}\right)n^2$$

Thus, $E_1 = \left(1.231\times 10^{-19}\,\text{J}\right)\left(1^2\right) = 1.2\times 10^{-19}\,\text{J}$, $E_2 = \left(1.231\times 10^{-19}\,\text{J}\right)\left(2^2\right) = 4.9\times 10^{-19}\,\text{J}$, and $E_3 = 1.1\times 10^{-18}\,\text{J}$.

(b) The energy is $E_2 - E_1 = 4.9\times 10^{-19}\ \text{J} - 1.2\times 10^{-19}\ \text{J} = 3.7\times 10^{-19}\ \text{J}$.

(c) Because energy is conserved, the photon will carry an energy of $E_2 - E_1 = 3.69\times 10^{-19}\ \text{J}$. That is,

$$E_2 - E_1 = E_{\text{photon}} = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_2 - E_1} = \frac{\left(6.63\times 10^{-34}\ \text{Js}\right)\left(3.0\times 10^8\ \text{m/s}\right)}{3.69\times 10^{-19}\ \text{J}} = 540\ \text{nm}$$

25.34. Model: A particle confined in a one-dimensional box has discrete energy levels.
Solve: (a) Equation 24.14 for the $n = 1$ state is

$$E_n = \frac{h^2}{8mL^2} = \frac{\left(6.63\times 10^{-34}\ \text{Js}\right)^2}{8\left(10\times 10^{-3}\ \text{kg}\right)\left(0.10\ \text{m}\right)^2} = 5.5\times 10^{-64}\ \text{J}$$

The minimum energy of the Ping-Pong ball is $E_1 = 5.5\times 10^{-64}\ \text{J}$.
(b) The speed is calculated as follows:

$$E_1 = 5.50\times 10^{-64}\,\text{J} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\left(10\times 10^{-3}\ \text{kg}\right)v^2 \Rightarrow v = \sqrt{\frac{2\left(5.50\times 10^{-64}\ \text{J}\right)}{10\times 10^{-3}\ \text{kg}}} = 3.3\times 10^{-31}\ \text{m/s}$$

25.39. Model: A particle confined in a one-dimensional box of length L has the discrete energy levels given by Equation 24.14.
Solve: (a) Since the energy is entirely kinetic energy,

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{p^2}{2m} = \tfrac{1}{2}mv_n^2 \Rightarrow v_n = \frac{h}{2mL}n \qquad n = 1,\ 2,\ 3,\ \ldots$$

(b) The first allowed velocity is

$$v_1 = \frac{6.63\times 10^{-34}\text{Js}}{2\left(9.11\times 10^{-31}\ \text{kg}\right)\left(0.20\times 10^{-9}\ \text{m}\right)} = 1.82\times 10^6\ \text{m/s}$$

For $n = 2$ and $n = 3$, $v_2 = 3.64\times 10^6$ m/s and $v_3 = 5.46\times 10^6$ m/s.