40.2. The relationship between probability and probability density is similar to the relationship between mass m and mass density ρ. Regions of higher mass density tell us where mass is concentrated. The mass itself is a more tangible quantity that depends both on the mass density and on the size of a specific piece of material. Similarly, probability density tells us regions in which a particle is more likely, or less likely, to be found. The probability is a definite number between 0 and 1. Probability depends both on the probability density and on the size of the specific region we are considering.

40.4. (a) The probability density is maximum at $x \pm 2$ mm.
(b) We cannot tell where the wave function is most positive; it could be at either $x = 2$ mm or $x = -2$ mm. It will be positive at one and negative at the other.

40.6. Particle 1 because it has a less definite Δx and therefore a more definite $\Delta p = \Delta mv$.

Problems and Exercises

40.4. Model: The probability that the outcome will be A or B is the sum of PA and PB. The expected value is your best possible prediction of the outcome of an experiment.
Solve: For each deck, there are 12 picture cards (4 Jacks, 4 Queens, and 4 Kings). Because the probability of drawing one card out of 52 cards is 1/52, the probability of drawing a card that is a picture card is 12/52 = 23.1%.
The number of picture cards that will be drawn is $0.231 \times 1000 = 231$.

40.7. Visualize: Combine Equations 40.10 and 40.11 to show that N is proportional to $|A(x)|^2\,\delta x$.

$$\frac{|A(x_2)|^2\,\delta x_2}{|A(x_1)|^2\,\delta x_1} = \frac{\dfrac{N(\text{in }\delta x_2\text{ at }x_2)}{N_{\text{tot}}}}{\dfrac{N(\text{in }\delta x_1\text{ at }x_1)}{N_{\text{tot}}}}$$

We are given $N_1 = 6000$, $\delta x_1 = 0.10$ mm, $A(x_1) = 200$ V/m, $N_2 = 3000$, and $\delta x_2 = 0.20$ mm. We are not given N_{tot} but it cancels anyway.

Solve: Solve the above equation for $|A(x_2)|$.

$$|A(x_2)| = |A(x_1)|\sqrt{\frac{\delta x_1 N(\text{in }\delta x_2\text{ at }x_2)}{\delta x_2 N(\text{in }\delta x_1\text{ at }x_1)}} = (200\text{ V/m})\sqrt{\frac{(0.10\text{ mm})(3000)}{(0.20\text{ mm})(6000)}} = 100\text{ V/m}$$

Assess: The answer is half of the wave amplitude at the other strip, which seems reasonable.

40.8. **Solve:** The probability that a photon arrives at this 0.10-mm-wide strip is

$$\text{Prob(in 0.10 mm at } x\text{)} = \frac{N}{1.0\times10^{10}} = P(x)\delta x$$

where N is the number of photons detected in the strip and the total number of photons is 1.0×10^{10}. We have

$$N = (1.0\times10^{10})(20\ \text{m}^{-1})(0.10\times10^{-3}\ \text{m}) \Rightarrow N = 2.0\times10^{7}$$

40.11. Model: The probability of finding a particle at position x is determined by $|\psi(x)|^2$.

Solve: **(a)** The probability of detecting an electron is Prob(in δx at x) $= |\psi(x)|^2 \delta x$. At $x = 0$ mm, the number of electrons landing is calculated as follows:

$$\frac{N}{1.0\times10^6} = |\psi(0\text{ mm})|^2 \delta x \Rightarrow N = \left(\tfrac{1}{3}\text{ mm}^{-1}\right)(0.010\text{ mm})\left(1.0\times10^6\right) = 3333$$

(b) Likewise, the number of electrons landing at $x = 2.0$ mm is

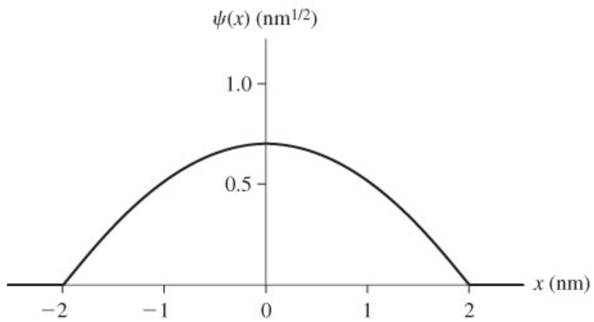
$$N = |\psi(2.0\text{ mm})|^2 \delta x N_{\text{total}} = \left(0.111\text{ mm}^{-1}\right)(0.010\text{ mm})\left(1.0\times10^6\right) = 1111$$

40.10. Solve: $|\psi(x)|^2 \delta x$ is a probability, which is dimensionless. The units of δx are m, so the units of $|\psi(x)|^2$ are m^{-1} and thus the units of ψ are $m^{-1/2}$.

40.14. Model: The probability of finding a particle is determined by the probability density $P(x) = |\psi(x)|^2$.

Solve: **(a)** The normalization condition for a wave function: $\int_{-\infty}^{\infty} |\psi(x)|^2 dx$ = area under the curve = 1. In the present case, the area under the $|\psi(x)|^2$-versus-x graph is $2a$ nm. Hence, $a = \frac{1}{2}$ nm^{-1}.

(b) Each point on the $\psi(x)$ graph is the square root of the corresponding point on the $|\psi(x)|^2$ graph. Where the $|\psi(x)|^2$ graph has dropped to 1/2 its maximum value at x = 1 nm, the $\psi(x)$ graph will have dropped only to $1/\sqrt{2} = 0.707$ of its maximum value. Thus the graph shape is convex upward. Since $a = \frac{1}{2}$ nm^{-1}, the peak value of $\psi(x)$ is $\sqrt{a} = 1/\sqrt{2}$ nm$^{-1/2}$ = 0.707 nm$^{-1/2}$. The graph is shown below. The negative of this graph, curving downward, would also be an acceptable wave function.



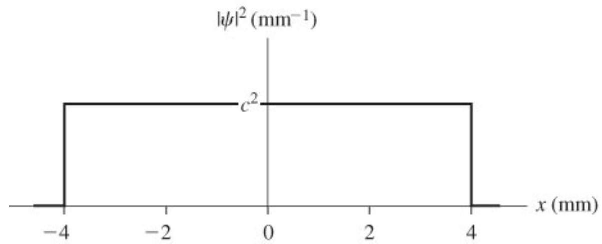
(c) The probability of the electron being located in the interval $1.0 \le x \le 2.0$ nm is

Prob(1.0 nm $\le x \le$ 2.0 nm) = area under the curve between 1.0 nm and 2.0 nm

$$= \frac{1}{2}\left(\frac{a}{2}\right)(2.0 \text{ nm} - 1.0 \text{ nm}) = \frac{(0.50 \text{ nm}^{-1})(1.0 \text{ nm})}{4} = 0.125$$

40.17. Model: The probability of finding the particle is determined by the probability density $P(x)=|\psi(x)|^2$.

Solve: **(a)** According to the normalization condition, $\int_{-\infty}^{\infty}|\psi(x)|^2\,dx=1$. From the given $\psi(x)$-versus-x graph, we first generate a $|\psi(x)|^2$-versus-x graph and then find the area under the curve.



The area under the $|\psi(x)|^2$-versus-x graph is

$$\int_{-4.0\text{ mm}}^{4.0\text{ mm}} c^2dx=(8.0\text{ mm})c^2=1\Rightarrow c=\sqrt{\frac{1}{8.0\text{ mm}}}=0.354\text{ mm}^{-1/2}$$

(b) The graph is shown above.
(c) The probability is

$$\text{Prob}(1.0\text{ mm}\le x\le 1.0\text{ mm})=\text{area}=c^2(2.0\text{ mm})=\left(\frac{1}{8.0}\text{mm}^{-1}\right)(2.0\text{ mm})=0.25$$

40.19. Model: A radio-frequency pulse is an electromagnetic wave packet, hence it must satisfy the relationship $\Delta f\,\Delta t \approx 1$.

Solve: The waves that must be superimposed to create the pulse of smallest duration span the frequency range
$f - \Delta f/2 \le f \le f + \Delta f/2$. Because Δf = 120 MHz – 80 MHz = 40 MHz, f = 100 MHz. Using Equation 40.20,

$$\Delta t \approx \frac{1}{\Delta f} = \frac{1}{40\text{ MHz}} = 2.5\times10^{-8}\text{ s} = 25\text{ ns}$$

Thus, a radio wave centered at 100 MHz and having a frequency span 80 MHz to 120 MHz can be used to create a wave of duration 25 ns.

40.20. Model: The beating of two waves of different frequencies produces a series of wave packets.

Solve: The beat frequency is $f_{beat} = f_1 - f_2 = 502\text{ Hz} - 498\text{ Hz} = 4\text{ Hz}$. The period of one beat is

$$T_{beat} = \frac{1}{f_{beat}} = \frac{1}{4\text{ Hz}} = 0.25\text{ s}$$

During 0.25 s, the wave moves forward $\Delta x = v_{sound} T_{beat} = (340\text{ m/s})(0.25\text{ s}) = 85\text{ m}$. Thus the length of each wave packet is 85 m.

40.21. Model: A laser pulse is an electromagnetic wave packet, hence it must satisfy the relationship $\Delta f\,\Delta t \approx$ 1.

Solve: Because $c = \lambda f$, the frequency and period are

$$f = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{-6}\ \text{m}} = 2.0\times10^{14}\ \text{Hz} \Rightarrow T = \frac{1}{f} = \frac{1}{2.0\times10^{14}\ \text{Hz}} = 5.0\times10^{-15}\ \text{s}$$

Since Δf = 2.0 GHz, the minimum pulse duration is

$$\Delta t \approx \frac{1}{\Delta f} = \frac{1}{2.0\times10^{9}\ \text{Hz}} = 5.0\times10^{-10}\ \text{s}$$

The number of oscillations in this laser pulse is

$$\frac{5.0\times10^{-10}\ \text{s}}{T} = \frac{5.0\times10^{-10}\ \text{s}}{5.0\times10^{-15}\ \text{s}} = 1.0\times10^{5}\ \text{oscillations}$$

40.22. Visualize: The uncertainty in velocity is $\Delta v_x = 3.58\times10^5$ m/s $-$ 3.48×10^5 m/s $= 1.0\times10^4$ m/s. We recall (or look up) the mass of an electron: $m_e = 9.11\times10^{-31}$ kg.

Solve: Solve for the position uncertainty Δx in Equation 40.28.

$$\Delta x \approx \frac{h}{2\Delta p_x} = \frac{h}{2m_e\Delta v_x} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2(9.11\times10^{-31}\ \text{kg})(1.0\times10^4\ \text{m/s})} = 3.6\times10^{-8}\ \text{m} = 36\ \text{nm}$$

Assess: The answer is a few dozen atomic diameters.

40.24. Model: Electrons are subject to the Heisenberg uncertainty principle.

Solve: Uncertainty in our knowledge of the position of the electron as it passes through the hole is $\Delta x = 10$ μm. With a finite Δx, the uncertainty Δp_x cannot be zero. Using the uncertainty principle,

$$\Delta p_x = m\Delta v_x = \frac{h}{2\Delta x} \Rightarrow \Delta v_x = \frac{h}{2m\Delta x} = \frac{6.63\times10^{-34}\ \text{J s}}{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(10\times10^{-6}\ \text{m}\right)} = 36\ \text{m/s}$$

Because the *average* velocity is zero, the best we can say is that the electron's velocity is somewhere in the interval

$-18\ \text{m/s} \le v_x \le 18\ \text{m/s}$.

40.25. Model: Protons are subject to the Heisenberg uncertainty principle.

Solve: We know the proton is somewhere within the nucleus, so the uncertainty in our knowledge of its position is at most $\Delta x = L = 4.0$ fm. With a finite Δx, the uncertainty Δp_x is given by the uncertainty principle:

$$\Delta p_x = m\Delta v_x = \frac{h/2}{\Delta x} \Rightarrow \Delta v_x = \frac{h}{2mL} = \frac{6.63\times10^{-34}\ \text{J s}}{2\left(1.67\times10^{-27}\ \text{kg}\right)\left(4.0\times10^{-15}\ \text{m}\right)} = 5.0\times10^{7}\ \text{m/s}$$

Because the average velocity is zero, the best we can say is that the proton's velocity is somewhere in the range -2.5×10^7 m/s to 2.5×10^7 m/s. Thus the smallest range of speeds is 0 to 2.5×10^7 m/s.

40.29. **Model:** The radio-wave pulses are wave packets, so each packet satisfies the relationship $\Delta f\,\Delta t \approx 1$.
Visualize: Please refer to Figure P40.29.
Solve: Because the frequency bandwidth is $\Delta f = 200$ kHz, the shortest possible pulse width is

$$\Delta t \approx \frac{1}{\Delta f} = \frac{1}{200\ \text{kHz}} = 5 \times 10^{-6}\ \text{s}$$

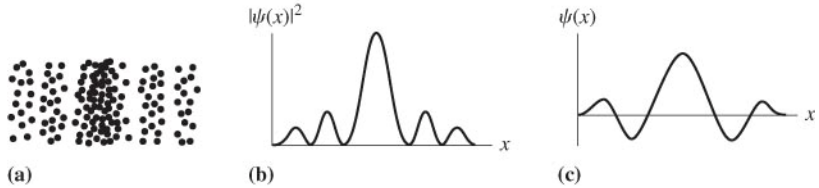
This means the time period of the pulse train is

$$T = 2\Delta t = 2(5 \times 10^{-6}\ \text{s}) = 10 \times 10^{-6}\ \text{s}$$

So, the frequency of the pulse train is $f = 1/T = 1.0 \times 10^{5}$ Hz. That is, the maximum transmission rate is 1.0×10^{5} pulses/s.

40.30. **Model:** The probability of finding a particle at position x is determined by $|\psi(x)|^2$.

Visualize:



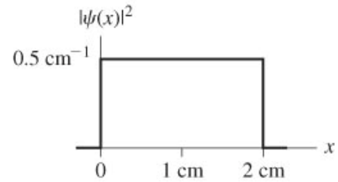
Solve: **(a)** Electrons are most likely to arrive at the points of maximum intensity. No electrons will arrive at points of zero intensity.

(b) The graph of $|\psi(x)|^2$ looks just like the classical intensity pattern of single-slit diffraction.

(c) The wave function $\psi(x)$ is square root of $|\psi(x)|^2$. It oscillates because it alternates between the positive and negative roots.

40.31. Model: The probability of finding a particle at position x is determined by $P(x) = |\psi(x)|^2$.

Visualize:



Solve: **(a)** Since the electrons are uniformly distributed over the interval $0 \le x \le 2$ cm, the probability density $P(x) = |\psi(x)|^2$ is constant over this interval. $P(x) = 0$ outside this interval because no electrons are detected. Thus $|\psi(x)|^2$ is a square function, as shown in the figure. To be normalized, the area under the probability curves must be 1. Hence, the peak value of $|\psi(x)|^2$ must be 0.5 cm^{-1}.

(b) The interval is $\delta x = 0.02$ cm. The probability is

$$\text{Prob(in } \delta x \text{ at } x = 0.80 \text{ cm)} = |\psi(x = 0.80 \text{ cm})|^2 \delta x = (0.5 \text{ cm}^{-1})(0.02 \text{ cm}) = 0.01 = 1\%$$

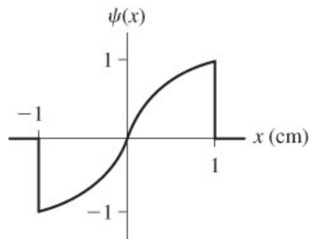
(c) From Equation 39.7, the number of electrons is

$$N(\text{in } \delta x \text{ at } x = 0.80 \text{ cm}) = N_{\text{total}}\text{Prob(in } \delta x \text{ at } x = 0.80 \text{ cm)} = 10^6 \times (0.01) = 10^4$$

(d) The probability density is $P(x = 0.80 \text{ cm}) = |\psi(x = 0.80 \text{ cm})|^2 = 0.5 \text{ cm}^{-1}$.

40.33. Model: The probability of finding a particle at position x is determined by $P(x) = |\psi(x)|^2$.

Visualize:



Solve: **(a)** Yes, because the area under the $|\psi(x)|^2$ curve is equal to 1.

(b) There are two things to consider when drawing $\psi(x)$. First $\psi(x)$ is an *oscillatory* function that changes sign every time it reaches zero. Second, $\psi(x)$ must have the right shape. Each point on the $\psi(x)$ curve is the square root of the corresponding point on the $|\psi(x)|^2$ curve. The values $|\psi(x)|^2 = 1\ \text{cm}^{-1}$ and $|\psi(x)|^2 = 0\ \text{cm}^{-1}$ clearly give $\psi(x) = \pm 1\ \text{cm}^{-1/2}$ and $\psi(x) = 0\ \text{cm}^{-1/2}$, respectively. But consider $x = 0.5$ cm, where $|\psi(x)|^2 = 0.5\ \text{cm}^{-1}$. Because $\sqrt{0.5} = 0.707$, $\psi(x = 0.5\ \text{cm}) = 0.707\ \text{cm}^{-1/2}$. This tells us that the $\psi(x)$ curve is not linear but *bows upward* over the interval $0 \le x \le 1$ cm. Thus, $\psi(x)$ has the shape shown in the above figure.

(c) $\delta x = 0.0010$ cm is a very small interval, so we can use Prob(in δx at x) = $|\psi(x)|^2\,\delta x$. The values of $|\psi(x)|^2$ can be read from Figure P40.33. Thus,

$$\text{Prob(in } \delta x \text{ at } x = 0.0\text{ cm)} = |\psi(x = 0.0\text{ cm})|^2\,\delta x = (0.0\ \text{cm}^{-1})(0.0010\ \text{cm}) = 0.000$$

$$\text{Prob(in } \delta x \text{ at } x = 0.5\text{ cm)} = |\psi(x = 0.5\text{ cm})|^2\,\delta x = (0.5\ \text{cm}^{-1})(0.0010\ \text{cm}) = 0.0005$$

$$\text{Prob(in } \delta x \text{ at } x = 0.999\text{ cm)} = |\psi(x = 1.0\text{ cm})|^2\,\delta x = (1.0\ \text{cm}^{-1})(0.0010\ \text{cm}) = 0.0010$$

(d) The number of electrons in the interval $-0.3\text{ cm} \le x \le 0.3$ cm is

$$N(\text{in } -0.3\text{ cm} \le x\ 0.3\text{ cm}) = N_{\text{total}} \times \text{Prob(in } -0.3\text{ cm} \le x \le 0.3\text{ cm)}$$

The probability is the area under the probability density curve. We have

$$\text{Prob(in } -0.3\text{ cm} \le x \le 0.3\text{ cm)} = \int_{-0.3\text{ cm}}^{0.3\text{ cm}} |\psi(x)|^2 dx = 2\times\left(\tfrac{1}{2}\times 0.3\text{ cm}\times 0.3\text{ cm}^{-1}\right) = 0.090$$

Thus, the number of electrons expected to land in the interval $-0.3\text{ cm} \le x \le 0.3$ cm is $10{,}000 \times 0.090 = 900$.

40.35. Model: The probability of finding a particle at position x is determined by the probability density $P(x) = |\psi(x)|^2$.

Solve: **(a)** The wave function is a straight line passing through the origin such that it is $+c$ at $x = +4$ mm and $-c$ at $x = -4$ mm. That is, the wave function is

$$\psi(x) = cx/4, \text{ where } x \text{ is in mm and } c \text{ is in } \text{mm}^{-1/2}.$$

Note that the units of c must be that of $\psi(x)$. Thus

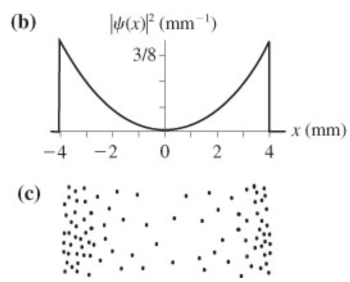
$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = \int_{-4}^{4} \left(c^2x^2/16\right) dx = 2\int_{0}^{4} \left(c^2x^2/16\right) dx = \frac{2c^2}{16}\left[\frac{x^3}{3}\right]_0^4 = \frac{8}{3}c^2$$

Because $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1,\ \frac{8}{3}c^2 = 1 \Rightarrow c = \sqrt{\frac{3}{8}}\ \text{mm}^{-1/2}$

(b) From part (a), we have

$$|\psi(x)|^2 = c^2x^2/16 = 3x^2/128 = \frac{3x^2}{128}\left(\text{mm}^{-1}\right)$$

A $|\psi(x)|^2$-versus-x graph is shown in the figure.



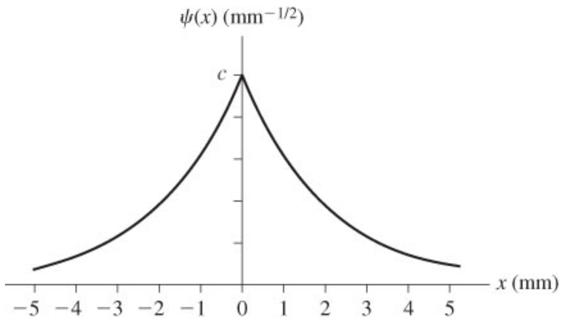
(c) The particle is most likely to be found at the positions where $|\psi(x)|^2$ is a maximum. The graph above gives a dot picture of the first few particles.

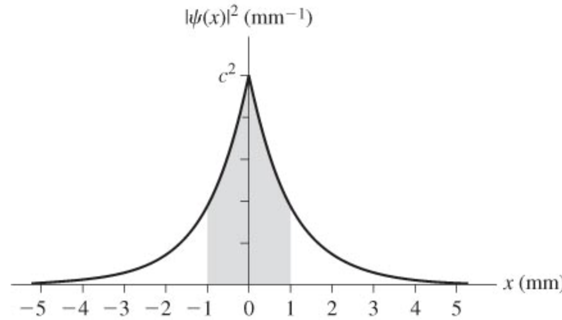
(d) $\text{Prob}(-2.0 \text{ mm} \le x \le 2.0 \text{ mm}) = 2\int_{-2.0 \text{ mm}}^{2.0 \text{ mm}} |\psi(x)|^2 dx = 2\int_0^{2.0} \frac{3x^2}{128} dx = 2\left(\frac{3}{128}\right)\left[\frac{x^3}{3}\right]_0^{2.0} = 0.125$

40.38. Model: The probability of finding a particle at position x is determined by the probability density $P(x) = |\psi(x)|^2$.

Solve: **(a)** $\psi(x) = ce^{x/L}$ for $x \le 0$ nm and $\psi(x) = ce^{-x/L}$ for $x \ge 0$ mm. The probability density will thus be $|\psi(x)|^2 = c^2 e^{2x/L}$ for $x \le 0$ mm and $|\psi(x)|^2 = c^2 e^{-2x/L}$ for $x \ge 0$ mm. With L = 2.0 mm, ψ and $|\psi|^2$ at various values of x are displayed in the table below.

x (mm)	0	0.5	1.0	1.5	2.0	3.0	4.0	5.0
$ce^{-x/L}$	c	$0.78c$	$0.61c$	$0.47c$	$0.37c$	$0.22c$	$0.14c$	$0.08c$
$c^2e^{-2x/L}$	c^2	$0.61c^2$	$0.37c^2$	$0.22c^2$	$0.14c^2$	$0.05c^2$	0.02^2	$0.01c^2$





(b) Normalization of the wave function requires that

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1 = 2\int_0^{\infty} |\psi(x)|^2 dx = 2\int_0^{\infty} c^2 e^{-2x/L} dx = 2c^2\left(-\frac{L}{2}\right)\left[e^{-2x/L}\right]_0^{\infty} \Rightarrow c = \frac{1}{\sqrt{L}} = \frac{1}{\sqrt{2.0\text{ mm}}} = 0.707\text{ mm}^{-1/2}$$

(c) The probability is

$$\text{Prob}(-1.0\text{ mm} \le x \le 1.0\text{ mm}) = \int_{-1.0\text{ mm}}^{1.0\text{ mm}} |\psi(x)|^2 dx$$

$$= 2\int_{0\text{ mm}}^{1.0\text{ mm}} c^2 e^{-2x/L} dx = 2c^2\left(-\frac{L}{2}\right)\left[e^{-2x/L}\right]_{0\text{ mm}}^{1.0\text{ mm}} = 2c^2\left(\frac{-2.0\text{ mm}}{2}\right)\left[e^{-1} - 1\right] = 0.632 = 63.2\%$$

(d) The region -1 mm $\le x \le 1$ mm is shaded on the probability density graph.

40.42. Model: A pulse is a wave packet, hence it must satisfy the relation $\Delta f\,\Delta t \approx 1$.

Solve: **(a)** The wavelength of 600 nm corresponds to a center frequency of

$$f_0 = \frac{c}{\lambda_0} = \frac{3\times10^8 \text{ m/s}}{600\times10^{-9} \text{ m}} = 5.0\times10^{14} \text{ Hz}$$

(b) The pulse duration is 6.0 fs, that is, $\Delta t = 6.0 \times 10^{-15}$ s. Because the time period of the center frequency is $T = f_0^{-1} = 2.0\times10^{-15}$ s, the number of cycles in the pulse is

$$\frac{\Delta t}{T} = \frac{6.0\times10^{-15} \text{ s}}{2.0\times10^{-15} \text{ s}} = 3$$

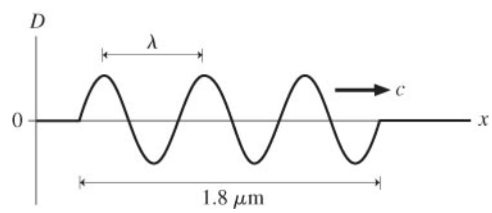
(c) The frequency bandwidth for a 6.0-fs-long pulse is

$$\Delta f = \frac{1}{\Delta t} = \frac{1}{6.0\times10^{-15} \text{ s}} = 1.67\times10^{14} \text{ Hz}$$

This bandwidth is centered on $f_0 = 5.00 \times 10^{14}$ Hz, so the necessary range of frequencies from $f_0 - \frac{1}{2}\Delta f$ to $f_0 + \frac{1}{2}\Delta f$ is from 4.17×10^{14} Hz to 5.83×10^{14} Hz.

(d) The pulse travels at speed c, so the length is $\Delta x = c\Delta t = (3.0 \times 10^8 \text{ m/s})(6.0 \times 10^{-15} \text{ s}) = 1.8 \times 10^{-6} \text{ m} = 1.8$ μm. This is 3λ, in agreement with the finding that there are 3 cycles in the pulse.

(e) The graph has three oscillations spanning 1.8 μm = 3λ.



40.44. Model: A dust speck is a particle and is thus subject to the Heisenberg uncertainty principle.

Solve: The uncertainty in our knowledge of the position of the dust speck is $\Delta x = 10\ \mu\text{m}$. The uncertainty in the dust speck's momentum is

$$\Delta p_x = \frac{h/2}{\Delta x} = \frac{6.63 \times 10^{-34}\ \text{J s}}{2\left(10 \times 10^{-6}\ \text{m}\right)} = 3.32 \times 10^{-29}\ \text{kg m/s}$$

Equivalently, the uncertainty in the dust particle velocity is

$$\Delta v_x = \frac{\Delta p_x}{m} = \frac{3.32 \times 10^{-29}\ \text{kg m/s}}{1.0 \times 10^{-16}\ \text{kg}} = 3.32 \times 10^{-13}\ \text{m/s}$$

The average velocity is 0 m/s, so the range of possible velocities is -1.66×10^{-13} m/s to $+1.66 \times 10^{-13}$ m/s. The particle could have a top speed of up to 1.66×10^{-13} m/s. The maximum kinetic energy the speck has is

$$K = \frac{1}{2}\ mv^2 = \frac{1}{2}\left(1.0 \times 10^{-16}\ \text{kg}\right)\left(1.66 \times 10^{-13}\ \text{m/s}\right)^2$$
$$= 1.4 \times 10^{-42}\ \text{J}$$

To get out of the hole, the particle would have to acquire potential energy

$$U = mgh = \left(1.0 \times 10^{-16}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\left(1.0 \times 10^{-6}\ \text{m}\right)$$
$$= 9.8 \times 10^{-22}\ \text{J}$$

The energy gain needed to get out of the hole is much larger than the available kinetic energy. The particle does not have anywhere near enough kinetic energy that it could, by any process, transform into potential energy and escape. Using $K = mgh$, the deepest hole from which the dust speck could have a good chance of escaping is

$$h = \frac{K}{mg} = \frac{1.4 \times 10^{-42}\ \text{J}}{\left(1.0 \times 10^{-16}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)} = 1.4 \times 10^{-27}\ \text{m}$$

Assess: This is not a very deep hole.

40.47. Solve: (a) For a photon, $E = hf$ which means $\Delta E = h\Delta f$. Assuming the photon is a wave packet, the relationship that is applicable to a wave packet $\Delta f\,\Delta t \approx 1$ becomes

$$\left(\frac{\Delta E}{h}\right)\Delta t \approx 1 \Rightarrow \Delta E\Delta t \approx h$$

(b) The energy of a photon cannot be exactly known. The uncertainty in our knowledge of a photon's energy depends on the length of time Δt that is available to measure it.

(c) The uncertainty in the energy is

$$\Delta E \cong \frac{h}{\Delta t} \cong \frac{6.63\times10^{-34}\ \text{J s}}{10\times10^{-9}\ \text{s}} = 6.63\times10^{-26}\ \text{J} = 4.14\times10^{-7}\ \text{eV}$$

(d) The energy of the photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J s})(3.0\times10^{8}\ \text{m/s})}{500\times10^{-9}\ \text{m}} = 3.978\times10^{-19}\ \text{J}\times\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 2.49\ \text{eV}$$

$$\Rightarrow \frac{\Delta E}{E} = \frac{4.14\times10^{-7}\ \text{eV}}{2.49\ \text{eV}} = 1.7\times10^{-7}$$