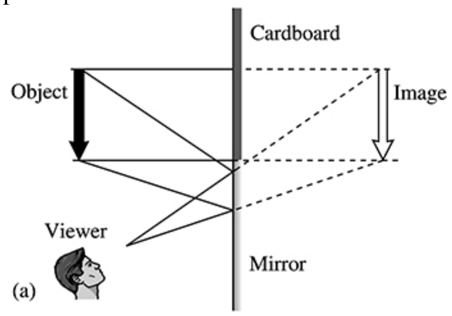
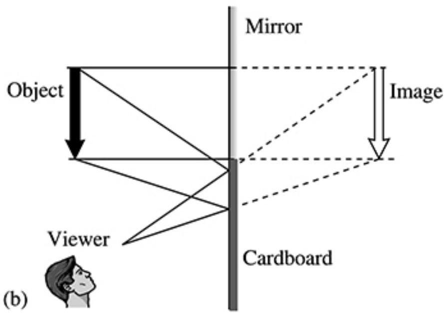
Chapter 23

Conceptual questions.

23.3. Light is scattered off all points of the pencil and into all directions of space. If light directed toward the mirror is reflected into your eye, you see the image of the pencil. **(a)** As part (a) of the figure shows, if the top half of the mirror is covered, light scattered from the pencil and reflected off the mirror can enter your eye and you will see the image of the pencil. **(b)** As part (b) of the figure shows, if the bottom half of the mirror is covered, light scattered from the pencil cannot be reflected off the mirror in such a matter that it enters your eye. You cannot see the image of the pencil.





23.5.

The light that enters the plastics (color filters) is white which suggests that it is a mixture of all coloirs in the spectrum. Therefore the colour of the sections would be

Section 1: Blue; all other colors are absorbed. Except blue light

Section 2: It filters all but red initially and then the blue plasyic filters red. Hence no light passes through . Therefore Black

Section 3: Red; all other colors are absorbed.

23.6. The card is red because it reflects red light and absorbs the other colors. When it is illuminated by red light the red light reflects off the card into your eyes and you see the red card as red. If the card is illuminated with blue light the light is all absorbed. No light is reflected, so the card looks black. If you illuminate the card with white light and look at it through a blue filter it will again look black because the red light reflected by the card is not passed by the blue filter.

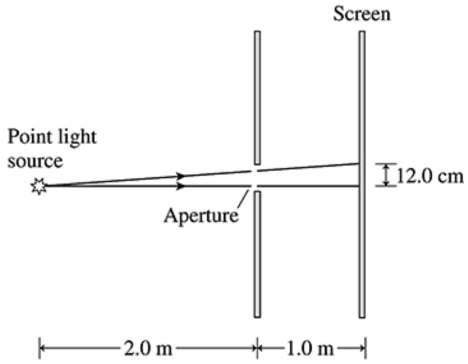
23.8. **(a)** Two rays cross at the image point. Since the image point has to be found by the intersection of two or more lines (in ray diagram they are the reflected/refracted rays) hence at least 2 rays are required. We in general select the 2 rays of whose path we know. One is parallel to the principal axis and the other passes through the optical center of the lens.

(b) An infinite number! All those that strike the lens from the object point will converge to the image point.

23.9. You will still see the entire image, but it will be dimmer as less light passes through the lens. Rays originating from the object move in all possible directions. All there rays are refracted by the lens to give the final image. Rays strike the opper half of the lens as much as they strike the lower half. Hence just covering the upper half will remove those rays from contributing to the final image but the rays that pass through the lower half will form an image which would then be dimmer than the original since almost half of the rays don't reach the screen.

Exercises and problems

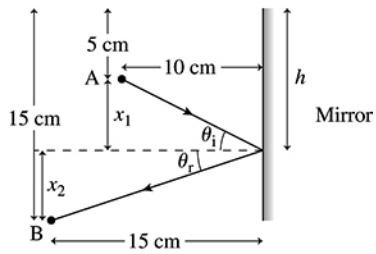
23.3. Model: Light rays travel in straight lines. The light source is a point source.
Visualize:



Solve: Let w be the width of the aperture. Then from the geometry of the figure,

$$\frac{w}{2.0\text{ m}} = \frac{12.0\text{ cm}}{2.0\text{ m} + 1.0\text{ m}} \Rightarrow w = 8.0\text{ cm}$$

23.7. Model: Light rays travel in straight lines and follow the law of reflection.
Visualize:



Solve: We are asked to obtain the distance $h = x_1 + 5.0$ cm. From the geometry of the diagram,

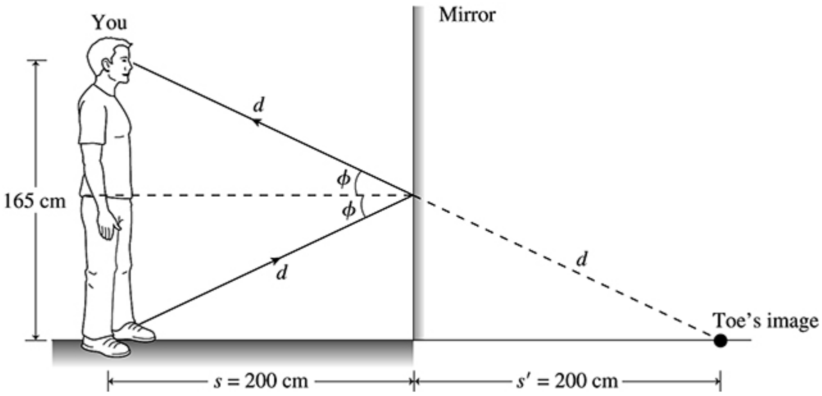
$$\tan\theta_i = \frac{x_1}{10\text{ cm}} \qquad \tan\theta_r = \frac{x_2}{15\text{ cm}} \qquad x_1 + x_2 = 10\text{ cm}$$

Because $\theta_r = \theta_i$, we have

$$\frac{x_1}{10\text{ cm}} = \frac{x_2}{15\text{ cm}} = \frac{10\text{ cm} - x_1}{15\text{ cm}} \Rightarrow (15\text{ cm})x_1 = 100\text{ cm}^2 - 10x_1 \Rightarrow x_1 = 4.0\text{ cm}$$

Thus, the ray strikes a distance 9.0 cm below the top edge of the mirror.

23.11. Model: Use the ray model of light and the law of reflection.
Visualize:



We only need one ray of light that leaves your toes and reflects in your eye.

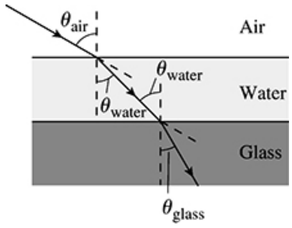
Solve: From the geometry of the diagram, the distance from your eye to the toes' image is

$$2d = \sqrt{(400\text{ cm})^2 + (165\text{ cm})^2} = 433\text{ cm}$$

Assess: The light appears to come from your toes' image.

23.12. Model: Use the ray model of light and Snell's law.

Visualize:



Solve: According to Snell's law for the air-water and water-glass boundaries,

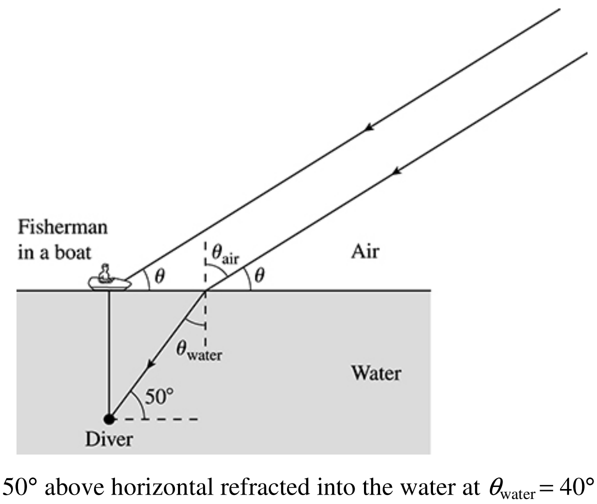
$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{water}}\sin\theta_{\text{water}} \qquad n_{\text{water}}\sin\theta_{\text{water}} = n_{\text{glass}}\sin\theta_{\text{glass}}$$

From these two equations, we have

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{glass}}\sin\theta_{\text{glass}} \Rightarrow \sin\theta_{\text{glass}} = \frac{n_{\text{air}}}{n_{\text{glass}}}\sin\theta_{\text{air}} = \left(\frac{1.0}{1.50}\right)\sin 60° \Rightarrow \theta_{\text{glass}} = \sin^{-1}\left(\frac{\sin\ 60°}{1.5}\right) = 35°$$

23.14. Model: Use the ray model of light. The sun is a point source of light.

Visualize:



A ray that arrives at the diver 50° above horizontal refracted into the water at $\theta_{\text{water}} = 40°$.

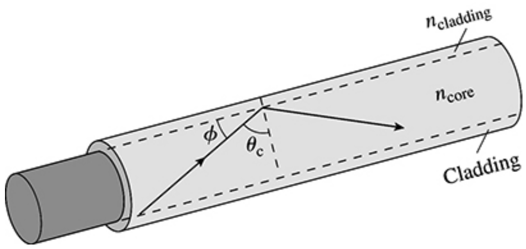
Solve: Using Snell's law at the water-air boundary

$$n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{water}}\sin\theta_{\text{water}} \Rightarrow \sin\theta_{\text{air}} = \frac{n_{\text{water}}}{n_{\text{air}}}\sin\theta_{\text{water}} = \left(\frac{1.33}{1.0}\right)\sin 40°$$

$$\Rightarrow \theta_{\text{air}} = 58.7°$$

Thus the height above the horizon is $\theta = 90° - \theta_{\text{air}} = 31.3° \approx 31°$. Because the sun is far away from the fisherman (and the diver), the fisherman will see the sun at the same angle of $31°$ above the horizon.

23.16. Model: Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection.
Visualize:

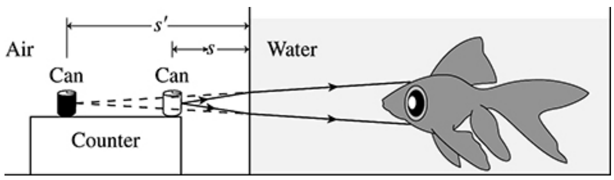


Solve: The critical angle of incidence is given by Equation 23.9:

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{cladding}}}{n_{\text{core}}}\right) = \sin^{-1}\left(\frac{1.48}{1.60}\right) = 67.7°$$

Thus, the maximum angle a light ray can make with the wall of the core to remain inside the fiber is 90° − 67.7° = 23.3°.
Assess: We can have total internal reflection because $n_{\text{core}} > n_{\text{cladding}}$.

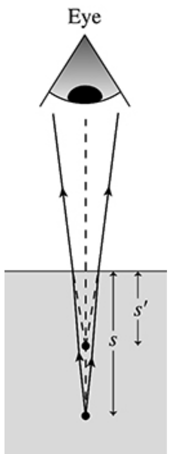
23.18. Model: Represent the can as a point source and use the ray model of light.
Visualize:



Paraxial rays from the can refract into the water and enter into the fish's eye.
Solve: The object distance from the edge of the aquarium is s. From the water side, the can appears to be at an image distance $s' = 30$ cm. Using Equation 23.13,

$$s' = \frac{n_2}{n_1}s = \frac{n_{\text{water}}}{n_{\text{air}}}s = \left(\frac{1.33}{1.0}\right)s \Rightarrow s = \frac{30\text{ cm}}{1.33} = 23\text{ cm}$$

23.19. Model: Represent the beetle as a point source and use the ray model of light.
Visualize:



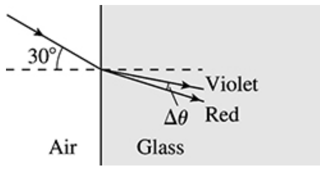
Paraxial rays from the beetle refract into the air and then enter into the observer's eye. The rays in the air when extended into the plastic appear to be coming from the beetle at a shallower location, a distance s' from the plastic-air boundary.

Solve: The actual object distance is s and the image distance is $s' = 2.0$ cm. Using Equation 23.13,

$$s' = \frac{n_2}{n_1}s = \frac{n_{\text{air}}}{n_{\text{plastic}}}s \Rightarrow 2.0\text{ cm} = \frac{1.0}{1.59}s \Rightarrow s = 3.2\text{ cm}$$

Assess: The beetle is much deeper in the plastic than it appears to be.

23.22. Model: Use the ray model of light.
Visualize:



Solve: Using Snell's law,

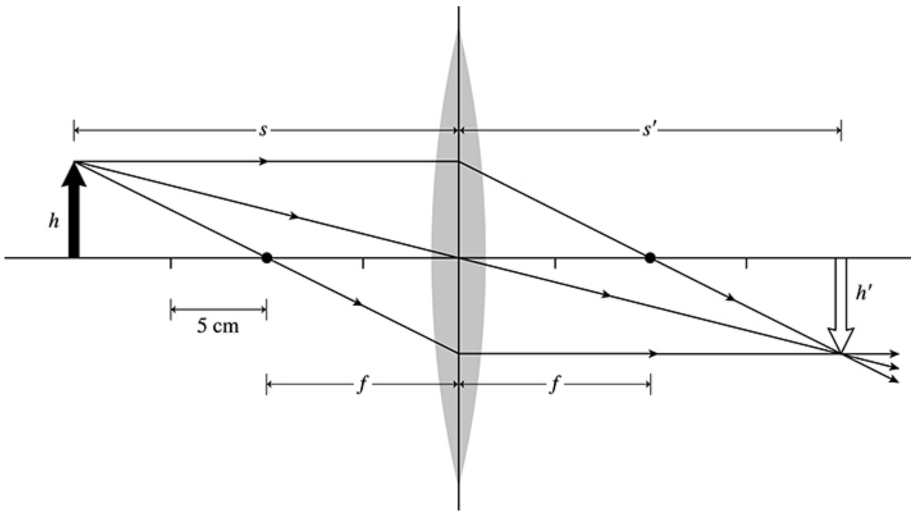
$$n_{\text{air}}\sin 30° = n_{\text{red}}\sin\theta_{\text{red}} \Rightarrow \theta_{\text{red}} = \sin^{-1}\left(\frac{\sin 30°}{1.52}\right) = 19.2°$$

$$n_{\text{air}}\sin 30° = n_{\text{violet}}\sin\theta_{\text{violet}} \Rightarrow \theta_{\text{violet}} = \sin^{-1}\left(\frac{\sin 30°}{1.55}\right) = 18.8°$$

Thus the angular spread is

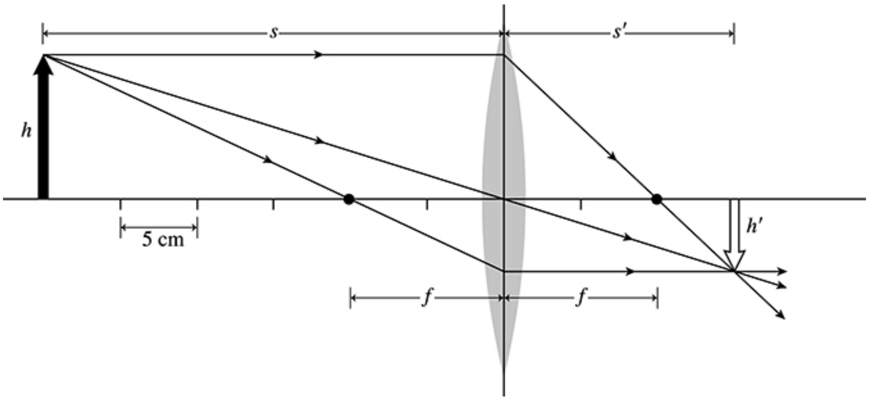
$$\Delta\theta = \theta_{\text{red}} - \theta_{\text{violet}} = 19.2° - 18.8° = 0.4°$$

23.26. Model: Use ray tracing to locate the image.
Solve:



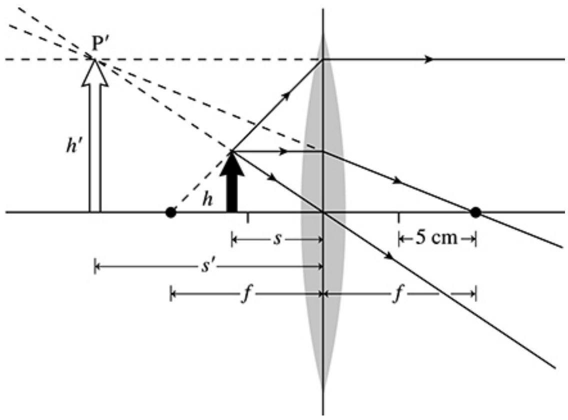
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. You can see from the diagram that the image is in the plane where the three special rays converge. The image is inverted and is located at $s' = 20.0$ cm to the right of the converging lens.

23.27. Model: Use ray tracing to locate the image.
Solve:



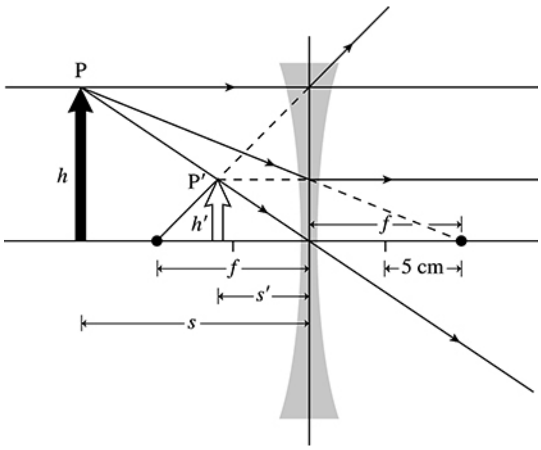
The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. You can see from the diagram that the image is in the plane where the three special rays converge. The image is located at $s' = 15$ cm to the right of the converging lens, and is inverted.

23.28. Model: Use ray tracing to locate the image.
Solve:



The figure shows the ray-tracing diagram using the steps of Tactics Box 23.2. You can see that the rays after refraction do not converge at a point on the refraction side of the lens. On the other hand, the three special rays, when extrapolated backward toward the incidence side of the lens, meet at P′, which is 15 cm from the lens. That is, $s' = -15$ cm. The image is upright.

23.29. Model: Use ray tracing to locate the image.
Solve:



The figure shows the ray-tracing diagram using the steps of Tactics Box 23.3. The three rays after refraction do not converge at a point, but they appear to come from P′. P′ is 6 cm from the diverging lens, so $s' = -6$ cm. The image is upright.

23.30. Model: Assume the biconvex lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = +40$ cm (convex toward the object) and the second surface has $R_2 = -40$ cm (concave toward the object). The index of refraction of glass is $n = 1.50$, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{40\text{ cm}} - \frac{1}{-40\text{ cm}}\right) \Rightarrow f = 40\text{ cm}$$

23.31. Model: Assume the planoconvex lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = \infty$ and the second surface has $R_2 = -40$ cm (concave toward the object). The index of refraction of polystyrene plastic is 1.59, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.59-1)\left(\frac{1}{\infty} - \frac{1}{-40\text{ cm}}\right) \Rightarrow \frac{1}{f} = \frac{0.59}{40\text{ cm}} \Rightarrow f = 68\text{ cm}$$

23.32. Model: Assume the biconcave lens is a thin lens.

Solve: If the object is on the left, then the first surface has $R_1 = -40$ cm (concave toward the object) and the second surface has $R_2 = +40$ cm (convex toward the object). The index of refraction of glass is 1.50, so the lensmaker's equation is

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{-40\text{ cm}} - \frac{1}{+40\text{ cm}}\right) = (0.50)\left(-\frac{1}{20\text{ cm}}\right) \Rightarrow f = -40\text{ cm}$$

23.34. Model: The water is a spherical refracting surface. Consider the paraxial rays that refract from the air into the water.

Solve: If the cat's face is 20 cm from the edge of the bowl, then $s = +20$ cm. The spherical fish bowl surface has $R = +25$ cm, because it is the convex surface that is toward the object. Also $n_1 = 1$ (air) and $n_2 = 1.33$ (water). Using Equation 23.21,

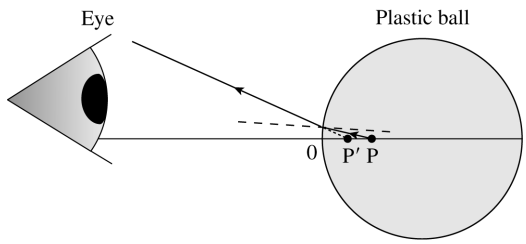
$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{20\text{ cm}} + \frac{1.33}{s'} = \frac{1.33-1}{25\text{ cm}} = \frac{0.33}{25\text{ cm}} = 0.0132\text{ cm}^{-1}$$

$$\Rightarrow \frac{1.33}{s'} = (0.0132 - 0.050)\text{ cm}^{-1} \Rightarrow s' = -36\text{ cm}$$

This is a virtual image located 36 cm outside the fishbowl. The fish, inside the bowl, sees the virtual image. That is, the fish sees the cat's face 36 cm from the bowl.

23.35. Model: Model the bubble as a point source and consider the paraxial rays that refract from the plastic into the air. The edge of the plastic is a spherical refracting surface.

Visualize:



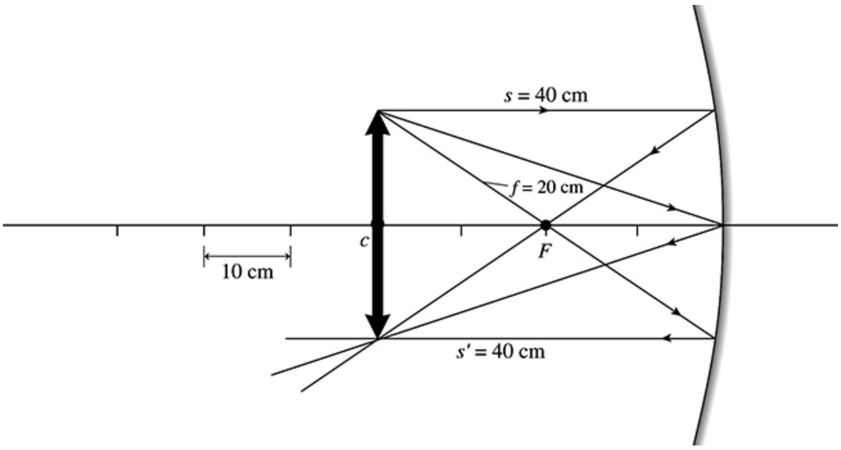
Solve: The bubble is at P, a distance of 2.0 cm from the surface. So, $s = 2.0$ cm. A ray from P after refracting from the plastic-air boundary bends away from the normal axis and enters the eye. This ray appears to come from P′, so the image of P is at P′ and it is a virtual image. Because P faces the concave side of the refracting surface, $R = -4.0$ cm. Furthermore, $n_1 = 1.59$ and $n_2 = 1.0$. Using Equation 23.21,

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R} \Rightarrow \frac{1.59}{2.0\text{ cm}}+\frac{1.0}{s'}=\frac{1.0-1.59}{-4.0\text{ cm}}=+\frac{0.59}{4.0\text{ cm}}=0.1475\text{ cm}^{-1}$$

$$\Rightarrow \frac{1}{s'}=0.1475\text{ cm}^{-1}-0.795\text{ cm}^{-1} \Rightarrow s'=-1.54\text{ cm}$$

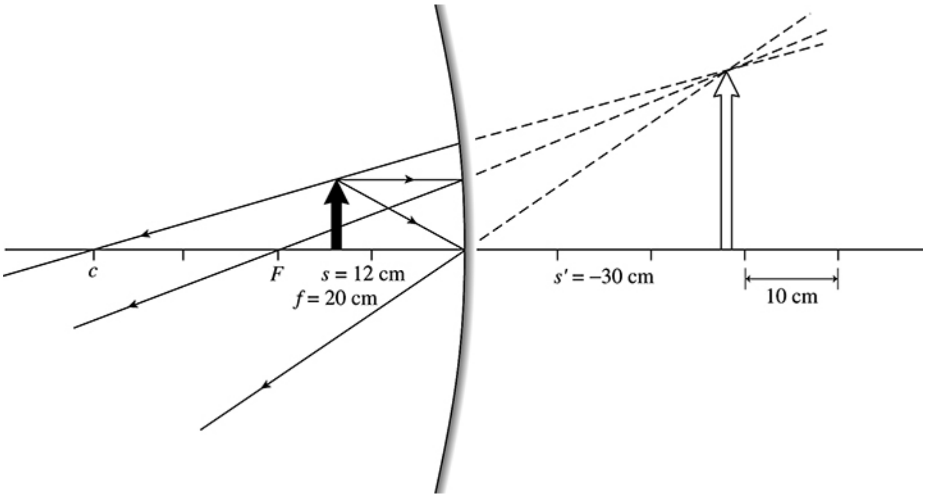
That is, the bubble appears $1.54\text{ cm} \approx 1.5\text{ cm}$ beneath the surface.

23.37. Solve: The image is at 40 cm as seen in the figure. It is inverted.



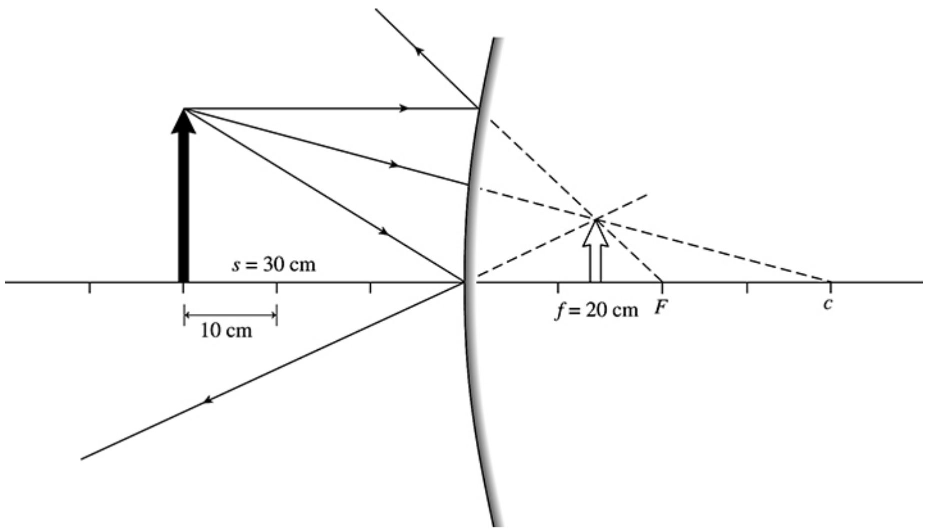
Assess: When the object is outside the focal length we get an inverted image.

23.38. Solve: The image is at -30 cm as seen in the figure. It is upright.



Assess: When the object is within the focal length we get a magnified upright image.

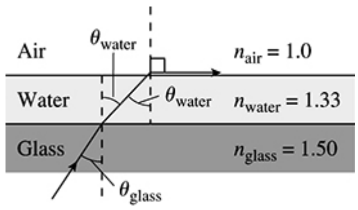
23.39. Solve: The image is at -12 cm as seen in the figure. It is upright.



Assess: We expected an upright virtual image from the convex mirror.

23.45. Use the ray model of light. For an angle of incidence greater than the critical angle, the ray of light undergoes total internal reflection. We have glass having the largest refractive index. We need to take care of total internal refletion at the glass-water interface as well as water air interface. However as we keep increasing the angle of incidence in glass we can be sure that the total internal reflection would not take place at the glass water interface earlier than it takes place in water-air interface because the critical angle for glass water interface is larger than that between water and air. Hence in the limiting condition we have the following situation as shown in the ray diagram below.

Visualize:



For angles θ_{water} that are less than the critical angle, light will be refracted into the air.

Solve: Snell's law at the water-air boundary is $n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{water}} \sin\theta_{\text{water}}$. Because the maximum angle of θ_{air} is $90°$, we have

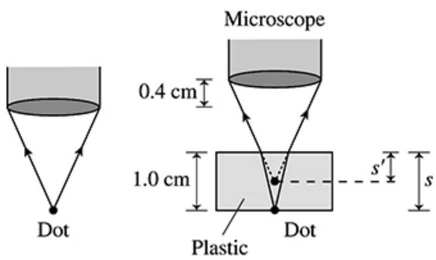
$$(1.0)\sin 90° = 1.33\sin\theta_{\text{water}} \Rightarrow \theta_{\text{water}} = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$$

Applying Snell's law again to the glass-water boundary,

$$n_{\text{glass}} \sin\theta_{\text{glass}} = n_{\text{water}} \sin\theta_{\text{water}} \Rightarrow \theta_{\text{glass}} = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\text{glass}}}\sin\theta_{\text{water}}\right) = \sin^{-1}\left(\frac{1.33(\sin 48.75°)}{1.50}\right) = 42°$$

23.46. Model: Use the ray model of light.

Visualize:

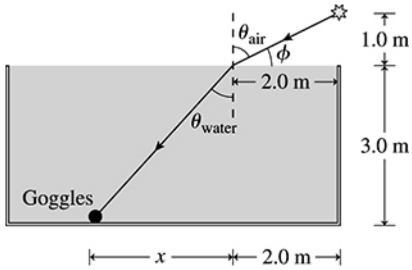


Solve: When the plastic is in place, the microscope focuses on the virtual image of the dot. From the figure, we note that s = 1.0 cm and s' = 1.0 cm − 0.4 cm = 0.6 cm. The rays are paraxial, and the object and image distances are measured relative to the plastic-air boundary. Using Equation 23.13,

$$s' = \frac{n_{\text{air}}}{n_{\text{plastic}}} s \Rightarrow 0.6 \text{ cm} = \frac{1.0}{n_{\text{plastic}}}(1.0 \text{ cm}) \Rightarrow n_{\text{plastic}} = \frac{1.0 \text{ cm}}{0.6 \text{ cm}} = 1.67$$

Thus $42°$ is the maximum angle of incidence onto the glass for which the ray emerges into the air.

23.51. Model: Use the ray model of light and the law of refraction. Assume that the laser beam is a ray of light.
Visualize:



The laser beam enters the water 2.0 m from the edge, undergoes refraction, and illuminates the goggles. The ray of light from the goggles then retraces its path and enters your eyes.
Solve: From the geometry of the diagram,

$$\tan\phi = \frac{1.0 \text{ m}}{2.0 \text{ m}} \Rightarrow \phi = \tan^{-1}(0.50) = 26.57° \Rightarrow \theta_{\text{air}} = 90° - 26.57° = 63.43°$$

Snell's law at the air-water boundary is $n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{water}} \sin\theta_{\text{water}}$. Using the above result,

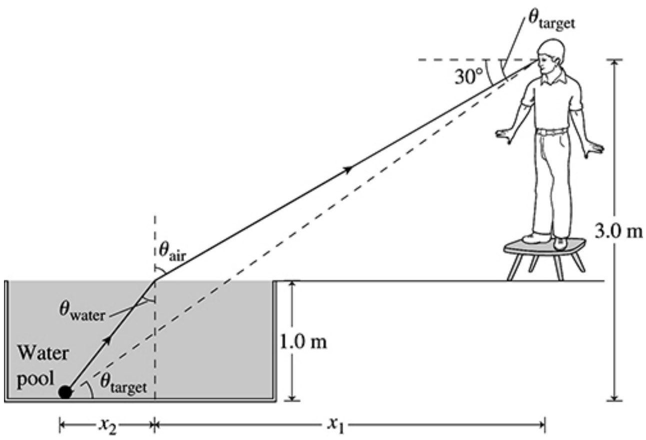
$$(1.0)\sin 63.43° = 1.33 \sin\theta_{\text{water}} \Rightarrow \theta_{\text{water}} = \sin^{-1}\left(\frac{\sin 63.43°}{1.33}\right) = 42.26°$$

Taking advantage of the geometry in the diagram again,

$$\frac{x}{3.0 \text{ m}} = \tan\theta_{\text{water}} \Rightarrow x = (3.0 \text{ m})\tan 42.26° = 2.73 \text{ m}$$

The distance of the goggles from the edge of the pool is 2.73 m + 2.0 m = 4.73 m ≈ 4.7 m.

23.54. Model: Use the ray model of light. Assume that the target is a point source of light.
Visualize:



Solve: From the geometry of the figure with $\theta_{\text{air}} = 60°$,

$$\tan\theta_{\text{air}} = \frac{x_1}{2.0 \text{ m}} \Rightarrow x_1 = (2.0 \text{ m})(\tan 60°) = 3.464 \text{ m}$$

Let us find the horizontal distance x_2 by applying Snell's law to the air-water boundary. We have

$$n_{\text{water}} \sin\theta_{\text{water}} = n_{\text{air}} \sin\theta_{\text{air}} \Rightarrow \theta_{\text{water}} = \sin^{-1}\left(\frac{\sin 60^\circ}{1.33}\right) = 40.63^\circ$$

Using the geometry of the diagram,

$$\frac{x_2}{1.0 \text{ m}} = \tan\theta_{\text{water}} \Rightarrow x_2 = (1.0 \text{ m})\tan 40.63^\circ = 0.858 \text{ m}$$

To determine θ_{target}, we note that

$$\tan\theta_{\text{target}} = \frac{3.0 \text{ m}}{x_1 + x_2} = \frac{3.0 \text{ m}}{3.464 \text{ m} + 0.858 \text{ m}} = 0.6941 \Rightarrow \theta_{\text{target}} = 35^\circ$$