22.2. Because $y_m = \frac{m\lambda L}{d}$ increasing λ and L increases the fringe spacing. Increasing d decreases the fringe spacing. Submerging the experiment in water decreases λ and decreases the fringe spacing. So the answers are (a) and (c).

22.4. (a) The equation for gratings does not contain the number of slits, so increasing the number of slits can't affect the angles at which the bright fringes appear as long as d is the same. So the number of fringes on the screen stays the same. (b) The number of slits does not appear in the equation for the fringe spacing, so the spacing stays the same. (c) Decreases; the fringes become narrower. (d) The equation for intensity does contain the number of slits, so each fringe becomes brighter: $I_{max} = N^2 I_1$.

22.5. $\lambda < a$. Several secondary maxima appear. For $a \sin \theta_p = p\lambda$, the first minima from the central maximum require $a \sin \theta_1 = \lambda$, which must be less than 1.

22.6. (a) The width increases because $\theta_1 = \frac{1.22\lambda}{D}$. (b) The width decreases because $\theta_1 = \frac{1.22\lambda}{D}$. (c) Almost uniformly gray with no minima.

22.7. When the experiment is immersed in water the frequency of the light stays the same. But the speed is slower, so the wavelength is smaller. When the wavelength is smaller the fringes get closer together because λL

$$\Delta y = \frac{\pi L}{d}.$$

22.10. If the wavelength is changed to $\lambda/2$ there will still be crests everywhere there were crests before (plus new crests halfway between, where there used to be troughs). If the interferometer was set up to display constructive interference originally, then crests were arriving from each arm of the interferometer together at the detector. That will still be true when the wavelength is halved. So the central spot remains bright.

22.2. Model: Two closely spaced slits produce a double-slit interference pattern. Visualize: The interference pattern looks like the photograph of Figure 22.3(b). It is symmetrical, with the m = 2 fringes on both sides of and equally distant from the central maximum.

Solve: The two paths from the two slits to the m = 2 bright fringe differ by $\Delta r = r_2 - r_1$, where

$$\Delta r = m\lambda = 2\lambda = 2(500 \text{ nm}) = 1000 \text{ nm}$$

Thus, the position of the m = 2 bright fringe is 1000 nm farther away from the more distant slit than from the nearer slit.

22.3. Model: Two closely spaced slits produce a double-slit interference pattern.

Visualize: The interference pattern looks like the photograph of Figure 22.3(b).

Solve: The bright fringes are located at positions given by Equation 22.4, $d \sin \theta_m = m\lambda$. For the m = 3 bright orange fringe, the interference condition is $d \sin \theta_3 = 3(600 \times 10^{-9} \text{ m})$. For the m = 4 bright fringe the condition is $d \sin \theta_4 = 4\lambda$. Because the position of the fringes is the same,

 $d\sin\theta_3 = d\sin\theta_4 = 4\lambda = 3(600 \times 10^{-9} \text{ m}) \Longrightarrow \lambda = \frac{3}{4}(600 \times 10^{-9} \text{ m}) = 450 \text{ nm}$

22.4. Model: Two closely spaced slits produce a double-slit interference pattern. Visualize: The interference pattern looks like the photograph of Figure 22.3(b). Solve: The formula for fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Longrightarrow 1.8 \times 10^{-3} \text{ m} = (600 \times 10^{-9} \text{ m}) \frac{L}{d} \Longrightarrow \frac{L}{d} = 3000$$

The wavelength is now changed to 400 nm, and L/d, being a part of the experimental setup, stays the same. Applying the above equation once again,

$$\Delta y = \frac{\lambda L}{d} = (400 \times 10^{-9} \text{ m})(3000) = 1.2 \text{ mm}$$

22.8. Model: Two closely spaced slits produce a double-slit interference pattern. **Visualize:** The interference pattern looks like the photograph of Figure 22.3(b). **Solve:** In a span of 12 fringes, there are 11 gaps between them. The formula for the fringe spacing is

$$\Delta y = \frac{\lambda L}{d} \Rightarrow \left(\frac{52 \times 10^{-3} \text{ m}}{11}\right) = \frac{(633 \times 10^{-9} \text{ m})(3.0 \text{ m})}{d} \Rightarrow d = 0.40 \text{ mm}$$

Assess: This is a reasonable distance between the slits, ensuring $d/L = 1.34 \times 10^{-4} << 1$.

22.10. Model: A diffraction grating produces a series of constructive-interference fringes at values of θ_m determined by Equation 22.15. Solve: We have

 $d\sin\theta_m = m\lambda$ $m = 0, 1, 2, 3, ... \Rightarrow d\sin 20.0^\circ = 1\lambda$ and $d\sin\theta_2 = 2\lambda$

Dividing these two equations,

 $\sin \theta_2 = 2\sin 20.0^\circ = 0.6840 \implies \theta_2 = 43.2^\circ$

22.11. Model: A diffraction grating produces an interference pattern.Visualize: The interference pattern looks like the diagram in Figure 22.8.Solve: The bright constructive-interference fringes are given by Equation 22.15:

$$d\sin\theta_m = m\lambda \implies d = \frac{m\lambda}{\sin\theta_m} = \frac{(2)(600 \times 10^{-9} \text{ m})}{\sin(39.5^\circ)} = 1.89 \times 10^{-6} \text{ m}$$

The number of lines in per millimeter is $(1 \times 10^{-3} \text{ m})/(1.89 \times 10^{-6} \text{ m}) = 530$.

22.13. Model: A diffraction grating produces an interference pattern.Visualize: The interference pattern looks like the diagram of Figure 22.8.Solve: The bright interference fringes are given by

$$d\sin\theta_m = m\lambda$$
 $m = 0, 1, 2, 3, ...$

The slit spacing is $d = 1 \text{ mm}/500 = 2.00 \times 10^{-6} \text{ m}$ and m = 1. For the red and blue light,

$$\theta_{1 \text{ red}} = \sin^{-1} \left(\frac{656 \times 10^{-9} \text{ m}}{2.00 \times 10^{-6} \text{ m}} \right) = 19.15^{\circ}$$
 $\theta_{1 \text{ blue}} = \sin^{-1} \left(\frac{486 \times 10^{-9} \text{ m}}{2.00 \times 10^{-6} \text{ m}} \right) = 14.06^{\circ}$

The distance between the fringes, then, is $\Delta y = y_{1 \text{ red}} - y_{1 \text{ blue}}$ where

$$y_{1 \text{ red}} = (1.5 \text{ m}) \tan 19.15^\circ = 0.521 \text{ m}$$

 $y_{1 \text{ blue}} = (1.5 \text{ m}) \tan 14.06^\circ = 0.376 \text{ m}$

So, $\Delta y = 0.145 \text{ m} = 14.5 \text{ cm}.$

22.15. Model: A narrow single slit produces a single-slit diffraction pattern.Visualize: The intensity pattern for single-slit diffraction will look like Figure 22.14.Solve: The minima occur at positions

$$y_p = p \frac{L\lambda}{a}$$

So
$$\Delta y = y_2 - y_1 = \frac{2\lambda L}{a} - \frac{1\lambda L}{a} = \frac{\lambda L}{a} \implies a = \frac{\lambda L}{\Delta y} = \frac{(633 \times 10^{-9} \text{ m})(1.5 \text{ m})}{0.00475 \text{ m}} = 2.0 \times 10^{-4} \text{ m} = 0.20 \text{ mm}$$

22.21. Model: The crack in the cave is like a single slit that causes the ultrasonic sound beam to diffract. **Visualize:**



Solve: The wavelength of the ultrasound wave is

$$\lambda = \frac{340 \text{ m/s}}{30 \text{ kHz}} = 0.0113 \text{ m}$$

Using the condition for complete destructive interference with p = 1,

$$a\sin\theta_1 = \lambda \Rightarrow \theta_1 = \sin^{-1}\left(\frac{0.0113 \text{ m}}{0.30 \text{ m}}\right) = 2.165^\circ$$

From the geometry of the diagram, the width of the sound beam is

 $w = 2y_1 = 2(100 \text{ m} \times \tan \theta_1) = 200 \text{ m} \times \tan 2.165^\circ = 7.6 \text{ m}$

Assess: The small-angle approximation is almost always valid for the diffraction of light, but may not be valid for the diffraction of sound waves, which have a much larger wavelength.

22.24. Model: Light passing through a circular aperture leads to a diffraction pattern that has a circular central maximum surrounded by a series of secondary bright fringes.Visualize: The intensity pattern will look like Figure 22.15.Solve: From Equation 22.24, the diameter of the circular aperture is

$$D = \frac{2.44\lambda L}{w} = \frac{2.44(633 \times 10^{-9} \text{ m})(4.0 \text{ m})}{2.5 \times 10^{-2} \text{ m}} = 0.25 \text{ mm}$$

22.26. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the pathlength difference Δr to increase by λ . Visualize: Please refer to the interferometer in Figure 22.20. Solve: From Equation 22.33, the wavelength is $\lambda = \frac{2\Delta L_2}{\Delta m} = \frac{2(100 \times 10^{-6} \text{ m})}{500} = 4.0 \times 10^{-7} \text{ m} = 400 \text{ nm}$

22.29. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the pathlength difference Δr to increase by λ .

Visualize: Please refer to the interferometer in Figure 22.20.

Solve: For sodium light of the longer wavelength (λ_1) and of the shorter wavelength (λ_2) ,

$$\Delta L = m \frac{\lambda_1}{2} \qquad \Delta L = (m+1) \frac{\lambda_2}{2}$$

We want the same path difference $2(L_2 - L_1)$ to correspond to one extra wavelength for the sodium light of shorter wavelength (λ_2). Thus, we combine the two equations to obtain:

$$m\frac{\lambda_1}{2} = (m+1)\frac{\lambda_2}{2} \Longrightarrow m(\lambda_1 - \lambda_2) = \lambda_2 \Longrightarrow m = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{589.0 \text{ nm}}{589.6 \text{ nm} - 589.0 \text{ nm}} = 981.67 \cong 982$$

Thus, the distance by which M₂ is to be moved is

$$\Delta L = m \frac{\lambda_1}{2} = 982 \left(\frac{589.6 \text{ nm}}{2}\right) = 0.2895 \text{ mm}$$

22.32. Model: Two closely spaced slits produce a double-slit interference pattern.Visualize: The interference fringes are equally spaced on both sides of the central maximum. The interference pattern looks like Figure 22.3(b).Solve: In the small-angle approximation

$$\Delta \theta = \theta_{m+1} - \theta_m = (m+1)\frac{\lambda}{d} - m\frac{\lambda}{d} = \frac{\lambda}{d}$$

Since $d = 200\lambda$, we have

$$\Delta \theta = \frac{\lambda}{d} = \frac{1}{200} \text{ rad} = 0.286^{\circ}$$

22.43. Model: A diffraction grating produces an interference pattern that is determined by both the slit spacing and the wavelength used. The visible spectrum spans the wavelengths 400 nm to 700 nm. Solve: According to Equation 22.16, the distance y_m from the center to the *m*th maximum is $y_m = L \tan \theta_m$. The angle of diffraction is determined by the constructive-interference condition $d \sin \theta_m = m\lambda$, where $m = 0, 1, 2, 3, \dots$ The width of the rainbow for a given fringe order is thus $w = y_{red} - y_{violet}$. The slit spacing is

$$d = \frac{1 \text{ mm}}{600} = \frac{1.0 \times 10^{-5} \text{ m}}{600} = 1.6667 \times 10^{-6} \text{ m}$$

For the red wavelength and for the m = 1 order,

$$d\sin\theta_{1} = (1)\lambda \Longrightarrow \theta_{1} = \sin^{-1}\frac{\lambda}{d} = \sin^{-1}\frac{700 \times 10^{-9} \text{ m}}{1.6667 \times 10^{-6} \text{ m}} = 24.83^{\circ}$$

From the equation for the distance of the fringe,

$$y_{\rm red} = L \tan \theta_1 = (2.0 \text{ m}) \tan (24.83^\circ) = 92.56 \text{ cm}$$

Likewise for the violet wavelength,

$$\theta_1 = \sin^{-1} \left(\frac{400 \times 10^{-9} \text{ m}}{1.6667 \times 10^{-6} \text{ m}} \right) = 13.88^\circ \Rightarrow y_{\text{violet}} = (2.0 \text{ m}) \tan(13.88^\circ) = 49.42 \text{ cm}$$

The width of the rainbow is thus 92.56 cm – 49.42 cm = 43.14 cm \approx 43 cm .

22.48. Visualize: The relationship between the diffraction grating spacing *d*, the angle at which a particular order of constructive interference occurs θ_m , the wavelength of the light, and the order of the constructive interference *m* is $d \sin \theta_m = m\lambda$. Also note N = 1/d.

Solve: The first order diffraction angle for green light is

$$\theta_1 = \sin^{-1}(\lambda/d) = \sin^{-1}(5.5 \times 10^{-7} \text{ m}/2.0 \times 10^{-6} \text{ m}) = \sin^{-1}(0.275) = 0.278 \text{ rad} = 16^{\circ}$$

Assess: This is a reasonable angle for a first order maximum.

22.52. Model: A narrow slit produces a single-slit diffraction pattern.

Visualize: The diffraction-intensity pattern from a single slit will look like Figure 22.14.

Solve: As given by Equation 22.19, the dark fringes in the pattern are located at $a \sin \theta_p = p\lambda$, where p = 1, 2, 3,

... For the diffraction pattern to have no minima, the first minimum must be located at least at $\theta_1 = 90^\circ$. From the constructive-interference condition $a \sin \theta_p = p\lambda$, we have

$$a = \frac{p\lambda}{\sin\theta_p} \Rightarrow a = \frac{\lambda}{\sin90^\circ} = \lambda = 633 \text{ nm}$$

22.58. Model: The antenna is a circular aperture through which the microwaves diffract. Solve: (a) Within the small-angle approximation, the width of the central maximum of the diffraction pattern is $w = 2.44 \lambda L/D$. The wavelength of the radiation is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{12 \times 10^9 \text{ Hz}} = 0.025 \text{ m} \Rightarrow w = \frac{2.44(0.025 \text{ m})(30 \times 10^3 \text{ m})}{2.0 \text{ m}} = 920 \text{ m}$$

That is, the diameter of the beam has increased from 2.0 m to 915 m, a factor of 458. **(b)** The average microwave intensity is

$$I = \frac{P}{\text{area}} = \frac{100 \times 10^3 \text{ W}}{\pi \left[\frac{1}{2} (915 \text{ m})\right]^2} = 0.15 \text{ W/m}^2$$

22.60. Model: The laser light is diffracted by the circular opening of the laser from which the beam emerges. **Solve:** The diameter of the laser beam is the width of the central maximum. We have

$$w = \frac{2.44\lambda L}{D} \Rightarrow D = \frac{2.44\lambda L}{w} = \frac{2.44(532 \times 10^{-9} \text{ m})(3.84 \times 10^{8} \text{ m})}{1000 \text{ m}} = 0.50 \text{ m}$$

In other words, the laser beam must emerge from a laser of diameter 50 cm.

22.63. Model: An interferometer produces a new maximum each time L_2 increases by $\frac{1}{2}\lambda$ causing the pathlength difference Δr to increase by λ .

Visualize: Please refer to the interferometer in Figure 22.20.

Solve: The path-length difference between the two waves is $\Delta r = 2L_2 - 2L_1$. The condition for constructive interference is $\Delta r = m\lambda$, hence constructive interference occurs when

$$2(L_2 - L_1) = m\lambda \Longrightarrow L_2 - L_1 = \frac{1}{2}m\lambda_2 = 1200(\frac{1}{2}\lambda) = 600\lambda$$

where $\lambda = 632.8$ nm is the wavelength of the helium-neon laser. When the mirror M₂ is moved back and a hydrogen discharge lamp is used, 1200 fringes shift again. Thus,

$$L'_2 - L_1 = 1200(\frac{1}{2}\lambda') = 600\lambda'$$

where $\lambda' = 656.5$ nm. Subtracting the two equations,

$$(L_2 - L_1) - (L'_2 - L_1) = 600(\lambda - \lambda') = 600(632.8 \times 10^{-9} \text{ m} - 656.5 \times 10^{-9} \text{ m})$$

 $\Rightarrow L'_2 = L_2 + 14.2 \times 10^{-6} \text{ m}$

That is, M_2 is now 14.2 μm closer to the beam splitter.

22.66. Model: The piece of glass increases the number of wavelengths in one arm of the interferometer. Each additional wavelength causes one bright-dark-bright fringe shift. Solve: We can rearrange Equation 22.36 to find that the index of refraction of glass is

$$n = 1 + \frac{\lambda_{\text{vac}} \Delta m}{2d} = 1 + \frac{(500 \times 10^{-9} \text{ nm})(200)}{2(0.10 \times 10^{-3} \text{ m})} = 1.50$$