Solutions for HW 8

Chapter 25

Conceptual Questions

25.1. θ_1 decreases. As the crystal is compressed, the spacing d between the planes of atoms decreases. For the first order diffraction m =1. The Bragg condition is $m\lambda = 2d\cos\theta_m$ so as d decreases, $\cos\theta_m$ must increase for the condition to be satisfied . But $\cos\theta$ increases as θ decreases. Hence there will be a decrease in the angle of incidence.

25.2. (a) $E_a > E_b > E_c$ because the energy per photon depends only on the frequency so $E = hf = hc/\lambda$. The smaller wavelengths correspond to higher frequencies.
(b) $N_c > N_b > N_a$ because the powers are equal, there must be more photons when the energy per photon is less.

25.3.

The energy of a photon is given by $E = hf = hc/\lambda$. Therefore the ratio of energies is,

$$\frac{E_2}{E_1} = \frac{hc/\lambda_2}{hc/\lambda_1} = \frac{\lambda_1}{2\lambda_1} = \frac{1}{2}$$

25.5. Fast electrons will have a shorter wavelength leading to less diffraction spreading and better resolution.

25.7. Because $E_n = n^2\frac{h^2}{8mL^2}$ we see that for a given n, E_n is inversely proportional to L^2. If L is doubled then E_n is decreased by a factor of 4. So the new $E_1 = 1\times 10^{-19}$ J.

25.8. It is the same, or 1.0×10^{-20} J.

$$E_{\mathrm{H}_1} = \frac{h^2}{8m_0L_0^2} \qquad E_{\mathrm{He}_1} = \frac{h^2}{8(4m_0)\left(\frac{L_0}{2}\right)^2} = E_{\mathrm{H}_1}$$

Exercises and Problems

25.4. Model: The angles of incidence for which diffraction from parallel planes occurs satisfy the Bragg condition.
Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where m = 1, 2 For first and second order diffraction, respectively

$$2d\cos\theta_1 = (1)\lambda \qquad 2d\cos\theta_2 = (2)\lambda$$

Dividing these two equations,

$$\frac{\cos\theta_2}{\cos\theta_1} = 2 \Rightarrow \theta_2 = \cos^{-1}(2\cos\theta_1) = \cos^{-1}(2\cos 68^\circ) = 41^\circ$$

25.7. Model: The angles corresponding to the various diffraction orders satisfy the Bragg condition.

Solve: The Bragg condition is $2d\cos\theta_m = m\lambda$, where m = 1, 2, 3, … gives the order of diffraction. The maximum possible value of m is the number of possible diffraction orders. The maximum value of $\cos\theta_m$ is 1. Thus, we tend to find the value of m for the limiting value of $\cos\theta_m$.

$$2d = m\lambda \Rightarrow m = \frac{2d}{\lambda} = \frac{2(0.180 \text{ nm})}{(0.085 \text{ nm})} = 4.2$$

We can observe up to the fourth diffraction order.

25.10. Model: Use the photon model of light.

Solve: The energy of a photon with wavelength λ_1 is $E_1 = hf_1 = hc/\lambda_1$. Similarly, $E_2 = hc/\lambda_2$. Since E_2 is equal to $2E_1$,

$$\frac{hc}{\lambda_2} = 2\frac{hc}{\lambda_1} \Rightarrow \lambda_2 = \frac{\lambda_1}{2} = \frac{600 \text{ nm}}{2} = 300 \text{ nm}$$

Assess: A photon with λ = 300 nm has twice the energy of a photon with λ = 600 nm. This is an expected result, because energy is inversely proportional to the wavelength.

25.12. We need to know the precise momentum to obtain the De Broglie wavelength. However this problem only asks the order of the value of wavelength of a much massive body weighing in kgs. Assuming an estimate of your mass being, say, $m \approx 70$ kg and your velocity is 1 m/s. Thus, your momentum is $p = mv \approx$ (70 kg)(1 m/s) = 70 kg m/s. Your de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times 10^{-34} \text{ Js}}{70 \text{ kg m/s}} \approx 9\times 10^{-36} \text{ m}$$

Which is infact too small to show any wavelike properties like diffraction etc.

25.14. Visualize: We'll employ Equations 25.8 $(\lambda = h/p)$ and 25.9 $(E = p^2/2m)$ to express the wavelength in terms of kinetic energy.

Solve: First solve Equation 25.9 for p: $p = \sqrt{2mE}$.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63\times 10^{-34} \text{J}\cdot\text{s}}{\sqrt{2(9.11\times 10^{-31} \text{ kg})(2.4\times 10^{-19} \text{J})}} = 1.0 \text{ nm}$$

Assess: The energy given is about 1.5 eV, which is a reasonable amount of energy. The resulting wavelength is a few to a few dozen times the size of an atom.

25.18. Model: Model the 5.0-fm-diameter nucleus as a box of length L = 5.0 fm.

Solve: The proton's energy is restricted to the discrete values

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{(6.63\times 10^{-34} \text{ Js})^2 n^2}{8(1.67\times 10^{-27} \text{ kg})(5.0\times 10^{-15} \text{ m})^2} = (1.316\times 10^{-12} \text{ J})n^2$$

where n = 1, 2, 3, … For n = 1, $E_1 = 1.3\times 10^{-12}$ J, for n = 2, $E_2 = (1.316\times 10^{-12} \text{ J})4 = 5.3\times 10^{-12}$ J, and for n = 3, $E_3 = 9E_1 = 1.2\times 10^{-11}$ J. We can observe that the energy of the nucleons in an atom is of the order of MeV which is much larger that that of an electron in the atom which is of the order of eV.

25.21. **Model:** **Use the photon model of light.**

Solve: **(a)** The wavelength is calculated as follows:

$$E_{\text{gamma}} = hf = h\left(\frac{c}{\lambda}\right) \Rightarrow \lambda = \frac{\left(6.63\times 10^{-34}\ \text{Js}\right)\left(3.0\times 10^{8}\ \text{m/s}\right)}{1.0\times 10^{-13}\ \text{J}} = 2.0\times 10^{-12}\ \text{m}$$

(b) The energy of a visible-light photon of wavelength 500 nm is

$$E_{\text{visible}} = h\left(\frac{c}{\lambda}\right) = \frac{\left(6.63\times 10^{-34}\ \text{Js}\right)\left(3.0\times 10^{8}\ \text{m/s}\right)}{500\times 10^{-9}\ \text{m}} = 3.978\times 10^{-19}\ \text{J}$$

The number of photons n such that $E_{\text{gamma}} = nE_{\text{visible}}$ is

$$n = \frac{E_{\text{gamma}}}{E_{\text{visible}}} = \frac{1.0\times 10^{-13}\ \text{J}}{3.978\times 10^{-19}\ \text{J}} = 2.5\times 10^{5}$$

25.22. **Model:** Use the photon model. The emitten EM radiation(photons) have the same frequency as the transmitting antenna. Therefore the energy of a 1000 kHz photon is

$$E_{\text{photon}} = hf = \left(6.63\times 10^{-34}\ \text{Js}\right)\left(1000\times 10^{3}\ \text{Hz}\right) = 6.63\times 10^{-28}\ \text{J}$$

The energy transmitted each second is 20×10^{3} J. The number of photons transmitted each second is $20\times 10^{3}\ \text{J}/6.63\times 10^{-28}\ \text{J} = 3.0\times 10^{31}$.

25.25. Model: Use the photon model of light and the Bragg condition for diffraction.

Solve: The Bragg condition for the reflection of x-rays from a crystal is $2d\cos\theta_m = m\lambda$. To determine the angles of incidence θ_m, we need to first calculate λ. The wavelength is related to the photon's energy as $E = hc/\lambda$. Thus,

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{Js})(3.0\times10^{8}\ \text{m/s})}{1.50\times10^{-15}\ \text{J}} = 1.326\times10^{-10}\ \text{m}$$

From the Bragg condition,

$$\theta_m = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left[\frac{(1.326\times10^{-10}\ \text{m})m}{2(0.21\times10^{-9}\ \text{m})}\right] = \cos^{-1}(0.3157m) \Rightarrow \theta_1 = \cos^{-1}(0.3157) = 71.6^\circ$$

Likewise, $\theta_2 = \cos^{-1}(0.3157\times2) = 50.8^\circ$ and $\theta_3 = 18.7^\circ$. Note that $\theta_4 = \cos^{-1}(0.3157\times4)$ is not allowed because the $\cos\theta$ cannot be larger than 1. Thus, the x-rays will be diffracted at angles of incidence equal to 18.7°, 50.8°, and 71.6°.

25.29. Model: Particles have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the particles causes an interference pattern in a double-slit apparatus.

Solve: **(a)** Since the speed of the neutron and electron are the same, the neutron's momentum is

$$p_\text{n} = m_\text{n}v_\text{n} = \frac{m_\text{n}}{m_\text{e}}m_\text{e}v_\text{n} = \frac{m_\text{n}}{m_\text{e}}m_\text{e}v_\text{e} = \frac{m_\text{n}}{m_\text{e}}p_\text{e}$$

where m_n and m_e are the neutron's and electron's masses. The de Broglie wavelength for the neutron is

$$\lambda_\text{n} = \frac{h}{p_\text{n}} = \frac{h}{p_\text{e}}\frac{p_\text{e}}{p_\text{n}} = \lambda_\text{e}\frac{m_\text{e}}{m_\text{n}}$$

From Section 22.2 on double-slit interference, the fringe spacing is $\Delta y = \lambda L/d$. Thus, the fringe spacing for the electron and neutron are related by

$$\Delta y_\text{n} = \frac{\lambda_\text{n}}{\lambda_\text{e}}\Delta y_\text{e} = \frac{m_\text{e}}{m_\text{n}}\Delta y_\text{e} = \left(\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right)(1.5\times10^{-3}\ \text{m}) = 8.18\times10^{-7}\ \text{m} = 0.818\ \mu\text{m}$$

(b) If the fringe spacing has to be the same for the neutrons and the electrons,

$$\Delta y_\text{e} = \Delta y_\text{n} \Rightarrow \lambda_\text{e} = \lambda_\text{n} \Rightarrow \frac{h}{m_\text{e}v_\text{e}} = \frac{h}{m_\text{n}v_\text{n}} \Rightarrow v_\text{n} = v_\text{e}\frac{m_\text{e}}{m_\text{n}} = (2.0\times10^{6}\ \text{m/s})\left(\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 1.1\times10^{3}\ \text{m/s}$$

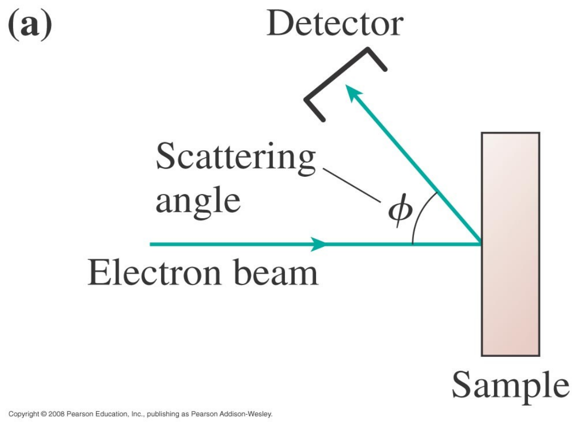
25.30. Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$. The wave nature of the electrons causes a diffraction pattern.

Solve: The width of the central maximum of a single-slit diffraction pattern is given by Equation 22.22:

$$w = \frac{2\lambda L}{a} = \frac{2Lh}{ap} = \frac{2Lh}{amv} = \frac{2(1.0\ \text{m})(6.63\times10^{-34}\ \text{Js})}{(1.0\times10^{-6}\ \text{m})(9.11\times10^{-31}\ \text{kg})(1.5\times10^{6}\ \text{m/s})} = 9.7\times10^{-4}\ \text{m} = 0.97\ \text{mm}$$

25.32. Model: Electrons have a de Broglie wavelength given by $\lambda = h/p$.

Visualize:



Notice that a scattering angle $\phi = 60^\circ$ corresponds to an angle of incidence $\theta = 30^\circ$.

Solve: Equation 25.6 describes the Davisson-Germer experiment: $D\sin(2\theta_m) = m\lambda$. Assuming m = 1, this equation simplifies to $D\sin 2\theta = \lambda$. Using $\lambda = h/mv$, we have

$$D = \frac{h}{mv\sin 2\theta} = \frac{6.63\times 10^{-34}\ \text{Js}}{(9.11\times 10^{-31}\ \text{kg})(4.30\times 10^{6}\ \text{m/s})\sin(60^\circ)} = 1.95\times 10^{-10}\ \text{m} = 0.195\ \text{nm}$$

25.33. Model: A confined particle can have only discrete values of energy.

Solve: **(a)** Equation 25.14 simplifies to

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{(6.63\times 10^{-34}\ \text{Js})^2}{8(9.11\times 10^{-31}\ \text{kg})(0.70\times 10^{-9}\ \text{m})^2} = (1.231\times 10^{-19}\ \text{J})n^2$$

Thus, $E_1 = (1.231\times 10^{-19}\,\text{J})(1^2) = 1.2\times 10^{-19}\,\text{J}$, $E_2 = (1.231\times 10^{-19}\,\text{J})(2^2) = 4.9\times 10^{-19}\,\text{J}$, and $E_3 = 1.1\times 10^{-18}\,\text{J}$.

(b) The energy is $E_2 - E_1 = 4.9\times 10^{-19}\ \text{J} - 1.2\times 10^{-19}\ \text{J} = 3.7\times 10^{-19}\ \text{J}$.

(c) Because energy is conserved, the photon will carry an energy of $E_2 - E_1 = 3.69\times 10^{-19}\ \text{J}$. That is,

$$E_2 - E_1 = E_{\text{photon}} = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_2 - E_1} = \frac{(6.63\times 10^{-34}\ \text{Js})(3.0\times 10^{8}\ \text{m/s})}{3.69\times 10^{-19}\ \text{J}} = 540\ \text{nm}$$

25.34. Model: A particle confined in a one-dimensional box has discrete energy levels.

Solve: **(a)** Equation 24.14

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1,\ 2,\ 3,\ 4,\ldots$$ for the n = 1 state is

$$E_n = \frac{h^2}{8mL^2} = \frac{(6.63\times 10^{-34}\ \text{Js})^2}{8(10\times 10^{-3}\ \text{kg})(0.10\ \text{m})^2} = 5.5\times 10^{-64}\ \text{J}$$

The minimum energy of the Ping-Pong ball is $E_1 = 5.5\times 10^{-64}\ \text{J}$.

(b) The only form of energy that the ball can have is considered to be the kinetic energy of the ball. Therefore the speed is calculated as follows:

$$E_1 = 5.50\times 10^{-64}\,\text{J} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(10\times 10^{-3}\ \text{kg})v^2 \Rightarrow v = \sqrt{\frac{2(5.50\times 10^{-64}\ \text{J})}{10\times 10^{-3}\ \text{kg}}} = 3.3\times 10^{-31}\ \text{m/s}$$

25.39. Model: A particle confined in a one-dimensional box of length L has the discrete energy levels given by Equation 24.14.

$$E_n = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2}{8mL^2}n^2 \qquad n = 1,\ 2,\ 3,\ 4, \ldots$$

Solve: **(a)** Since the energy is entirely kinetic energy,

$$E_n = \frac{h^2}{8mL^2}n^2 = \frac{p^2}{2m} = \tfrac{1}{2}mv_n^2 \Rightarrow v_n = \frac{h}{2mL}n \qquad n = 1,\ 2,\ 3,\ \ldots$$

(b) The first allowed velocity is

$$v_1 = \frac{6.63\times 10^{-34}\text{Js}}{2(9.11\times 10^{-31}\ \text{kg})(0.20\times 10^{-9}\ \text{m})} = 1.82\times 10^{6}\ \text{m/s}$$

For $n = 2$ and $n = 3$, $v_2 = 3.64 \times 10^6$ m/s and $v_3 = 5.46 \times 10^6$ m/s.