

Final Exam Review

Monday, December 07, 2009
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Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Gauss's law

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's law for magnetism

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$$

Faraday's law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt}$$

Ampère-Maxwell law

Lorentz Force Law:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

Chapter 33:

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$d\vec{F} = I d\vec{s} \times \vec{B}, \quad \vec{F} = I \int d\vec{s} \times \vec{B}$$

Force on current segment Force on wire

$$\vec{F} = I \vec{L} \times \vec{B}, \text{ Force on straight wire in uniform } B\text{-field}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}, \quad \vec{\mu} = I \vec{A}, \quad U = -\vec{\mu} \cdot \vec{B}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

- ① B-field from finite straight wire
- ② B-field from ∞ -ite " "
- ③ B-field from circular arc (@ $r=0$)
- ④ B-field from loop on-axis ($r=0$)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

- ① ∞ -ite straight wire
- ② ∞ -ite straight wire, finite thickness
- ③ Solenoid
- ④ toroid

Applications: • Force between two straight current carrying wires
• Hall effect

Chapter 34:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

- Motional $\mathcal{E}$ mf:
- Bar being dragged through uniform B-field @ Constant Velocity:
- $q\vec{E} = qv\vec{B}$, $\mathcal{E} = E\ell \Rightarrow \mathcal{E} = vBl$

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d\Phi_B}{dt}$$

- $\Phi_B = N \int \vec{B} \cdot d\vec{a}$, $\oint \vec{E} \cdot d\vec{s} \equiv \mathcal{E}$
 $\uparrow$ # of turns
- Lenz's Law
- $\frac{dB}{dt}$, $\frac{dA}{dt}$, angle between $\vec{B}$ & $\vec{A}$ changes with time

- E-field exists without physical "ring"

- Back $\mathcal{E}$ mf for inductor, transformers

Applications:

Inductors & self inductance

$$\mathcal{E}_{\text{back}} = -N \frac{d\Phi_B}{dt}, \quad \Phi_B = BA, \quad B \propto I \text{ for solenoid}$$

$$\Rightarrow \mathcal{E}_{\text{back}} \propto \frac{dI}{dt} \Rightarrow \mathcal{E}_{\text{back}} = -L \frac{dI}{dt}$$

$\uparrow$
defines self inductance

Calculating L for air-core solenoid:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}} \Rightarrow B \cdot \ell = \mu_0 \left(\frac{N}{\ell} I_0 \ell \right) \Rightarrow B = \mu_0 \eta I_0$$

$\uparrow$ N/ℓ

$$\mathcal{E}_{\text{back}} = -N \frac{d\Phi_B}{dt} = -N \underbrace{\mu_0 \eta A_{\perp}}_{\equiv L} \frac{dI}{dt}$$

$\equiv L = \mu_0 \eta^2 V$

Energy stored in inductor (B-field):

$$\frac{dW}{dt} = \frac{d}{dt} \mathcal{E} = I \mathcal{E} \Rightarrow dW = L I dI$$

$$\Rightarrow W = L \int_0^I I dI = \frac{1}{2} L I^2, \quad B = \mu_0 \eta I \Rightarrow I = \frac{B}{\mu_0 \eta}$$

$$\Rightarrow \mathcal{U}_B = \frac{W}{V} = \frac{1}{2} \mu_0 \eta^2 I^2 = \frac{1}{2\mu_0} B^2$$

Energy stored in capacitor (E-field):

$$V = E d, \int E \cdot d\vec{r} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow E a = \frac{1}{\epsilon_0} \frac{Q}{A} a \Rightarrow E = \frac{1}{\epsilon_0} \frac{Q}{A}$$

$$C = \frac{Q}{V} = \frac{(\epsilon_0 A) E}{E d} = \frac{\epsilon_0 A}{d}$$

$$dw = dq V = dq \frac{Q}{C} \Rightarrow W = \frac{d}{\epsilon_0 A} \int_0^Q q dq = \frac{d}{\epsilon_0 A} \frac{1}{2} q^2$$

or

$$dw = dq V = dq E d = \frac{d}{\epsilon_0 A} dq q \Rightarrow W = \frac{d}{\epsilon_0 A} \frac{1}{2} q^2$$

$\hookrightarrow (\epsilon_0 A E)^2$

$$\Rightarrow W = \frac{1}{2} \epsilon_0 \underbrace{A d}_V E^2 \Rightarrow u_E = \frac{1}{2} \epsilon_0 E^2$$

More Applications:

RC Circuits: $-IR - L \frac{dI}{dt} = 0$ Discharge

$$\Rightarrow \frac{dI}{dt} = -\frac{R}{L} I \Rightarrow I = I_0 e^{-t/\tau} ; I_0 = \mathcal{E}/R$$

$$\Rightarrow -\frac{1}{\tau} = -\frac{R}{L} \Rightarrow \tau = \underline{\underline{L/R}}$$

$$\mathcal{E} - IR - L \frac{dI}{dt} = 0 \quad \text{charging}$$

$$\Rightarrow \frac{\mathcal{E}}{R} - I - L \frac{dI}{dt} = 0 ; X = \frac{\mathcal{E}}{R} - I \Rightarrow \frac{dX}{dt} = -\frac{dI}{dt}$$

$$\Rightarrow \frac{dX}{dt} = -\frac{1}{\tau} X \Rightarrow X = X_0 e^{-t/\tau} ; \text{at } t=0 \quad I=0 \Rightarrow X_0 = \mathcal{E}/R$$

$$\Rightarrow \frac{\mathcal{E}}{R} - I = \frac{\mathcal{E}}{R} e^{-t/\tau} \Rightarrow I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$$

LC Circuits: Charge circuit: $C = \frac{Q}{\mathcal{E}} \Rightarrow Q_0 = \mathcal{E} C$

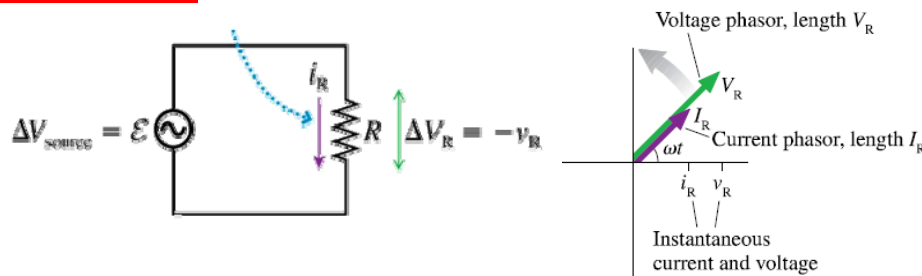
$$-\frac{Q}{C} - L \frac{dI}{dt} = 0 \Rightarrow \frac{d^2 Q}{dt^2} = -\frac{1}{LC} Q$$

$$\Rightarrow Q = A \cos \omega t, \quad Q_0 = \mathcal{E} C = A ; \quad \frac{d^2 Q}{dt^2} = -\omega^2 Q$$

$$\Rightarrow \omega^2 = \frac{1}{LC}, \quad \omega = \sqrt{1/LC}, \quad Q = \mathcal{E} C \cos \omega t$$

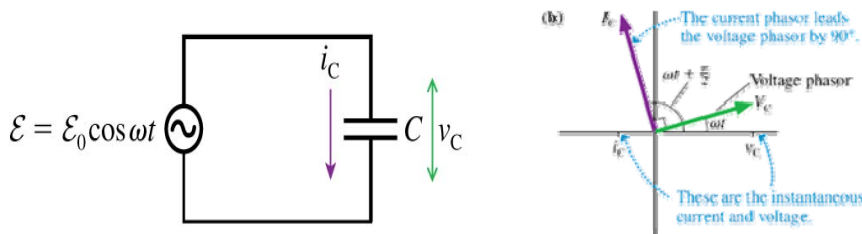
Chapter 36: More circuits

Last Time



$$V_R = i_R R$$

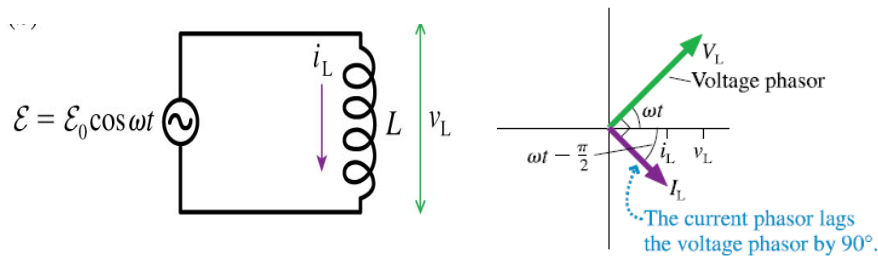
$$V_R = R I_R$$



$$V_C \neq X_C i_C$$

$$V_C = X_C I_C$$

$$X_C = \frac{1}{\omega C}$$

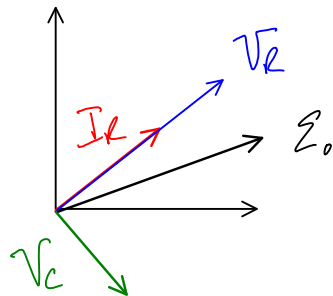


$$V_L \neq X_L i_L$$

$$V_L = X_L I_L$$

$$X_L = \omega L$$

RC filters:



$$\varepsilon_0^2 = V_R^2 + V_C^2$$

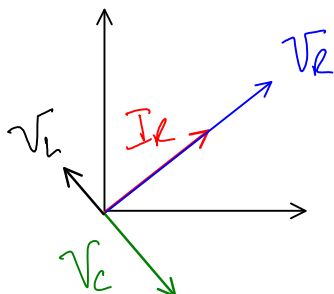
$$= I^2 (R^2 + X_C^2) \Rightarrow I = \frac{\varepsilon_0}{Z} = \frac{\varepsilon_0}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$$

$$V_C = I X_C = \frac{\varepsilon_0 (\frac{1}{\omega C})}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}, \quad V_R = I R = \frac{\varepsilon_0 R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$$

$$\text{As } \omega \rightarrow 0 \rightarrow V_C = \varepsilon_0, \quad V_R \rightarrow 0$$

$$\omega \rightarrow \infty \Rightarrow V_C \rightarrow 0, \quad V_R = \varepsilon_0$$

RLC Filter:



$$\varepsilon_0^2 = I^2 (V_R^2 + (V_L - V_C)^2)$$

$$= I^2 (R^2 + (\omega L - \frac{1}{\omega C})^2)$$

$$Z^2$$

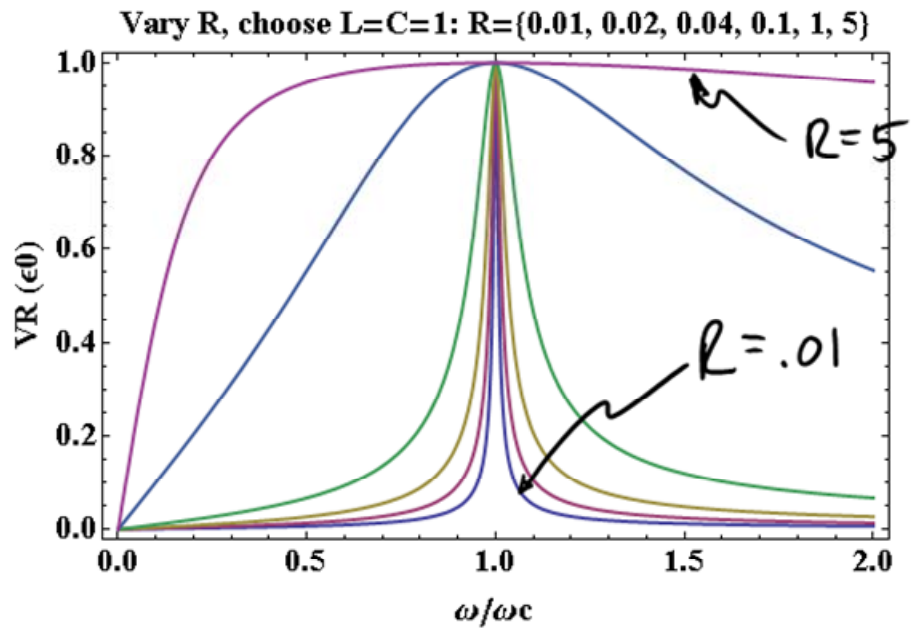
$$\Rightarrow I = \frac{\varepsilon_0}{Z}; \quad V_R = I R = \frac{\varepsilon_0 R}{Z}$$

$$V_C = I X_C = \frac{\varepsilon_0 (\frac{1}{\omega C})}{Z}, \quad V_L = I X_L = \frac{\varepsilon_0 (\omega L)}{Z}$$

$$V_R = \frac{Z_0 R}{\sqrt{R^2 - (X_L - X_C)^2}} \quad \text{Max when } X_L = X_C \Rightarrow \frac{1}{\omega L} = \omega C$$

$$\text{or } \omega_0 = \sqrt{\frac{1}{LC}}$$

$$V_R \text{ Max} \Rightarrow I_R \text{ Max} \Rightarrow P_R = I_R V_R \text{ Max @ } \omega_0$$



Chapter 35:

$$\vec{F} = q \vec{E} + q \vec{v} \times \vec{B} \quad \text{yields:}$$

Galilean Transformation of $\vec{E}$ & $\vec{B}$ -fields ($v \ll c$)

$$\vec{E}' = \vec{E} - \vec{v} \times \vec{B}$$

$$\vec{B}' = \vec{B} - \frac{\vec{v}}{c^2} \times \vec{E}$$

- Biot-Savart Law
derivable from $\vec{E}$ -field
of pt. charge

$$\oint \vec{B} \cdot d\vec{a} = 0$$

Magnetic Gauss's Law,
No magnetic monopoles

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I_{enc} + I_d), \quad \text{Displacement Current.}$$

For capacitor:

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \vec{E} = \frac{Q}{A \epsilon_0}$$

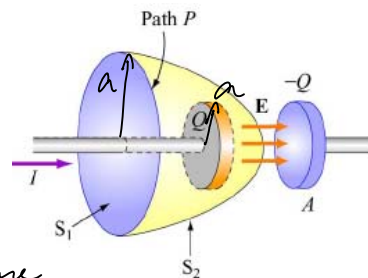
$$\oint \vec{B} \cdot d\vec{s} = B \cdot 2\pi a = \mu_0 \underbrace{I_{enc}}_{= I, \text{ blue surface}}$$

Yellow surface, require: $B \cdot 2\pi a = \mu_0 I_d$

$$\frac{d\Phi_E}{dt} = A \frac{dE}{dt} = \frac{1}{\epsilon_0} \frac{dQ}{dt} = \frac{1}{\epsilon_0} I_d \Rightarrow I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

- Direction of $\vec{B}$ found from "opposite of Lenz's Law"



$\vec{E}$ & $\vec{B}$ Plane waves, free space

- Meaning of plane wave:

$$\vec{E} = E_0 \cos(kx - \omega t) \hat{y}$$

$$\Rightarrow \vec{B} = B_0 \cos(kx - \omega t) \hat{z}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}, \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$B_0 = \frac{E_0}{c}, \quad v = c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

- Flow of energy: Poynting vector is Intensity = Power/Area

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}, \text{ instantaneous, } \vec{S}_{\text{avg}} = \frac{1}{2} \left(\frac{1}{\mu_0} \right) \vec{E}_0 \times \vec{B}_0$$

$$\underline{E^2 - (pc)^2 = E_0^2}$$

- $E = pc$ for $m_0 = 0 \Rightarrow p = E/c$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\Delta P / \Delta t}{\text{Area}} = \frac{1}{c} \underbrace{\frac{\Delta E}{\Delta t \cdot \text{Area}}}_{\text{Intensity}} = \frac{S}{c} \quad \text{absorber}$$

$$= \frac{2S}{c} \quad \text{reflector}$$

- Polarization, $E = E_0 \cos$ through Polarizer
 $I = I_0 \cos^2 \theta$ Malus's Law

Chapter 22: Physical Optics

$$E_1 = E_0 \cos(kx_1 - \omega t + \phi_{10}) \quad , \quad E_2 = E_0 \cos(kx_2 - \omega t + \phi_{20})$$

$$E_1 + E_2 = 2 E_0 \cos\left(\frac{\Delta \Phi}{2}\right) \cos(kx_{avg} - \omega t_{avg})$$

$$\Delta \Phi = (kx_2 - \omega t + \phi_{20}) - (kx_1 - \omega t + \phi_{10})$$

$$\Delta \Phi = k\delta + \Delta\phi \quad , \quad \delta = x_2 - x_1 \quad , \quad \Delta\phi = \phi_{20} - \phi_{10}$$

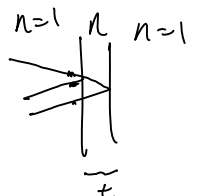
Assume $\Delta\phi = 0$;

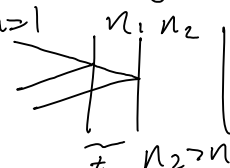
$$\text{Constructive} \Rightarrow \cos\left(\frac{k\delta}{2}\right) = \pm 1 \Rightarrow \frac{k\delta}{2} = m\pi \Rightarrow \delta = m\lambda$$

$$\text{Destructive} \Rightarrow \cos\left(\frac{k\delta}{2}\right) = 0 \Rightarrow \frac{k\delta}{2} = \left(m + \frac{1}{2}\right)\pi \Rightarrow \delta = \left(m + \frac{1}{2}\right)\frac{\lambda}{2}$$

$$I_{avg} = I_{0,avg} \cos^2\left(\frac{\Delta \Phi}{2}\right)$$

• Double slit:  $\delta = d \sin \theta$, $I = I_0 \cos^2\left(\frac{d \sin \theta}{\lambda} \pi\right)$
or $\sin \theta \approx \tan \theta \approx \frac{y}{L}$

• Thin film in air:  $\Phi_1 = \pi$, $\Phi_2 = k' \cdot 2t$, $k' = \frac{2\pi}{\lambda/n} = n k_0$
 $\frac{\Delta \Phi}{2} = \frac{k_0 2nt - \pi}{2}$

• AR Coating:  $\Phi_1 = \pi$, $\Phi_2 = k_1 \cdot 2t + \pi$, $k_1 = \frac{2\pi}{\lambda} n_2$
 $\Rightarrow \frac{\Delta \Phi}{2} = \frac{k_0 2n_2 t}{2} = \left(m + \frac{1}{2}\right)\pi$
 $\Rightarrow t = \frac{\lambda_0}{n_2} 2\pi \left(m + \frac{1}{2}\right)$

• Michelson Interferometer:  $\Phi_1 = k 2L_1$, $\Phi_2 = k 2L_2$
 $k = \frac{2\pi}{\lambda}$

• Single Slit: $I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$, $\beta = \left(\frac{y}{L} \right) \pi \frac{a}{\lambda}$

Short Cut destructive interfere: $\frac{a}{2} (\sin \theta) = \delta = \frac{\lambda}{2}$

$\rightarrow \frac{a}{2m} \sin \theta = \frac{\lambda}{2}$

• Resolution: Max of 1 = Min of original peak

Angle to first min: $\frac{a}{2} \sin \Delta \theta = \frac{\lambda}{2} \Rightarrow \sin \Delta \theta \approx \frac{\lambda}{a}$ Slit

$\sin \Delta \theta = 1.22 \frac{\lambda}{a}$ hole

Chapter 23:

$\theta_i = \theta_r$, Law of Reflection

$v = \frac{c}{n}$, Velocity of light in medium of index n

$n_1 \sin \theta_1 = n_2 \sin \theta_2$, Snell's Law,

Critical angle & total internal reflection
($\theta_2 \rightarrow 90^\circ$ for $n_2 < n_1$)

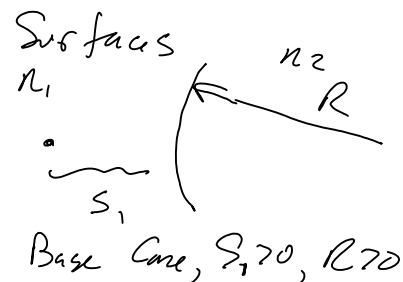
Dispersion: $n(\omega) \rightarrow$ Prisms

Applications:

Image formation from

- Flat mirror
- n_1, n_2 Flat interface
- Image @ Refractory Spherical

$$\frac{n_1}{s_1} + \frac{n_2}{s_2} = \left(\frac{n_2 - n_1}{R} \right)$$



Note: Flat surface from $R \rightarrow \infty$

- Thin lens: 2 Refracting Surfaces

$$\frac{n_1}{s_1} + \frac{n_2}{s_2} = (n_2 - n_1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{s_1} + \frac{1}{s_2} = \left(\frac{n_2 - n_1}{n_1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$f = s_2 \text{ when } s_1 \rightarrow \infty \Rightarrow \frac{1}{f} = \left(\frac{n_2 - n_1}{n_1} \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\Rightarrow \frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}$$

- Spherical mirrors:

$$\frac{1}{s_1} + \frac{1}{s_2} = \frac{1}{f}, \quad \frac{1}{f} = \frac{2}{R}$$

↑

Base case, $R > 0$,
 $s_1 > 0$, $s_2 > 0$...

Ray Tracing for All Cases

Chapter 24: Lens systems

- Camera - zoom lenses, $f/\# \equiv \frac{f}{D}$, $I \propto \frac{D^2}{f^2}$,

Exposure = $I \Delta t$, Depth of Field

- Farsighted / Hyperopia, Nearsighted / myopia

How to find corrective lenses: find lens that moves object placed @ ∞ or N.P. to an image where the person can see it.

$$\text{Power of lens} = \frac{1}{f}, \text{ Diopter } \left(\frac{1}{m} \right)$$

- Angular Magnification: $M = \frac{\theta_2}{\theta_1}$
- Eye piece, telescope, microscope
- $NA = n \sin \alpha_o = \frac{D/2}{f}$

- Diffraction from lens

$$\frac{a}{2} \sin \theta = \frac{\lambda}{2}, \sin \theta = \frac{w/2}{f} \Rightarrow w = 2f \sin \theta = 2f \frac{\lambda}{a}$$

for "slit" like lens

$$\Rightarrow w = 1.22 \left(\frac{2f\lambda}{a} \right) \text{ for circular lens}$$

↳ Smallest Spot Size

Chapter 25

- $p = \frac{h}{\lambda}$, de Broglie

- X-ray Diffraction

Final exam iv relativity

Monday, December 07, 2009
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Chapter 37

- Galilean transformation

$$\begin{aligned}x' &= (x - vt) \\t' &= t\end{aligned}$$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v \Rightarrow u' = u - v$$

$$\frac{\partial^2 x'}{\partial t'^2} = \frac{\partial^2 x}{\partial t^2} \Rightarrow F' = F$$

c is the same in all reference frames
Laws of physics the same in all reference frames

- Peggy & Ryan example

- Simultaneity is relative

- Derived Length Contraction: $L = L_0/\gamma$, L_0 proper length

- Light Clock $\rightarrow$ analysis of time dilation

$$\Delta t = \Delta \tau \gamma \quad \Delta \tau \text{ proper time}$$

- Twin paradox: Earth measures proper time

- Invariant s in light clock

$$\rightarrow s^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

- Lorentz transformation (implying 1st event is
Coordinate system origin lining
up @ $t = t' = 0$)

$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$dx' = \gamma(dx - vdt), \quad dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)$$

$$\Rightarrow \frac{dx'}{dt'} = \frac{dx - v dt}{dt - (v/c^2) dx} = \frac{dx/dt - v}{1 - \frac{v}{c^2} \frac{dx}{dt}}$$

$$\Rightarrow u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

• Conservation of momentum requires:

$$p \equiv \gamma_p m_0 u, \quad \gamma_p = \frac{1}{\sqrt{1 - (u/c)^2}}$$

• Using invariant S^2

$$\Rightarrow \gamma_p m_0 c^2 - (pc)^2 = \text{invariant}$$

Choose frame where $p' = 0, u' = 0 (\Rightarrow \gamma_p \rightarrow 1)$

$$\Rightarrow \underbrace{\gamma_p m_0 c^2}_{\equiv E} - (pc)^2 = \underbrace{m_0 c^2}_{\equiv E_0}$$

$$\Rightarrow \boxed{E - (pc)^2 = E_0}$$

$$\star E = E_0 + KE \Rightarrow KE = E - E_0 \Rightarrow KE = (\gamma_p - 1) m_0 c^2$$

Chapters 38 + 39

Quantization : photons ($E = \hbar\omega$, $p = \hbar k$), Charge
 $p = E/c = \hbar\omega/c$

- J. J. Thompson, crossed field experiment $\rightarrow \frac{e}{m}$ electrons
- Milikan oil drops : quantized charge
- mass spectrometer
- Photo electric effect : $h\nu = eV_{\text{stop}} + \Phi \rightarrow$ Concept of photon validation
 - KE of $e^- \propto \nu$, not I
- Planck's Black Body radiation $\rightarrow$ Energy of walls quantized

Defn : 1eV ; $\begin{matrix} 10^9 & 1 \\ \text{V} & \text{V} \\ 1\text{e} & 1\text{V} \end{matrix} = 1\text{eV} = 1.6 \cdot 10^{-19} \text{J}$

- Particle in a Box: $\sin k_n L = 0 \Rightarrow k_n = \frac{n\pi}{L}$
 $\& p_n = \hbar k_n$, $\& E_n = \frac{p_n^2}{2m} = \frac{(\hbar k_n)^2}{2m}$

- Bohr Model of atom:

$$\textcircled{1} F = m \frac{v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad \textcircled{2} p = mv = \hbar k$$

$$\textcircled{3} \text{ Stationary state: } 2\pi r = n\lambda$$

$$\Rightarrow r_n = a_B n^2, \quad V_n = \frac{1}{n} V_0, \quad E_n = -\frac{R_y}{n^2}, \quad L_n = n\hbar$$

$$\lambda = \frac{\lambda_0}{\left(\frac{1}{m}\right)^2 + \left(\frac{1}{m}\right)^2}$$

Chapter 40:

- Prob (in dx at x) = $\underbrace{P(x)}_{\text{Probability density}} dx$, $P(x) = |\psi|^2$
- Normalization: $\int |\psi(x)|^2 dx = 1$
- Wave packet: $\Delta f \Delta t \approx 1 \Rightarrow \Delta \omega \Delta t = 2\pi$
- Heisenberg Uncertainty: $\Delta E \Delta x \approx 2\pi \Rightarrow \Delta x \Delta p \approx h$
$$\Delta x \Delta p_x \geq \frac{h}{2}$$
- Single slit diffraction $\Rightarrow \Delta x \Delta p_x \approx h$

Chapter 41

- Schrodinger's Equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U \psi$$

If U independent of time:

$$\Rightarrow E \psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U \psi$$

$$\Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} = (E - U(x)) \psi$$

- $\psi + \psi'$ cts
- ψ normalizable

- If $U = \text{Constant}$, $\psi = A e^{bx} \Rightarrow$

$$-\frac{\hbar^2}{2m} b^2 = E - U \Rightarrow b = \sqrt{-\frac{2m}{\hbar^2} (E - U)}$$

$$\Rightarrow b = \begin{cases} \pm i k & E > U \\ \pm K & E < U \end{cases} \quad k = K = \sqrt{\frac{2m}{\hbar^2} |E - U|}$$

- infinite well: $\psi = Ae^{ikx} + Be^{-ikx}$
 $= A' \cos kx + B' \sin kx$

$\Rightarrow \sin kx = 0$ at $x=0$ & $x=L$

$\Rightarrow kL = n\pi$ or $k_n = \frac{n\pi}{L}$

$p_n = \hbar k_n$ & $E_n = \frac{p_n^2}{2m} = \frac{(\hbar k_n)^2}{2m}$

Normalization: $\int \psi^2 dx = \int_0^L (B' \sin kx)^2 dx = 1 \Rightarrow$
 $B' = \sqrt{\frac{2}{L}}$

- Correspondence Principle
- Finite potential well, Harmonic Oscillator
- Penetration depth: $\kappa = \frac{1}{\lambda}$
- Sketching $\psi(x)$ for different $U(x)$
 - $KE = E - U = \frac{p^2}{2m} \sim \frac{1}{\lambda^2} \therefore KE \uparrow \Rightarrow \lambda \downarrow, \kappa \uparrow$
 - If $KE \uparrow$, Amplitude $\downarrow$
- Capacitor, Harmonic Oscillator
- H_2^+ : Bonding & Antibonding bands
- Quantum Tunneling: $\psi(x) = e^{-\kappa x}$ inside barrier

Chapter 42:

• $E_n = -\frac{R_y}{n^2}$, $L = \sqrt{l(l+1)} \hbar$ $l=0, \dots, n-1$
 $L_z = m\hbar$, $m=-l, \dots, l$
 $\dots$

- $P(r) = (4\pi r^2 dr) R_{ne}^2(r)$
- Stern Gerlach + electron Spin: $S_z = \pm \frac{1}{2} \hbar$
- Pauli Exclusion Principle
- Multi Electron Atom
- Optical Selection Rules: $\Delta l = \pm 1$
- Lifetime: $\frac{N}{N_0} = e^{-t/\tau}$
- Stimulated Emission

- Copenhagen vs Realist: What is a measurement
- Stern Gerlach filters
- EPR Experiment
- Bell's Inequality: What does it prove in context of EPR?