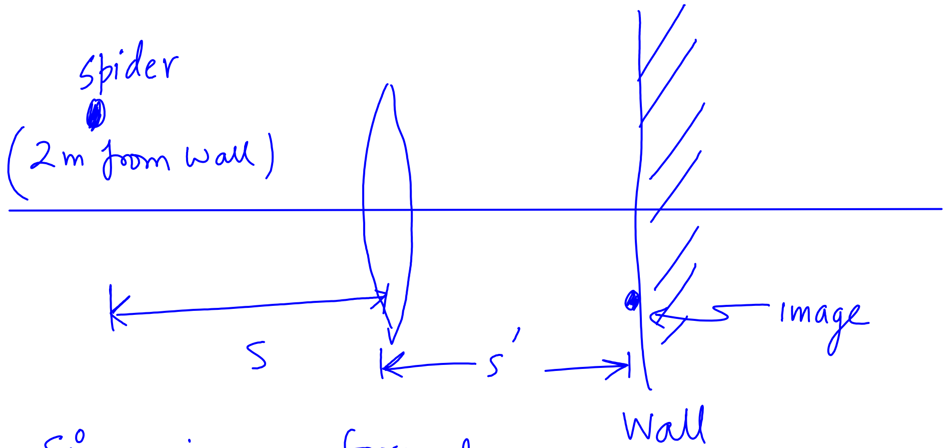
Tuesday, October 13, 2009
5:02 PM

Solution to selected problems in HW#7

23.69 A diagram might be helpful to visualize



Since image formed will be real, M is negative

$$\Rightarrow \frac{-s'}{s} = \frac{-1}{2} \qquad \text{—— (1)}$$

While, the lens equation is:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{—— (2)}$$

and since the spider is 2m from wall

$$s + s' = 2 \qquad \text{—— (3)}$$

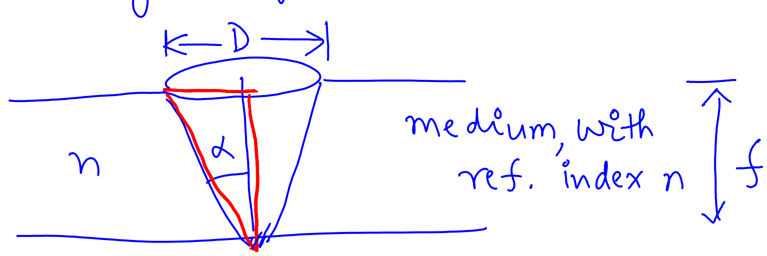
(1) & (3) give $s = 4/3$ m

$\Rightarrow$ (2) & (1) give $f = 4/9$ m

≈ 44 cm

24.48

The diagram of a microscope objective is



α is half the angular size

In the triangle in red

$$\tan\alpha = \frac{D/2}{f} \Rightarrow \alpha = \tan^{-1}\frac{D/2}{f}$$

While $NA = n\sin\alpha = n\sin\tan^{-1}\frac{D/2}{f}$

The minimum angle resolved is

$\theta = \frac{1.22\lambda}{D}$, which gives the minimum resolvable distance

$$x = \frac{0.61\lambda}{NA}$$

$$= \frac{0.61\lambda}{n\sin\tan^{-1}\frac{D/2}{f}} \quad \text{——— (1)}$$

Now, we are told, $x = 400$ nm

$\lambda = 550$ nm

$n = 1$ (air) and

$f = 1.6$ mm

Given these values and our eq^n (1), we can invert eq^n (1) to get D

$$n\sin\tan^{-1}\frac{D/2}{f} = \frac{0.61\lambda}{x}$$

$$\Rightarrow \frac{D/2}{f} = \tan\sin^{-1}\frac{0.61\lambda}{nx}$$

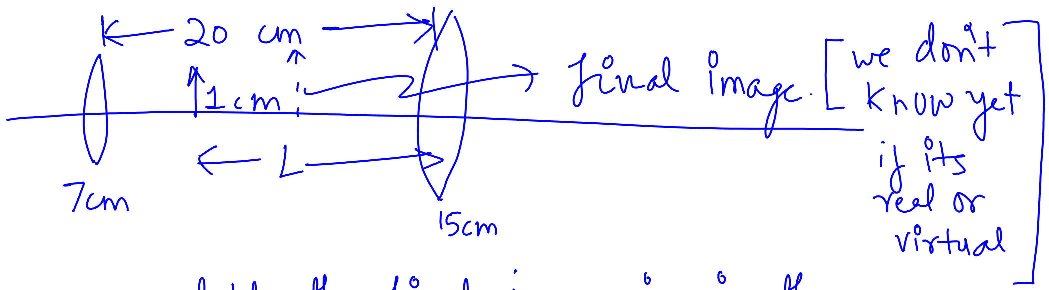
$$\Rightarrow D = 2f\tan\sin^{-1}\frac{0.61\lambda}{nx}$$

We can plug the numbers to get

a numerical answer for D.

24.32

A diagram might be helpful here to visualize.



We are told the final image is in the middle of the two lenses

$$\Rightarrow \quad s_2' = -10 \qquad \text{(since it is on left)}$$

$$f = 15$$

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{s_2} = \frac{1}{f} - \frac{1}{s_2'} = \frac{1}{15} + \frac{1}{10} = \frac{5}{30} = \frac{1}{6}$$

$$\Rightarrow s_2 = 6$$

So, for the 7 cm lens, the image is at $20 - 6 = 14$ cm

$$\Rightarrow s_1' = 14 \text{ cm}, \quad f = 7 \text{ cm}$$

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{s_1} = \frac{1}{f} - \frac{1}{s_1'} = \frac{1}{7} - \frac{1}{14} = \frac{1}{14}$$

$$\Rightarrow s_1 = 14 \text{ cm}$$

and since we were given this as L, we immidiately get that L = 14 cm.

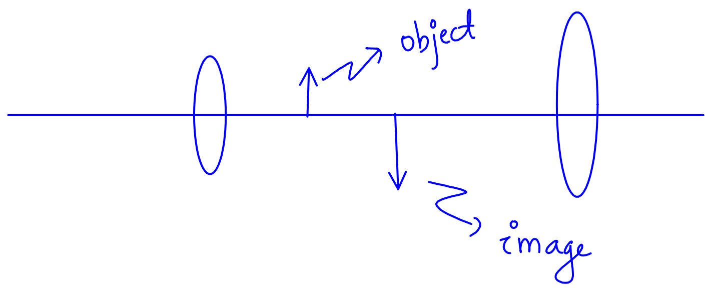
The magnification calculation is straightforward, once we know the s_1, s_1', s_2, s_2' etc. We just need to keep in mind that the just need to keep in mind that the total magnification is the product of individual magnifications. So,

$$m = m_1 \times m_2 = \frac{-s_1'}{s_1} \times \frac{-s_2'}{s_2} \quad \text{etc}$$

Once we know m, the height of image is given as

$$h_{image} = m \times h_{object}$$

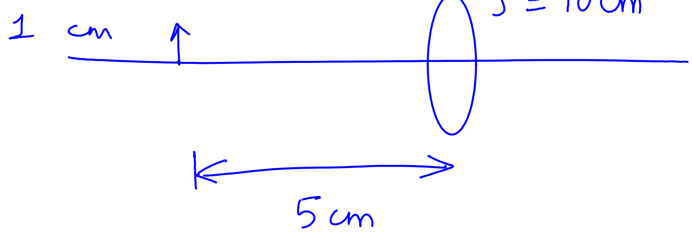
Numerical value of m comes out to be negative, which means $h_{image} < 0$, so the image is inverted. So, our initial sketch was wrong! We can correct it now :)



24.28

To solve this, we can use the concept that the image from one lens acts as the object for the other.

Hence, let's just consider them one by one



$s_1 = 5\,cm$; $f = 10\,cm$

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f} \Rightarrow \frac{1}{s_1'} = \frac{1}{f} - \frac{1}{s_1}$$

$$= \frac{1}{10} - \frac{1}{5} = \frac{-1}{10}$$

$$\Rightarrow s_1' = -10$$

Now, the effective object for the mirror is at $s_2 = 10 + 5 = 15\,cm$. It'll be then a virtual image

$$\frac{1}{s_2'} = \frac{1}{f} - \frac{1}{s_2} = \frac{1}{-30} - \frac{1}{15} = \frac{-3}{30}$$

$$= -\frac{1}{10}$$

$$\Rightarrow s_2' = -10\,cm$$

Hence final image will be 10 cm to the right of the mirror.