

41.6. Wave function a is an antibonding orbital and wave function b is a bonding orbital. You can tell because in the bottom one the electron has a high probability of being found between adjacent protons indicating that the adjacent protons are sharing the electron.

41.7. $(P_{\text{tunnel}})_d > (P_{\text{tunnel}})_b > (P_{\text{tunnel}})_a > (P_{\text{tunnel}})_c$ because $P_{\text{tunnel}} = e^{\frac{-2w}{\eta}}$ and $\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$.

41.16. Model: The electron is a quantum harmonic oscillator.

Solve: (a) The energy levels of a harmonic oscillator are $E = (n + \frac{1}{2})\hbar\omega = \frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \dots$. The classical angular frequency of a mass on a spring is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.0 \text{ N/m}}{9.11 \times 10^{-31} \text{ kg}}} = 1.48 \times 10^{15} \text{ rad/s}$$

$$\Rightarrow \hbar\omega = (1.05 \times 10^{-34} \text{ J s})(1.48 \times 10^{15} \text{ rad/s}) = 1.56 \times 10^{-19} \text{ J} = 0.972 \text{ eV}$$

The first three energy levels are $E_1 = 0.49 \text{ eV}$, $E_2 = 1.46 \text{ eV}$, and $E_3 = 2.43 \text{ eV}$.

(b) The photon energy equals the energy lost by the electron: $E_{\text{photon}} = \Delta E_{\text{elec}} = E_3 - E_1 = 1.94 \text{ eV} = 3.10 \times 10^{-19} \text{ J}$. The wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E_{\text{elec}}} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.0 \times 10^8 \text{ m/s})}{3.10 \times 10^{-19} \text{ J}} = 640 \text{ nm}$$

41.17. Model: See Example 41.9.

Visualize: For a harmonic oscillator the energy difference between adjacent energy levels is $\Delta E = \hbar\omega_e$. We also know the emitted photon has energy $E_{\text{photon}} = hf_{\text{photon}} = \Delta E$. Combine these to get $\omega_e = 2\pi\frac{c}{\lambda}$.

Solve: We also recall that $\omega_e = \sqrt{\frac{k}{m}}$. Solve this for k and substitute for ω_e .

$$k = m\omega_e^2 = m\left(2\pi\frac{c}{\lambda}\right)^2 = (9.11 \times 10^{-31} \text{ kg})\left(2\pi\frac{3.0 \times 10^8 \text{ m/s}}{1200 \times 10^{-9} \text{ m}}\right)^2 = 2.25 \text{ N/m}$$

Assess: This answer is one-fourth of the answer in Example 41.9, which makes sense. An electron would emit a 1200 nm photon in any $n \rightarrow n-1$ jump in this quantum harmonic oscillator; not just the $3 \rightarrow 2$ jump.

41.20. Solve: Electrons are bound inside metals by an amount of energy called the work function E_0 . This is the energy that must be supplied to lift an electron out of the metal. In our case, E_0 is the amount of energy ($U_0 - E$) appearing in Equation 41.41 for the penetration distance. Thus,

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{(1.05 \times 10^{-34} \text{ J s})}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(4.0 \text{ eV} \times 1.60 \times 10^{-19} \text{ J/eV})}} = 9.72 \times 10^{-11} \text{ m}$$

The probability that an electron will tunnel through a $w = 4.54 \text{ nm}$ gap (from a metal to an STM probe) is

$$P_{\text{tunnel}} = e^{-2w/\eta} = e^{-2(0.45 \times 10^{-9} \text{ m})/(9.72 \times 10^{-11} \text{ m})} = 9.5 \times 10^{-5} = 0.0095\%$$

41.21. Solve: (a) The probability of an electron tunneling through a barrier is

$$P_{\text{tunnel}} = e^{-2w/\eta} \quad \eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

$$\Rightarrow \ln P_{\text{tunnel}} = -2w/\eta \Rightarrow [\ln P_{\text{tunnel}}]^2 = 4w^2 \frac{2m(U_0 - E)}{\hbar^2} \Rightarrow E = U_0 - \frac{\hbar^2}{8mw^2} [\ln P_{\text{tunnel}}]^2$$

$$\Rightarrow E = 5.0 \text{ eV} - \frac{(1.05 \times 10^{-34} \text{ J s})^2}{8(9.11 \times 10^{-31} \text{ kg})(1.0 \times 10^{-9} \text{ m})^2} \frac{1 \text{ eV}}{(1.60 \times 10^{-19} \text{ J})} [\ln P_{\text{tunnel}}]^2$$

$$= 5.0 \text{ eV} - (0.009455 \text{ eV}) [\ln P_{\text{tunnel}}]^2$$

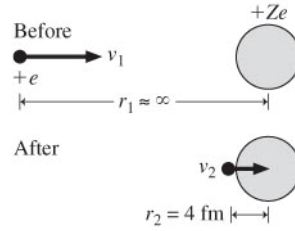
For $P_{\text{tunnel}} = 0.10$, $E = 5.0 \text{ eV} - 0.050 \text{ eV} = 4.95 \text{ eV}$.

(b) For $P_{\text{tunnel}} = 0.010$, $E = 5.0 \text{ eV} - 0.20 \text{ eV} = 4.80 \text{ eV}$.

(c) For $P_{\text{tunnel}} = 0.0010$, $E = 5.0 \text{ eV} - 0.45 \text{ eV} = 4.55 \text{ eV}$.

41.31. Model: The nucleus can be modeled as a potential well.

Visualize:



Please refer to Figure 41.17.

Solve: The gamma ray wavelength $\lambda = 1.73 \times 10^{-4} \text{ nm}$ corresponds to a photon energy of $E_{\text{photon}} = hc/\lambda = 7.2 \text{ MeV}$. From Fig. 41.17, we can see that a photon of this energy is emitted in a transition from the $n = 2$ to $n = 1$ energy level. This can happen after a proton-nucleus collision if the proton's impact excites the nucleus from the $n = 1$ ground state to the $n = 2$ excited state. To cause such an excitation, the proton's kinetic energy at the instant of impact must be $K \geq 7.2 \text{ MeV}$. Let v_1 be the proton's initial speed at the distance $r_1 \approx \infty$. If v_1 is the *minimum* speed that can excite the $n = 2$ state in the nucleus, then the proton has $K_2 = 7.2 \text{ MeV}$ at the distance r_2 equal to the radius of nucleus (4 fm). Its potential energy at this point is the electrostatic potential energy between the proton of charge $+e$ and the nucleus of charge $+Ze$, with $Z = 13$. The conservation of energy equation $K_1 + U_1 = K_2 + U_2$ is

$$\begin{aligned} \frac{1}{2}mv_1^2 + 0 \text{ J} &= K_2 + \frac{Ze^2}{4\pi\epsilon_0 r} \\ \Rightarrow v_1 &= \sqrt{\frac{2}{m} \left(K_2 + \frac{Ze^2}{4\pi\epsilon_0 r} \right)} = \sqrt{\frac{2}{1.67 \times 10^{-27} \text{ kg}} \left((7.2 \text{ MeV}) \left(\frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right) + \frac{(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2) 13 (1.6 \times 10^{-19} \text{ C})^2}{4.0 \times 10^{-15} \text{ m}} \right)} \\ &= 4.77 \times 10^7 \text{ m/s} \end{aligned}$$

This is the *minimum* speed, so any $v_1 \geq 4.77 \times 10^7 \text{ m/s}$ can cause the emission of a gamma ray.

41.34. Solve: (a) From Equation 41.46, $b = \sqrt{\hbar/m\omega}$. The units of b are

$$\sqrt{\frac{\text{J} \times \text{s}}{\text{kg} \times \text{s}^{-1}}} = \sqrt{\frac{\text{kg} \times \text{m}^2 \text{ s}^2}{\text{s}^2 \text{ kg}}} = \text{m}$$

(b) The classical turning point is the point where $U = E$ and $K = 0$ J. Since $U = \frac{1}{2}kx^2$ and $E = E_1 = \frac{1}{2}\hbar\omega$, the classical turning point for an oscillator in the $n = 1$ state is

$$\frac{1}{2}kx_{\text{tp}}^2 = \frac{1}{2}\hbar\omega \Rightarrow x_{\text{tp}} = \pm\sqrt{\frac{\hbar\omega}{k}} = \pm\sqrt{\frac{\hbar\omega}{\omega^2 m}} = \pm\sqrt{\frac{\hbar}{\omega m}} = \pm b$$

41.35. Solve: (a) The ground-state wave function of the quantum harmonic oscillator is $\psi_1(x) = A_1 e^{-x^2/2b^2}$. Normalization requires

$$\int_{-\infty}^{\infty} |\psi_1(x)|^2 dx = A_1^2 \int_{-\infty}^{\infty} e^{-x^2/2b^2} dx = 1$$

Change the variable to $u = x/b$. Then, $dx = bdu$. The integration limits don't change, so

$$1 = bA_1^2 \int_{-\infty}^{\infty} e^{-u^2} du$$

The definite integral can be looked up in a table of integrals. The result is $\sqrt{\pi}$. Hence,

$$1 = bA_1^2 \sqrt{\pi} = A_1^2 \sqrt{\pi b^2} \Rightarrow A_1 = \frac{1}{(\pi b^2)^{1/4}}$$

(b) The forbidden region is both $x < -b$ and $x > b$. $|\psi_1(x)|^2$ is symmetrical about $x = 0$ m, so

$$\text{Prob}(x < -b \text{ or } x > b) = (2)\text{Prob}(x > b) = 2 \int_b^{\infty} |\psi_1(x)|^2 dx = \frac{2}{\sqrt{\pi b^2}} \int_b^{\infty} e^{-x^2/2b^2} dx$$

(c) The integral of part (b) cannot be evaluated in closed form, but the answer can be found with a numerical integration. First, change the variable to $u = x/b$, making $dx = bdu$. But unlike the variable change in part (c), this *does* change the lower limit of integration. Thus,

$$\text{Prob}(x < -b \text{ or } x > b) = \frac{2}{\sqrt{\pi b^2}} b \int_1^{\infty} e^{-u^2} du = \frac{2}{\sqrt{\pi}} \int_1^{\infty} e^{-u^2} du$$

The definite integral can be evaluated numerically with a calculator or computer, giving

$$\int_1^{\infty} e^{-u^2} du = 0.139$$

The probability of finding the harmonic oscillator in the forbidden region is $2(\pi)^{-1/2}(0.139) = 0.157 = 15.7\%$.

41.37. Model: The collisions with the ground are perfectly elastic.

Solve: (a) The classical probability density at position y of finding a ball that bounces between the ground and height h is given by Equation 41.32:

$$P_{\text{class}}(y) = \frac{2}{Tv(y)}$$

where $v(y)$ is the ball's velocity as a function of y and T is the period of oscillation. For a freely falling object, energy conservation gives

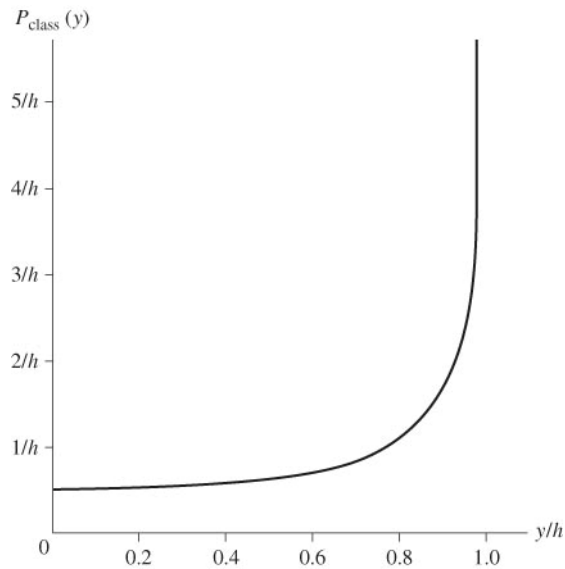
$$mgh = \frac{1}{2}mv^2 + mgy \Rightarrow v(y) = \sqrt{2g(h-y)}$$

The time $t = \frac{1}{2}T$ to reach a height h after a collision with the ground can be found from kinematics:

$$\Delta y = h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

$$\Rightarrow P_{\text{class}}(y) = \frac{2}{2\sqrt{2h/g}} \frac{1}{\sqrt{2g(h-y)}} = \sqrt{\frac{g}{2h}} \frac{1}{\sqrt{2g(h-y)}} = \frac{1}{2\sqrt{h}\sqrt{h-y}} = \left(\frac{1}{2h}\right) \frac{1}{\sqrt{1-(y/h)}}$$

(b)



(c) The ball is most likely to be found near the upper turning point at $y = h$. This is because $v \rightarrow 0$ m/s at $y = h$ so the ball spends more time at this point. For the same reason, the ball spends the least time near the ground, where it is moving fastest, and thus the probability density is the least at $y = 0$ m.

41.40. Solve: From Equation 41.41, the penetration distance is

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{(1.05 \times 10^{-34} \text{ J s})}{\sqrt{2(1.67 \times 10^{-27} \text{ kg})(1.0 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}} = 4.54 \times 10^{-15} \text{ m}$$

The probability the proton will tunnel through the barrier is

$$P_{\text{tunnel}} = e^{-2w/\eta} = e^{-2(10 \times 10^{-15} \text{ m})/(4.54 \times 10^{-15} \text{ m})} = 0.012 = 1.2\%$$

42.6. (a) Yes, this is the ground state of nitrogen. (b) No, only two electrons are allowed in the $2s$ state. (c) No, the $2s$ state would fill before the $2p$ state.

42.9. The red-orange colors in the neon emission spectrum are due to transitions from excited $3p$ states to the lower energy but still excited $3s$ states. This occurs because the ground states are collisionally excited by the electrical discharge. The absorption spectrum of a gas consists of *only* those spectral lines that start from the ground state. This is because essentially all the atoms are in the ground state and the population of excited states is too small for absorption from these states to be noticed. For neon in a glass tube, there's no population of atoms in the $3s$ state to absorb red-orange light. The only wavelengths that can be absorbed are those absorbed by ground-state neon atoms. The lowest excited states are greater than 16 eV above the ground state, so all absorption lines in neon will have wavelengths less than 100 nm. Neon is transparent to all visible wavelengths of light.

42.10. The radial probability density $P_r(r) = 4\pi r^2 |R_{nl}(r)|^2$ is the probability density for finding the electron at a *distance* r from the nucleus. The radial wave function $R_{1s}(r)$ is a maximum at the origin $r = 0$ m. Consequently, the origin is the most likely *point* for finding the electron. The probability density $P(r)$ is smaller at any *one* point having $r = a_B$ than it is at the origin. However, there are *many* points having $r = a_B$, whereas only a single point has $r = 0$ m. The increased number of points more than compensates for the decreased probability per point. As a result, it is more probable to find the electron at *distance* $r = a_B$ than it is at distance $r = 0$ m.

42.1. Solve: (a) A $4p$ state corresponds to $n = 4$ and $l = 1$. From Equation 42.3, the orbital angular momentum is $L = \sqrt{1(1+1)}\hbar = \sqrt{2}\hbar$.

(b) In the case of a $5f$ state, $n = 5$ and $l = 3$. So, $L = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar$.

42.6. Model: No two electrons can have exactly the same set of quantum numbers (n, l, m, m_s) .

Solve: For $n = 1$, there are a total of two states with the quantum numbers given by $(1, 0, 0, \pm \frac{1}{2})$. For $n = 2$, there are a total of eight states:

$$(2, 0, 0, \pm \frac{1}{2}) \quad (2, 1, -1, \pm \frac{1}{2}) \quad (2, 1, 0, \pm \frac{1}{2}) \quad (2, 1, 1, \pm \frac{1}{2})$$

For $n = 3$, there are a total of 18 states:

$$(3, 0, 0, \pm \frac{1}{2}) \quad (3, 1, -1, \pm \frac{1}{2}) \quad (3, 1, 0, \pm \frac{1}{2}) \quad (3, 1, 1, \pm \frac{1}{2}) \\ (3, 2, 2, \pm \frac{1}{2}) \quad (3, 2, 1, \pm \frac{1}{2}) \quad (3, 2, 0, \pm \frac{1}{2}) \quad (3, 2, -1, \pm \frac{1}{2}) \quad (3, 2, -2, \pm \frac{1}{2})$$

42.7. Solve: (a) A lithium atom has three electrons, two are in the $1s$ shell and one is in the $2s$ shell. The electron in the $2s$ shell has the following quantum numbers: $n = 2$, $l = 0$, $m = 0$, and m_s . m_s could be either $+\frac{1}{2}$ or $-\frac{1}{2}$. Thus, lithium atoms should behave like hydrogen atoms because lithium atoms could exist in the following two states: $(2, 0, 0, +\frac{1}{2})$ and $(2, 0, 0, -\frac{1}{2})$. Thus there are two lines.

(b) For a beryllium atom, we have two electrons in the $1s$ shell and two electrons in the $2s$ shell. The electrons in both the $1s$ and $2s$ states are filled. Because the two electron magnetic moments point in opposite directions, beryllium has *no* net magnetic moment and is not deflected in a Stern-Gerlach experiment. Thus there is only one line.

42.14. Model: Assume the hydrogen atom starts in the ground state.

Visualize: The electron gains 12.5 eV of energy in the acceleration. This is enough to excite the hydrogen atom to $n = 3$ (with the electron left with some energy) but no higher. (See Figure 42.4.)

Solve: The atom could emit a photon in the two possible quantum-jump transitions: $3 \rightarrow 2$ and $3 \rightarrow 1$. The corresponding energies of the emitted photons are $-1.51 \text{ eV} - (-3.40 \text{ eV}) = 1.89 \text{ eV}$ and $-1.51 \text{ eV} - (-13.60 \text{ eV}) = 12.09 \text{ eV}$. The corresponding wavelengths are given by $\lambda = hc/E_{\text{photon}}$.

transition	E_{photon} (eV)	λ (nm)
$3 \rightarrow 2$	1.89	656
$3 \rightarrow 1$	12.09	102

Assess: The wavelength for the $3 \rightarrow 2$ transition is in the visible Balmer series; while the one for the $3 \rightarrow 1$ transition is in the ultraviolet region.

42.15. Solve: (a) A $4p \rightarrow 4s$ transition is allowed because $\Delta l = 1$. Using the sodium energy levels from Figure 42.25, the wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV nm}}{3.75 \text{ eV} - 3.19 \text{ eV}} = 2210 \text{ nm} = 2.21 \mu\text{m}$$

(b) A $3d \rightarrow 4s$ transition is not allowed because $\Delta l = 2$ violates the selection rule that requires $\Delta l = 1$.

42.17. Solve: The interval $\Delta t = 0.50 \text{ ns}$ is very small in comparison with the lifetime $\tau = 25 \text{ ns}$, so we can write

$$\text{Prob}(\text{decay in } \Delta t) = r \Delta t$$

where r is the decay rate. The decay rate is related to the lifetime by $r = 1/\tau = 1/(25 \text{ ns}) = 0.040 \text{ ns}^{-1}$. Thus

$$\text{Prob}(\text{decay in } \Delta t) = (0.040 \text{ ns}^{-1})(0.50 \text{ ns}) = 0.020 = 2.0\%$$

42.19. Solve: (a) The number of atoms that have undergone a quantum jump to the ground state is

$$0.90N_0 = 0.90(1.0 \times 10^6) = 9.0 \times 10^5$$

Because each transition is accompanied by a photon, the number of emitted photons is also 9.0×10^5 .

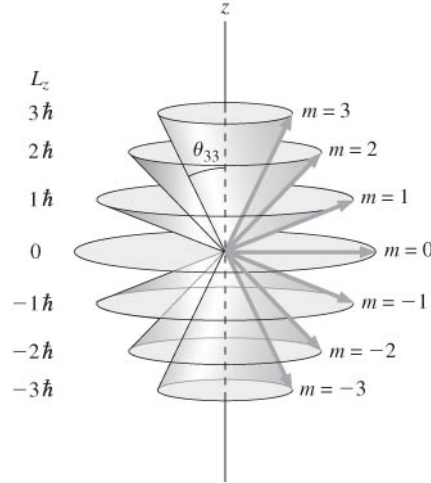
(b) 10% of the atoms remain excited at $t = 20$ ns. Thus

$$N_{\text{exc}} = N_0 e^{-t/\tau} \Rightarrow 0.10 N_0 = N_0 e^{-20 \text{ ns}/\tau} \Rightarrow \ln(0.10) = \frac{-20 \text{ ns}}{\tau} \Rightarrow \tau = 8.7 \text{ ns}$$

42.24. Solve: (a) For $l = 3$, the magnitude of the angular momentum vector is

$$L = \sqrt{l(l+1)}\hbar = \sqrt{3(3+1)}\hbar = \sqrt{12}\hbar$$

The angular momentum vector has a z -component $L_z = L\cos\theta$ along the z -axis. $L_z = m\hbar$ where m is an integer between $-l$ and l , that is, between -3 and 3 . The seven possible orientations of the angular momentum vector for $l = 3$ are shown in figure below.



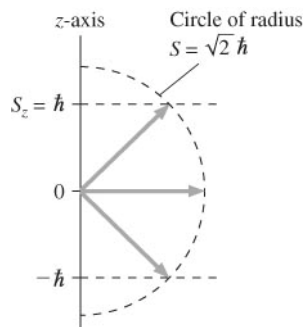
(b) From Equation 42.5, the minimum angle between $\vec{L}$ and the z -axis is

$$\theta_{33} = \cos^{-1}\left(\frac{3\hbar}{\sqrt{12}\hbar}\right) = 30^\circ$$

42.25. Solve: (a) For $s = 1$, $S = \sqrt{s(s+1)}\hbar = \sqrt{2}\hbar = 1.48 \times 10^{-34} \text{ J s}$.

(b) The spin quantum number is $m_s = -1, 0, \text{ or } 1$.

(c) The figure below shows the three possible orientations of $\vec{S}$.



42.26. Solve: (a) Since $L_x^2 + L_y^2 + L_z^2 = L^2$, $L_x^2 + L_y^2 = L^2 - L_z^2$. For a hydrogen atom with $l = 2$, the magnitude of L^2 is always $l(l + 1)\hbar^2$ or $6\hbar^2$. The value of L_z is $m\hbar$, where m is an integer between $-l$ and l . Hence, the maximum value of L_z is $2\hbar$ and the *minimum* value of $(L_x^2 + L_y^2)^{1/2}$ is

$$\sqrt{l(l+1)\hbar^2 - (2\hbar)^2} = \sqrt{6\hbar^2 - 4\hbar^2} = \sqrt{2}\hbar$$

(b) Because the minimum value of L_z is 0 J s, the maximum value of $\sqrt{L_x^2 + L_y^2}$ is

$$\sqrt{L^2 - L_z^2} = \sqrt{6\hbar^2 - 0 \text{ J s}} = \sqrt{6}\hbar$$

42.28. Solve: (a) From Equation 42.7, the radial wave function of hydrogen in the 1s state is

$$R_{1s}(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B} \Rightarrow R_{1s}\left(\frac{1}{2}a_B\right) = \frac{1}{\sqrt{\pi a_B^3}} e^{-\frac{1}{2}} = \frac{0.342}{\sqrt{a_B^3}}$$

(b) From Equation 42.10, the probability density is

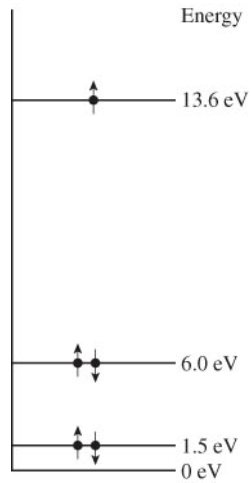
$$P_r(r) = 4\pi r^2 \left| R_{1s}(r) \right|^2 \Rightarrow P_{1s}\left(\frac{1}{2}a_B\right) = 4\pi \left(\frac{a_B}{2}\right)^2 \left(\frac{1}{\sqrt{\pi a_B^3}} e^{-\frac{1}{2}} \right)^2 = \frac{0.368}{a_B}$$

42.43. Model: We have a one-dimensional rigid box with infinite potential walls and a length 0.50 nm.

Solve: (a) From Equation 41.22, the lowest energy level is

$$E_1 = \frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34} \text{ J s})^2}{8(9.11 \times 10^{-31} \text{ kg})(5.0 \times 10^{-10} \text{ m})^2} \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} = 1.51 \text{ eV}$$

The next two levels are $E_2 = 4E_1 = 6.04 \text{ eV}$ and $E_3 = 9E_1 = 13.6 \text{ eV}$. The Pauli principle allows only two electrons in each of these energy levels, one with spin up and one with spin down. So five electrons fill the $n = 1$ and $n = 2$ levels, with the fifth electron going to $n = 3$.



(b) The ground-state energy of these five electrons is $E = 2E_1 + 2E_2 + E_3 = 28.7 \text{ eV}$

42.47. Solve: The number of excited atoms left at time t is given by Equation 42.25: $N_{\text{ext}} = N_0 e^{-t/\tau}$. If 1% of the atoms in the excited state decay in $t = 0.20$ ns, then 99% of the atoms remain in the excited state. So,

$$0.99N_0 = N_0 e^{-0.20 \text{ ns}/\tau} \Rightarrow 0.99 = e^{-0.20 \text{ ns}/\tau} \Rightarrow \ln(0.99) = -0.20 \text{ ns}/\tau \Rightarrow \tau = 19.90 \text{ ns}$$

Having determined τ , we can now find the time during which 25% of the sample of excited atoms would decay, leaving 75% still excited. Applying Equation 42.25 once again,

$$0.75N_0 = N_0 e^{-t/19.90 \text{ ns}} \Rightarrow t = 5.7 \text{ ns}$$

42.49. Solve: If there are N_{exc} atoms in the excited state, the *rate* of decay is rN_{exc} , where $r = 1/\tau$ is the decay rate. Each decay generates one photon, so the rate of photon emission (photons per second) is the same as the rate of decay (decays per second). Hence, the number of photons emitted per second is

$$rN_{\text{exc}} = \frac{N_{\text{exc}}}{\tau} = \frac{1.0 \times 10^9}{20 \times 10^{-9} \text{ s}} = 5.0 \times 10^{16}$$