38.5. (a) The atomic number of an element, represented by Z, is the number of electrons in a neutral atom and the number of protons in the nucleus. The atomic number of hydrogen is Z = 1, helium is Z = 2, and lithium is Z = 3. However, the masses of these three elements do not progress in this linear manner. The mass of helium is four times the mass of hydrogen and the mass of lithium is seven times the mass of hydrogen. If a nucleus contains only Z protons, then helium should be just twice as massive as hydrogen and lithium three times as massive as hydrogen. So, there is something else in the nucleus.

(b) Thomson and his student Aston found that many elements consist of atoms of differing mass. For example, neon (Z = 10) atoms are found to have integral masses 20 u, 21 u, and 22 u. Potassium (Z = 19) atoms are found to have integral masses 39 u, 40 u, and 41 u. Because the masses of the different isotopes differ by ≈ 1 u, the mass of neutron must be ≈ 1 u, the same as the mass of a proton.

38.6. Larger than because the force exerted by a magnetic field on the charged particle to deflect it is proportional to the amount of charge on the particle. An alpha particle has more charge than a cathode-ray particle.

38.7. Scientists at the time could not imagine the extremely high density of the tiny nucleus. They also had no idea what would hold protons together in a nucleus when there was a known repulsive force between protons.

38.8. Alpha particles scattered through large angles because they had near collisions with very massive and highly charged particles. Only small deflections were expected for an alpha particle passing through a Thomson atom.

38.9. (a) ${}^{6}\text{Li}^{+}$ (b) ${}^{13}\text{C}^{-}$

38.2. Model: Assume the fields between the electrodes are uniform and that they are zero outside the electrodes.

Visualize: Please refer to Figures 38.7 and 38.8. Solve: (a) The speed with which a particle can pass without deflection is

$$v = \frac{E}{B} = \frac{\Delta V/d}{B} = \frac{600 \text{ V}/5.0 \times 10^{-3} \text{ m}}{2.0 \times 10^{-3} \text{ T}} = 6.0 \times 10^7 \text{ m/s}$$

(b) The radius of cyclotron motion in a magnetic field is

$$r = \left(\frac{m}{e}\right)\frac{v}{B} = \left(\frac{9.11 \times 10^{-31} \text{ kg}}{1.60 \times 10^{-19} \text{ C}}\right) \left(\frac{6.0 \times 10^7 \text{ m}}{2.0 \times 10^{-3} \text{ T}}\right) = 0.17 \text{ m} = 17 \text{ cm}$$

38.3. Model: Assume the fields between the electrodes are uniform and that they are zero outside the electrodes.

Visualize: Without the external magnetic field *B*, the electrons will be deflected *up* toward the positive electrode. The magnetic field must therefore be directed *out of the page* to exert a balancing downward force on the negative electron.

Solve: In a crossed-field experiment, the magnitudes of the electric and magnetic forces on the electron are given by Equation 38.4. The magnitude of the magnetic field is

$$B = \frac{E}{v} = \frac{\Delta V/d}{v} = \frac{200 \text{ V}/8.0 \times 10^{-3} \text{ m}}{5.0 \times 10^{6} \text{ m}} = 5.0 \times 10^{-3} \text{ T}$$

Thus $\vec{B} = (5.0 \times 10^{-3} \text{ T}, \text{ out of page}).$

38.4. Model: Assume the electric field $(E = \Delta V/d)$ between the plates is uniform. Visualize: Please refer to Figure 38.9. Solve: (a) The mass of the droplet is

$$m_{\rm drop} = \rho V = \rho \left(\frac{4\pi}{3}R^3\right) = \left(885 \text{ kg/m}^3\right) \frac{4\pi}{3} \left(0.4 \times 10^{-6} \text{ m}\right)^3 = 2.37 \times 10^{-16} \text{ kg} \approx 2.4 \times 10^{-16} \text{ kg}$$

(b) In order for the upward electric force to balance the gravitational force, the charge on the droplet must be

$$q_{\rm drop} = \frac{m_{\rm drop}g}{E} = \frac{\left(2.37 \times 10^{-16} \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)}{20 \text{ V}/11 \times 10^{-3} \text{ m}} = 1.28 \times 10^{-18} \text{ C} \approx 1.3 \times 10^{-18} \text{ C}$$

(c) Because the electric force is directed toward the electrode at the higher potential (or more positive plate), the charge on the droplet is negative. The number of surplus electrons is

$$N = \frac{q_{\text{droplet}}}{e} = \frac{1.28 \times 10^{-18} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 8$$

38.5. Model: Assume the electric field $(E = \Delta V/d)$ between the plates is uniform. Visualize: Please refer to Figure 38.9. To balance the weight, the electric force must be directed toward the upper electrode, which is more positive than the lower electrode. Solve: Since $m_{e_{i}} = \rho V = \rho (\frac{4\pi}{2}) R^3$, the equation $m_{e_{i}} g = g_{e_{i}} E$ is

Solve: Since
$$m_{drop} = \rho V = \rho \left(\frac{4\pi}{3}\right) R^3$$
, the equation $m_{drop}g = q_{drop}E$ is
 $R^3 = \frac{(\Delta V/d)(15e)}{\frac{4\pi}{3}\rho g} = \frac{(25 \text{ V}/0.012 \text{ m})(15)(1.60 \times 10^{-19} \text{ C})}{\frac{4\pi}{3}(860 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 1.4163 \times 10^{-19} \text{ m}^3 \Rightarrow R = 0.52 \ \mu\text{m}$

38.10. Model: The electron volt is a unit of energy. It is defined as the energy gained by an electron if it accelerates through a potential difference of 1 volt. Visualize:



Solve: (a) The figure for part (a) shows a Li^{++} ion accelerating from rest across a parallel-plate capacitor. The energy conservation equation $K_f + qV_f = K_i + qV_i$ is

$$K_{\rm f} = K_{\rm i} + q(V_{\rm i} - V_{\rm f}) = 0 + (2e)\Delta V = 2e(5000 \text{ V}) = 10,000 \text{ eV} = 10 \text{ keV}$$

(b) The potential energy is

$$U = \frac{1}{4\pi\varepsilon_0} \frac{(e)(e)}{10 \text{ fm}} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{10.0 \times 10^{-15} \text{ m}} = 2.30 \times 10^{-14} \text{ J} \times \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} = 144 \text{ keV}$$

(c) The energy-conservation equation for the figure for part (c) $K_{\rm f} + U_{\rm f} = K_{\rm i} + U_{\rm i}$ is

$$K_{\rm f} = 0 \,\,{\rm J} + (U_{\rm i} - U_{\rm f}) = mg(y_{\rm i} - y_{\rm f}) = (0.200 \,\,{\rm kg})(9.8 \,\,{\rm m/s^2})(1.0 \,\,{\rm m} - 0 \,\,{\rm m})$$
$$= 1.96 \,\,{\rm J} \times \frac{1 \,\,{\rm eV}}{1.60 \times 10^{-19} \,\,{\rm J}} = 1.2 \times 10^{19} \,\,{\rm eV}$$

38.11. Visualize: The work-kinetic energy theorem is $\Delta K = W_{net}$. Work done on the proton in slowing it down will be $W = -q\Delta V$.

$$K_{\rm f} - K_{\rm i} = -q\Delta V$$

Solve: But since $K_{\rm f} = 0$, then

$$K_{\rm i} = q\Delta V = (1\rm{e})(75\rm{V}) = 75\rm{eV}$$

Assess: The plate separation doesn't matter.

38.14. Model: For a neutral atom, the number of electrons is the same as the number of protons. **Solve:** (a) Since Z = 5, the ¹⁰B atom has 5 electrons, 5 protons, and 10 - 5 = 5 neutrons. (b) Since Z = 7, ¹³N⁺ has 6 electrons, 7 protons, and 13 - 7 = 6 neutrons. (c) Since Z = 8, the triply charged ¹⁷O⁺⁺⁺ ion has 5 electrons, 8 protons, and 17 - 8 = 9 neutrons. **38.23.** Model: Assume the blackbodies obey Wein's law in Equation 38.9: $\lambda_{\text{peak}} = (2.90 \times 10^6 \text{ nm} \cdot \text{K})/T$. Visualize: We want to solve for T in the equation above. We are given $\lambda_{\text{peak}} = 300 \text{ nm}$, 3000 nm. Solve:

$$T = \frac{2.90 \times 10^6 \,\mathrm{nm} \cdot \mathrm{K}}{\lambda_{\mathrm{peak}}}$$

(a)

$$T = \frac{2.90 \times 10^6 \,\mathrm{nm} \cdot \mathrm{K}}{300 \,\mathrm{nm}} = 9667 \,\mathrm{K} = 9394^{\circ}\mathrm{C}$$

(b)

$$T = \frac{2.90 \times 10^6 \,\mathrm{nm} \cdot \mathrm{K}}{3000 \,\mathrm{nm}} = 966.7 \,\mathrm{K} = 694^{\circ}\mathrm{C}$$

38.24. Model: Assume the metal sphere is a blackbody (so the emissivity e=1). Visualize: First use Wein's law (Equation 38.9) to find the temperature, then use Stefan's law (Equation 38.8) to determine the power radiated. We are given R = 1.0 cm and $\lambda_{peak} = 2000$ nm. Solve:

$$T = \frac{2.90 \times 10^6 \text{ nm} \cdot \text{K}}{2000 \text{ nm}} = 1450 \text{ K}$$
$$\frac{Q}{\Delta t} = e\sigma A T^4 = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[4\pi (1.0 \text{ cm})^2](1450 \text{ K})^4 = 315 \text{ W}$$

Assess: The sphere radiates more than 3100W light bulbs, but it has a larger surface area than the filaments, so the answer is reasonable.

38.25. Model: Assume the ceramic cube is a blackbody (so the emissivity e=1).

Visualize: First use Stefan's law (Equation 38.8), $\frac{Q}{\Delta t} = e\sigma AT^4$, to find the temperature, then use Wein's law (Equation 38.9) to get the peak wavelength. We are given $A = 6(3.0 \text{ cm} \times 3.0 \text{ cm}) = 0.0054 \text{ m}^2$ and $Q/\Delta t = 630 \text{ W}$.

Solve: Solve Stefan's law for *T*.

$$T = \sqrt[4]{\frac{Q/\Delta t}{e\sigma A}} = \sqrt[4]{\frac{630 \text{ W}}{(1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.0054 \text{ m}^2)}} = 1198 \text{ K}$$

Now plug this temperature into Wein's law.

$$\lambda_{\text{peak}} = \frac{2.90 \times 10^6 \text{ nm} \cdot \text{K}}{T} = \frac{2.90 \times 10^6 \text{ nm} \cdot \text{K}}{1198 \text{ K}} = 2420 \text{ nm} = 2.42 \ \mu\text{m}$$

38.27. Model: Use the relativistic expression for the total energy.Solve: (a) The energy of the proton is

$$E = \gamma_{\rm p} mc^2 = 500 \text{ GeV} = \frac{\left(1.67 \times 10^{-27} \text{ kg}\right) \left(3.0 \times 10^8 \text{ m/s}\right)^2}{\sqrt{1 - v^2/c^2}} = 500 \times 10^9 \text{ eV} \times \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}$$
$$\Rightarrow \sqrt{1 - v^2/c^2} = 1.879 \times 10^{-3} \Rightarrow v = 0.999998c$$

(b) Likewise for the electron,

$$E = 2.0 \text{ GeV} = \frac{(9.11 \times 10^{-31} \text{ kg})(3.0 \times 10^8 \text{ m/s})^2}{\sqrt{1 - v^2/c^2}}$$
$$\Rightarrow \sqrt{1 - v^2/c^2} = 2.562 \times 10^{-4} \Rightarrow v = 0.999999997c$$

38.32. Model: Assume the fields between the electrodes are uniform and that they are zero outside the electrodes.

Solve: In a crossed-field experiment, the deflection is zero when the magnetic and electric forces exactly balance each other. From Equation 38.4,

$$B = \frac{E}{v} = \frac{\Delta V/d}{v} = \frac{500 \text{ V}/1.0 \times 10^{-2} \text{ m}}{v} = \frac{5.0 \times 10^4 \text{ V/m}}{v}$$

We need to obtain v before we can find B. We know that (i) a potential difference of 500 V across the plates causes the proton to deflect *vertically down* by 0.5 cm and (ii) the proton travels a horizontal distance of 5.0 cm in the same time (t) as it travels vertically down by 0.5 cm. From kinematics,

$$x_{\rm f} - x_{\rm i} = v_{\rm xi} (t_{\rm f} - t_{\rm i}) + \frac{1}{2} a_{\rm x} (t_{\rm f} - t_{\rm i})^2 = v (t - 0 \text{ s}) \Longrightarrow v = \frac{x_{\rm f} - x_{\rm i}}{t} = \frac{5.0 \text{ cm}}{t}$$

The time *t* can be found from the vertical motion as follows:

$$y_{\rm f} - y_i = v_{\rm yi} (t_{\rm f} - t_{\rm i}) + \frac{1}{2} a_y (t_{\rm f} - t_{\rm i})^2 \Longrightarrow 0.50 \text{ cm} = \frac{1}{2} a_y t^2$$

The force on the electron is

$$F_{\rm E} = eE = \frac{e\Delta V}{d} = ma_y \implies a_y = \frac{e\Delta V}{md}$$
$$\implies 0.50 \times 10^{-2} \text{ m} = \frac{1}{2} \frac{e\Delta V}{md} t^2 = \frac{1}{2} \frac{\left(1.6 \times 10^{-19} \text{ C}\right)(500 \text{ V})t^2}{\left(1.67 \times 10^{-27} \text{ kg}\right)\left(1.0 \times 10^{-2} \text{ m}\right)} \implies t = 4.569 \times 10^{-8} \text{ s}$$

The speed of the electron is

$$v = \frac{5.0 \text{ cm}}{t} = \frac{5.0 \times 10^{-2} \text{ m}}{4.569 \times 10^{-8} \text{ s}} = 1.094 \times 10^{6} \text{ m/s} \Rightarrow B = \frac{5.0 \times 10^{4} \text{ V/m}}{1.094 \times 10^{6} \text{ m/s}} = 46 \text{ mT}$$

Using the right hand rule we determine that this magnetic field must be directed *into the page* in order for the magnetic force to point opposite of the electric force. Thus $\vec{B} = (0.046 \text{ T}, \text{ into page})$.

38.33. Model: Assume the fields are uniform. Visualize:



Solve: Between the plates, the fields are crossed so that the particle passes through the plates without deflection. From Equation 38.4, the speed of the particle is

$$v = \frac{E}{B} = \frac{187,500 \text{ V/m}}{0.125 \text{ T}} = 1.50 \times 10^6 \text{ m/s}$$

After leaving the plates, the particle experiences only a magnetic force and thus follows a circular path of radius r. Thus the charge-to-mass ratio is

$$r = \frac{mv}{qB} \Rightarrow \frac{q}{m} = \frac{v}{Br} = \frac{1.50 \times 10^6 \text{ m/s}}{(0.125 \text{ T})(\frac{1}{2} \times 0.2505 \text{ m})} = 9.58 \times 10^7 \text{ C/kg}$$

This must be a proton because the proton's charge-to-mass ratio is

$$\frac{e}{m} = \frac{1.6 \times 10^{-19} \text{ C}}{1.67 \times 10^{-27} \text{ kg}} = 9.58 \times 10^7 \text{ C/kg}$$

38.35. Model: Model the Li atom as a single valence electron orbiting a sphere with net charge q = +1e due to the 3 protons and 2 inner electrons. A sphere of charge acts like a point charge with the total charge concentrated at the center of the sphere.

Visualize:



Solve: The electron's energy is both kinetic and potential: $E = \frac{1}{2} mv^2 + kq(-e)/r$. To say that the energy needed to ionize the atom is 5.14 eV means that you would need to increase the electron's energy by 5.14 eV to remove it from the atom, taking it to $r \approx \infty$. Since E = 0 for charged particles that are infinitely separated, the energy of the must be $E = -5.14 \text{ eV} = -8.224 \times 10^{-19} \text{ J}$. Negative energy indicates that the system is bound, and the absolute value is the *binding energy*. Thus from energy considerations we learn that

$$E = \frac{1}{2}mv^2 - \frac{Ke^2}{r} = -8.224 \times 10^{-19} \text{ J}$$

The Coulomb force on the electron provides the centripetal acceleration of circular motion. The force equation is

$$F_r = \frac{Ke^2}{r^2} = ma_r = \frac{mv^2}{r} \Longrightarrow \frac{1}{2}mv^2 = \frac{Ke^2}{2r}$$

Substitute this expression for the kinetic energy into the energy equation:

$$\frac{1}{2}mv^2 - \frac{Ke^2}{r} = \frac{Ke^2}{2r} - \frac{Ke^2}{r} = -\frac{Ke^2}{2r} = -8.224 \times 10^{-19} \text{ J}$$
$$\Rightarrow r = \frac{(8.99 \times 10^9 \text{ N m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{2(8.224 \times 10^{-19} \text{ J})} = 1.40 \times 10^{-10} \text{ m} = 0.140 \text{ nm}$$

With the radius now known, we can use the result of the force equation to find that

$$v = \sqrt{\frac{Ke^2}{mr}} = 1.34 \times 10^6 \text{ m/s}$$

38.41. Model: Assume the nucleus is at rest. Use the conservation of energy equation. Visualize:



Solve: The energy conservation equation $K_{\rm f} + U_{\rm f} = K_{\rm i} + U_{\rm i}$ is

$$0 \text{ J} + \frac{1}{4\pi\varepsilon_0} \frac{(2e)(Ze)}{(6 \text{ fm})} = 6.24 \text{ MeV} + 0 \text{ J}$$
$$\Rightarrow \frac{(9.0 \times 10^9 \text{ N m}^2/\text{kg}^2)(2)(1.60 \times 10^{-19} \text{ C})^2 Z}{6.0 \times 10^{-15} \text{ m}} = 6.24 \times 10^6 \text{ eV} \times \frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}$$

Solving for Z gives Z = 13. The element is aluminum.

38.44. Model: Assume the ¹²C nucleus is at rest. Energy is conserved. Visualize:



Solve: (a) A proton with an initial velocity v_i and zero electric potential energy (due to its infinite separation from a

¹²C nucleus) moves toward the ¹²C nucleus. It must impact the ¹²C nucleus, which has a radius of 2.75 fm, with an energy of 3.0 MeV. The energy conservation equation $K_f + U_f = K_i + U_i$ is

$$3.0 \text{ MeV} + \frac{1}{4\pi\varepsilon_0} \frac{e(6e)}{(2.75 \times 10^{-15} \text{ m})} = \frac{1}{2} m_{\text{proton}} v_i^2 + 0 \text{ J}$$

$$\Rightarrow 3.0 \times 10^6 \text{ eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} + \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(6)(1.60 \times 10^{-19} \text{ C})^2}{2.75 \times 10^{-15} \text{ m}} = \frac{1}{2} (1.67 \times 10^{-27} \text{ kg}) v_i^2$$

$$\Rightarrow v_i = 3.43 \times 10^7 \text{ m/s}$$

(**b**) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2 = e\Delta V$. Hence, $\Delta V = \frac{1}{2} \frac{(1.67 \times 10^{-27} \text{ kg})(3.43 \times 10^7 \text{ m/s})^2}{1.6 \times 10^{-19} \text{ C}} = 6.14 \times 10^6 \text{ V}$ **39.2.** (a) When $\Delta V > 0$, all the emitted electrons are attracted to and collected by the anode. This means a further increase in the voltage cannot change the number of electrons arriving per second and thus cannot increase the current.

(b) The work function E_0 is the *minimum* energy an electron needs to escape from the metal. Some electrons, such as those a bit further from the surface, need more than E_0 to escape. There is a *range* of escape energies, so the escaped electrons have a range of kinetic energies and not a single kinetic energy.

(c) If the anode potential is V, an electron leaving the cathode with kinetic energy K arrives at the anode with kinetic energy K' = K + eV. A negative V causes a decrease in kinetic energy. K' cannot become negative, so for $eV \le -K$ the electron is repelled by the anode and turned back toward the cathode. The emitted electrons have a maximum kinetic energy K_{max} . When $eV = -K_{\text{max}}$, all electrons are turned back and the current drops to zero. If the current reaches zero at $V = -V_{\text{stop}}$, then $V_{\text{stop}} = K_{\text{max}}/e$. The stopping voltage measures the maximum kinetic energy by causing the most energetic electrons, those with $K = K_{\text{max}}$, to be turned back from the anode.



According to classical physics, the photoelectric current is *not* dependent on the light's frequency. If the light intensity remains constant (same energy per second falling on the metal), the photoelectric current should be constant with *no threshold* as the frequency is changed. Thus, the graph would be a horizontal line, starting from f = 0 Hz.

39.3.



The graphs would still be horizontal for $\Delta V > 0$ V because all the electrons are being collected. Also, more intense light would still give a larger current. But classical physics postulates a *thermal* cause for the photoelectric effect. If this were true, more intense light would heat the electrons more and thus eject the electrons with more kinetic energy. In this case, more intense light would have a *larger stopping voltage*. This would appear on the graph as a more negative *x*-intercept.

39.5. The photoelectric current depends on the potential difference ΔV between the two electrodes, the nature of the cathode metal, and the intensity of the light.



Solve: (a) According to classical physics, there is no dependence on the light's wavelength. If the light intensity remains constant (same amount of energy falling on the metal cathode), the photocurrent will be unchanged.

(b) The maximum kinetic energy of the electrons emitted from a cathode is $K_{\text{max}} = E_{\text{elec}} - E_0$. If E_0 is smaller for a different metal, the emitted electrons will have a higher kinetic energy and thus the stopping potential will be larger.

39.8. The electron. With $\lambda = \frac{h}{mv}$ and $v = \sqrt{\frac{2K}{m}}$ then $\lambda = \frac{h}{m\sqrt{\frac{2K}{m}}} = \frac{h}{\sqrt{2Km}}$. So the particle with the smaller

mass has the larger wavelength.

39.5. Model: The threshold frequency for the ejection of photoelectrons is $f_0 = E_0 / h$ where E_0 is the work function.

Solve: The visible region of light extends from 400 nm to 700 nm. For $\lambda_0 = 400$ nm, the work function is

$$E_0 = f_0 h = \frac{hc}{\lambda_0} = \frac{\left(6.63 \times 10^{-34} \text{ J s}\right)\left(3.0 \times 10^8 \text{ m/s}\right)}{400 \times 10^{-9} \text{ m}} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 3.11 \text{ eV}$$

For $\lambda_0 = 700$ nm,

$$E_0 = \frac{\left(6.63 \times 10^{-34} \text{ J s}\right)\left(3.0 \times 10^8 \text{ m/s}\right)}{700 \times 10^{-9} \text{ m}} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 1.78 \text{ eV}$$

The cathode that will work in the entire visible range must have a work function of 1.78 eV or less.

39.15. Solve: The de Broglie wavelength is $\lambda = h/mv$. Thus,

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg})(500 \times 10^{-9} \text{ m})} = 1456 \text{ m/s}$$

A potential difference of ΔV will raise the kinetic energy of a rest electron by $\frac{1}{2}mv^2$. Thus,

$$e\Delta V = \frac{1}{2}mv^2 \Longrightarrow \Delta V = \frac{mv^2}{2e} = \frac{(9.11 \times 10^{-31} \text{ kg})(1456 \text{ m/s})^2}{2(1.6 \times 10^{-19} \text{ C})} = 6.0 \times 10^{-6} \text{ V}$$

Assess: A mere 6.0×10^{-6} V is able to increase an electron's speed to 1456 m/s.

39.19. Model: For a "particle in a box," the energy is quantized. Solve: The energy of the n = 1 state is

$$E_{1} = (1)^{2} \frac{h^{2}}{8mL^{2}} = E_{\text{photon}} = \frac{hc}{\lambda} \Longrightarrow L = \sqrt{\frac{h\lambda}{8mc}} = \sqrt{\frac{(6.63 \times 10^{-34} \text{ J s})(600 \times 10^{-9} \text{ m})}{8(9.11 \times 10^{-31} \text{ kg})(3.0 \times 10^{8} \text{ m/s})}} = 0.427 \text{ nm}$$

39.23. Model: The electron must have $k \ge \Delta E_{atom}$ to cause collisional excitation. The atom is initially in the n = 1 ground state. Visualize:



Solve: The kinetic energy of the incoming electron is

$$E = \frac{1}{2}mv^{2} = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(1.30 \times 10^{6} \text{ m/s})^{2} = 7.698 \times 10^{-19} \text{ J} = 4.81 \text{ eV}$$

The electron has enough energy to excite the atom to the n = 2 stationary state $(E_2 - E_1 = 4.00 \text{ eV})$. However, it does not have enough energy to excite the atom into the n = 3 state which requires a total energy of 6.00 eV.

39.25. Solve: (a) From Equation 39.25, the radius in state *n* of a hydrogen atom is $r_n = n^2 a_{\rm B}$. A 100 nm diameter atom has $r_n = 50$ nm. Thus, the quantum state is

$$n = \sqrt{\frac{r_n}{a_{\rm B}}} = \sqrt{\frac{50 \text{ nm}}{0.0529 \text{ nm}}} = 30.74$$

Since *n* has to be an integer, we obtain n = 31.

(b) From Equation 39.27, the electron's speed is

$$v_{31} = \frac{v_1}{31} = \frac{2.19 \times 10^6 \text{ m/s}}{31} = 7.06 \times 10^4 \text{ m/s}$$

From Equation 39.30, the electron's energy is

$$E_{31} = -\frac{E_1}{(31)^2} = -\frac{13.60 \text{ eV}}{(31)^2} = -0.0142 \text{ eV}$$

39.29. Solve: The units of \hbar are the units of angular momentum, L = mvr. These units are

$$J \cdot s = kg \cdot \frac{m^2}{s^2} \cdot s = kg \cdot \frac{m}{s} \cdot m$$

39.34. Solve: The laser light delivers 2.50×10^{17} photons per second and 100×10^{-3} J of energy per second. Thus, the energy of each photon is

$$\frac{100 \times 10^{-3} \text{ J/s}}{2.50 \times 10^{17} \text{ s}^{-1}} = 4.00 \times 10^{-19} \text{ J}$$

From Equation 39.4, the wavelength of the photons is

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ J s})(3.00 \times 10^8 \text{ m/s})}{4.00 \times 10^{-19} \text{ J}} = 4.97 \times 10^{-7} \text{ m} = 497 \text{ nm}$$

Assess: The wavelength is in the visible region.

39.37. Solve: (a) The threshold frequency is $f_0 = E_0/h$. The threshold frequency for potassium and gold are given in the table in part (b).

Metal	$E_0 (eV)$	f_0 (Hz)	λ_0 (nm)
Potassium	2.30	5.56×10^{14}	540
Gold	5.10	1.23×10^{15}	244
	Metal Potassium Gold	Metal E_0 (eV)Potassium2.30Gold5.10	Metal E_0 (eV) f_0 (Hz) Potassium 2.30 5.56×10^{14} Gold 5.10 1.23×10^{15}

(b) The corresponding threshold wavelength is $\lambda_0 = c/f_0$. The results of the calculations are in the table below.

(c) When light of wavelength λ is incident on the metal, the maximum kinetic energy of the photoelectrons is

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = hf - E_0 = \frac{hc}{\lambda} - E_0 \implies v_{\max} = \sqrt{\frac{2}{m}}\left(\frac{hc}{\lambda} - E_0\right)$$

 E_0 must be converted to SI units of joules before this formula can be used. v_{max} for potassium and gold are given in the table in part (d).

(d) The stopping potential is

$$V_{\rm stop} = \frac{K_{\rm max}}{e} = \frac{1}{e} \left(\frac{hc}{\lambda} - E_0 \right)$$

where again E_0 has to be joules. The results of the calculations are in the table below.

Metal	E_0 (J)	$v_{\rm max} ({\rm m/s})$	$V_{\rm stop}\left({ m V} ight)$
Potassium	3.68×10^{-19}	10.8×10^{5}	3.35
Gold	8.16×10^{-19}	4.4×10^{5}	0.55

39.40. Solve: (a) The stopping potential is

$$V_{\text{stop}} = \frac{h}{e}f - \frac{h}{e}f_0$$

A graph of V_{stop} versus frequency f should be linear with x-intercept f_0 and slope h/e. Since the x-intercept is $f_0 = 1.0 \times 10^{15}$ Hz, the work function is

$$E_0 = hf_0 = (4.14 \times 10^{-15} \text{ eV s})(1.0 \times 10^{15} \text{ Hz}) = 4.14 \text{ eV}$$

(b) The slope of the graph is

$$\frac{\Delta V_{\text{stop}}}{\Delta f} = \frac{8.0 \text{ V} - 0 \text{ V}}{3.0 \times 10^{15} \text{ Hz} - 1.0 \times 10^{15} \text{ Hz}} = 4.0 \times 10^{-15} \text{ V s}$$

Because the slope of the V_{stop} versus f graph is h/e, an experimental value of Planck's constant is

$$h = e(4.0 \times 10^{-15} \text{ V s}) = (1.6 \times 10^{-19} \text{ C})(4.0 \times 10^{-15} \text{ V s}) = 6.4 \times 10^{-34} \text{ J s}$$

Assess: This value of the Planck's constant is about 3.5% lower than the accepted value.

39.44. Model: Electrons have both particle-like and wave-like properties. Visualize:



Please refer to Figure 39.13.

Solve: (a) The kinetic energy is $K = \frac{1}{2}mv^2 = 50 \text{ keV} = 8.0 \times 10^{-15} \text{ J}$. Using this formula, the electron's speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(8.0 \times 10^{-15} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 1.32 \times 10^8 \text{ m/s} \approx 1.3 \times 10^8 \text{ m/s}$$

(b) From Equation 22.7, the fringe spacing in a double-slit interference experiment is $\Delta y = \lambda L/d$, where d is the slit separation and L is the distance to the viewing screen. The wavelength of the electrons is their de Broglie wavelength. We have

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg})(1.32 \times 10^8 \text{ m/s})} = 5.5 \times 10^{-12} \text{ m}$$
$$\Rightarrow \Delta y = \frac{\lambda L}{d} = \frac{(5.5 \times 10^{-12} \text{ m})(1.0 \text{ m})}{1.0 \times 10^{-6} \text{ m}} = 5.5 \times 10^{-6} \text{ m} = 5.5 \ \mu\text{m}$$

39.46. Model: Electrons have both particle-like and wave-like particles. Visualize:



Solve: The kinetic energy of the electrons is

$$K_{\rm f} = \frac{1}{2}mv^2 = K_{\rm i} + e\Delta V = 0 \,\mathrm{J} + e(250 \,\mathrm{V}) = (1.60 \times 10^{-19} \,\mathrm{C})(250 \,\mathrm{V}) = 4.00 \times 10^{-17} \,\mathrm{J}$$
$$\Rightarrow v = \sqrt{\frac{2(4.00 \times 10^{-17} \,\mathrm{J})}{9.11 \times 10^{-31} \,\mathrm{kg}}} = 9.37 \times 10^6 \,\mathrm{m/s}$$

The de Broglie wavelength at this speed is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J s}}{(9.11 \times 10^{-31} \text{ kg})(9.37 \times 10^6 \text{ m/s})} = 7.77 \times 10^{-11} \text{ m}$$

Circular-aperture diffraction produces the first minimum, defining the edge of the central maximum, at $\theta = 1.22\lambda/D$ for small angles, as is the case here. The diameter of the opening is

$$D = \frac{1.22 \ \lambda}{\theta} = \frac{1.22 \left(7.77 \times 10^{-11} \text{ m}\right)}{(0.5^{\circ}/180^{\circ})\pi \text{ rad}} = 1.09 \times 10^{-8} \text{ m}$$

Assess: The diameter of the hole that diffracts the electron beam is small indeed, and understandably so.

39.49. Model: The energy of the emitted gamma-ray photon is exactly equal to the energy between levels 1 and 2.

Solve: From Equation 39.14, the energy levels of the proton are

$$E_n = n^2 \frac{h^2}{8 m L^2}$$
 $n = 1, 2, 3, ...$

The energy of the emitted photon is

$$E_2 - E_1 = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2}$$
$$\Rightarrow L = \sqrt{\frac{3h^2}{8m(E_2 - E_1)}} = \sqrt{\frac{3(6.63 \times 10^{-34} \text{ J s})^2}{8(1.67 \times 10^{-27} \text{ kg})(2.0 \times 10^6 \text{ eV})}} \times \frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}} = 1.8 \times 10^{-14} \text{ m} = 18 \text{ fm}$$

Assess: This is roughly the size of a typical nucleus.