Exam III and IIIb GJ Solutions

Thursday, November 19, 2009
10:15 AM

NAME / Section #:	Exam III Problem #1 Phys270
______________________________ / ________	

1. [25 pts] In the earth's reference frame, a tree is at the origin and a pole is at x=50km. Lightning strikes both the tree and the pole at t=5 μsec. The same lightning strikes are observed by a rocket traveling in the +x-direction at 0.9c. Note that the origins of the two reference frames (earth and rocket) coincide at time t=0 in both reference frames.

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = \frac{1}{\sqrt{1-.81}} = 2.29$$

$$\frac{v}{c^2} = \frac{.9}{(3\cdot10^8)} = .3\cdot10^{-8} = 3\cdot10^{-9}\ \frac{sec}{m}$$

(a) [15 pts] What are the space-time coordinates for these two events in the rocket's reference frame?

$$x' = \gamma(x - vt) = \begin{cases} 2.29\,(0 - .9\cdot10^8\cdot3\cdot5\cdot10^{-6}) = -3.1\ km & \text{Tree} \\ 2.29\,(5\cdot10^4 - 3\cdot5\cdot(.9)\cdot10^8\cdot10^{-6}) = +112\ km & \text{Pole} \end{cases}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \begin{cases} 2.29\,(5\cdot10^{-6} - 0) \approx +11.5\ \mu sec & \text{Tree} \\ 2.29\,(5\cdot10^{-6} - 3\cdot10^{-9}\cdot5\cdot10^4) \approx -333\ \mu sec & \text{Pole} \end{cases}$$

(b) [5 pts] In the rocket's frame, which event occurs first and how much time elapses between the two events?

Pole is struck first, then the tree. $\Delta t = 333 + 11.5 \approx 345\ \mu sec$

(c) [5 pts] What is the distance between the two events in the rocket's reference frame?

$112 + 3\ km = 115\ km$

NAME / Section #:	Exam III Problem #2 Phys270
______________________________ / ________	

2. [25 pts] A planet is 500 light years away from earth as measured in the earth's reference frame. An astronaut makes the one-way trip in 20 years as measured in the rocket's frame. The rocket has a rest mass of 10,000 kg. Assume the rocket quickly accelerates and decelerates so that the time required to do so may be ignored.

Rocket is measuring proper time, $\Delta\tau$

$\Delta\tau = 20$ years

$\Delta t = \frac{\Delta x}{v}$, $\Delta x = 500\cdot c\cdot 1yr$] Earth frame

(a) [9 pts] How fast is the rocket moving as measured in earth's reference frame?

$$\Delta t = \frac{\Delta x}{v} = \Delta\tau\,\gamma = \frac{\Delta\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \Rightarrow \left(\frac{\Delta x}{\Delta\tau}\right)^2 = \frac{v^2}{1-\left(\frac{v}{c}\right)^2} \Rightarrow v^2\left(1+\left(\frac{\Delta x}{\Delta\tau}\right)^2\frac{1}{c^2}\right) = (\Delta x/\Delta\tau)^2$$

$$\Rightarrow v = \frac{\Delta x/\Delta\tau}{\sqrt{1+\left(\frac{\Delta x}{\Delta\tau}\right)^2\left(\frac{1}{c}\right)^2}};\quad \frac{\Delta x}{\Delta\tau} = \frac{500\,c}{20} = 25\,c$$

$$\therefore v = \frac{25\,c}{\sqrt{1+25^2}} = \boxed{.9992\ c}$$

(b) [8 pts] How long does the trip take as measured by people in the earth's reference frame?

$$\Delta t = \frac{\Delta x}{v} = \frac{500\ c\ 1yr}{.9992\,c} = \boxed{500.4\ years}$$

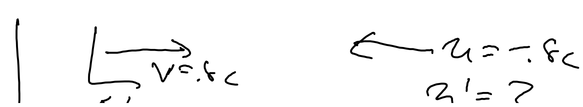
(c) [8 pts] How much energy is required to accelerate the rocket to this speed (as found in part a)?

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 25.02$$

$$KE = (\gamma - 1)m_0c^2 = 24\,(10^4)(3\cdot10^8)^2 = \boxed{2.2\cdot10^{22}\ J}$$

NAME / Section #:	Exam III Problem #3 Phys270
______________________________ / ________	

3. [25 pts] Two rockets, A and B, approach the earth from opposite directions at speed 0.8c. The length of each rocket measured in its rest frame is 200 m.



(a) [15 pts] What is the speed of rocket A with respect to rocket B?

$x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$

S S'

$$\frac{dx'}{dt'} = u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{-.8c - .8c}{1 + (.8)^2} = \frac{-1.6c}{1.64} = .976c$$

(b) [10 pts] What is the length of rocket A as measured by the crew of rocket B?

$$L = \frac{L_0}{\gamma}, \quad L_0 = 200\,m, \quad \gamma = \frac{1}{\sqrt{1 - (.976)^2}} = 4.56$$

$$\Rightarrow L = 43.9\ m$$

NAME / Section #:	Exam III
	Problem #4
________________________________ / _______	Phys270

4. [15 pts] A thin solid barrier in the xy-plane has a 5-μm-diameter circular hole. An electron traveling in the +z-direction (with v_x =0 m/s and v_y =0 m/s) passes through the hole. Afterward, within what range is v_x likely to be? Assume all velocities are small enough to be considered non-relativistic.

$$\Delta x \Delta p_x \sim \frac{h}{2}, \quad \Delta x = 5 \cdot 10^{-6}\,m$$

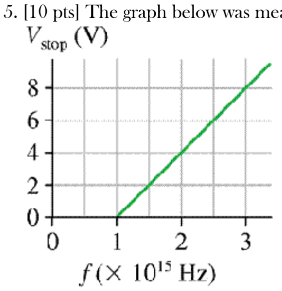
$$\Delta p_x = m_e \Delta v_x = \frac{h}{2\Delta x}$$

$$\Rightarrow \Delta v_x = \frac{h}{2 m_e \Delta x} = \frac{6.63 \cdot 10^{-34}\ J \cdot sec}{2 \cdot (9.11 \cdot 10^{-31})(5 \cdot 10^{-6})}$$

$$\Rightarrow \Delta v_x = 72\ m/sec$$

NAME / Section #:	Exam III
	Problem #5
________________________________ / _______	Phys270

5. [10 pts] The graph below was measured in a photoelectric-effect experiment.



(a) [5 pts] What is the work function (in eV) of the cathode?

$$eV_{stop} = hf - \phi$$

$$@\ V_{stop} = 0 \Rightarrow \phi = hf = \frac{6.63 \cdot 10^{-34}\ J \cdot s}{[1.6 \cdot 10^{-19}\ J/eV]} \times 1 \cdot 10^{15}\ 1/sec = \frac{6.63}{1.6}\ eV$$

$$= 4.1\ eV$$

(b) [5 pts] What experimental value of Planck's constant is obtained from these data?

$$V_{stop} = \left(\frac{h}{e}\right) f - \frac{\phi}{e}; \quad slope = \frac{h}{e} \Rightarrow \frac{8V}{2 \cdot 10^{15} Hz} \cdot 1.6 \cdot 10^{-19} = h$$

Pasted from <file:///C:\Greg\Teaching\Phys270\Sync\Quizzes\ExamIIIv3.doc>

$$\Rightarrow h = 4 \cdot 1.6 \cdot 10^{-34}\ J \cdot s$$
$$= 6.4 \cdot 10^{-34}\ J \cdot s$$

Exam IIIb starts Here!

NAME / Section #:	Exam III Problem #1 Phys270
______________________ / ______	

1. [20 pts] In the earth's reference frame, a tree is at the origin and a pole is at x=50km. Lightning strikes the tree at t=5 μsec and then the pole at t=10 μsec. The lightning strikes are observed by a rocket traveling in the +x-direction at 0.7c. Note that the origins of the two reference frames (earth and rocket) coincide at time t=0 in both reference frames.

(a) [10 pts] What are the spacetime coordinates for these two events in the rocket's reference frame?

$v = .7c$, $t_{tree} = 5 \cdot 10^{-6}$, $x_{tree} = 0$
$t_{pole} = 10 \cdot 10^{-6}$, $x_{pole} = 5 \cdot 10^4$
$\Rightarrow \gamma = 1.4$

$$x' = \gamma(x - vt) = \begin{cases} -1.47 \text{ km} & \text{tree} \\ 67 \text{ km} & \text{pole} \end{cases}$$

$$t' = \gamma\left(t - x\frac{v}{c^2}\right) = \begin{cases} 7.0 \ \mu s & \text{tree} \\ -149 \ \mu s & \text{pole} \end{cases}$$

(b) [10 pts] In the rocket's frame, which event occurrs first and how much time elapses between the two events?

pole is struck first : $\Delta t = 149 + 7 = 156 \ \mu s$

NAME / Section #:	Exam III Problem #2 Phys270
______________________ / ______	

2. [30 pts] A cow is running at a relativistic velocity towards a shed. The shed has two doors opposite one another. The cow is 2.5 m long as measured in the cow's rest frame and the shed is 1.0 m long as measured in the shed's rest frame. A farmer in the shed's reference frame observes the cow fit into the shed, and closes the two paper thin doors and traps the cow momentarily before the cow plows through the backdoor.

a. [10 pts]At what minimum speed is the cow running?

L, length of cow in sheds reference frame $= 1.0 \text{ m} = \dfrac{\overbrace{L_0}^{2.5 \text{ m}}}{\gamma} \Rightarrow \left(\dfrac{1}{\gamma}\right)^2 = \left(\dfrac{1}{2.5}\right)^2 = 1 - \left(\dfrac{v}{c}\right)^2$

$\Rightarrow v = .916 \ c$

b. [7 pts] From the cow's perspective, how long is the barn (that is, the spacing between the doors)?

$$L = \frac{1 \text{ m}}{\gamma} = \frac{1}{2.5} \cdot 1 \text{ m} = \underline{.4 \text{ m}}$$

c. [8 pts]Order the following events in the cow's reference frame (and state which events ocurr at the same time):
event a: Farmer closes back door
event b: Farmer closes front door
event c: Cow's tail reaches back door
event d: Cow's tail reaches front door
event e: Cow breaches the back door
event f: Cow's nose reaches front door

f, a + e, d + b, c

NAME / Section #:	Exam III Problem #2 Phys270
______________________ / ______	

Problem #2 ---- Continued

d. [5 pts] As observed in the cow's reference frame, is it possible for the cow to fit inside the barn? If so, what minimum speed must the cow be moving? If not, explain your reasoning.

No, the cow is 2.5 m long + the shed is 0.4 m long

NAME / Section #: ______________________________ / ______	Exam III Problem #3 Phys270

See problem 3 above + change #'s

3. [20 pts] Two rockets, A and B, approach the earth from opposite directions at speed 0.9c. The length of each rocket measured in its rest frame is 80 m.

(a) [10 pts] What is the speed of rocket A with respect to rocket B?

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}, \quad u = -.9c, \; v = .9c$$

$$\Rightarrow u' = \frac{-1.8c}{1.81} = .994\,c$$

(b) [10 pts] What is the length of rocket A as measured by the crew of rocket B?

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(.994)^2}} = 9.53$$

$$L = \frac{L_0}{\gamma} = \frac{80}{9.53} = 8.4\,m$$

NAME / Section #: ______________________________ / ______	Exam III Problem #4 Phys270

4. [5 pts] A 1.5-μm-wavelength laser is chopped into a short pulse and transmitted into a glass optical fiber with index n=1.33 . The optical fiber can support a maximum bandwidth of 2.0 GHz. How many oscillations are in the shortest-duration laser pulse that can be sustained by the fiber?

$$\Delta t \Delta f \approx 1 \Rightarrow \Delta t = \frac{1}{2\cdot 10^9\,Hz} = .5\cdot 10^{-9}\,sec = 5\cdot 10^{-10}\,sec$$

$$\lambda f = c, \; f = \frac{1}{T}, \; \#\text{ of oscillation} = \frac{\Delta t}{T} = \Delta t \frac{c}{\lambda}$$

$$\Rightarrow \#\text{ of oscillations} = \frac{3\cdot 10^8}{1.5\cdot 10^{-6}}\, 5\cdot 10^{-10} = 10\cdot 10^4 = 10^5$$

NAME / Section #: ______________________________ / ______	Exam III Problem #5 Phys270

10 nm 10^{-17} g

5. [15 pts] A small speck of dust with mass 10^{-14} g has fallen into a crack. The crack is infinitely deep and long with a spacing of 1 μm between the walls of the crack. Assume the particle size is very small compared to the crack spacing and that the particle has no extra kinetic energy above that expected from a confined particle. According to the Heisenberg uncertainty principle, what is the deepest the particle can be inside the crack and still have an appreciable chance of escaping?

$$\frac{1}{2}mv^2 = mgy \Rightarrow v = \sqrt{2gy}$$

$$\Delta x \Delta p_x = \frac{h}{2} = \Delta x (m \Delta v)$$

$$\Rightarrow \Delta v = \frac{h}{2m\Delta x} = \sqrt{2gy}$$

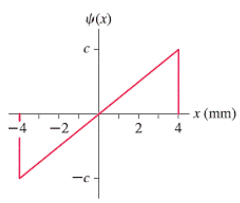
$$\Rightarrow 2gy = \left(\frac{h}{2m\Delta x}\right)^2$$

$$\Rightarrow y = \frac{1}{8}\,\frac{1}{g}\left(\frac{h}{m\Delta x}\right)^2$$

$$y = \frac{1}{8}\,\frac{1}{9.8}\left(\frac{6.6\cdot 10^{-34}}{(10^{-20}\,kg)\,10^{-8}}\right)^2 \approx 5.6\cdot 10^{-13}\,m$$

NAME / Section #: ______________________________ / ______	Exam III Problem #6 Phys270

6. [10 pts] The figure below shows the wave function of a particle confined between x=-4.0 mm and x=+4.0 mm. The wave function is zero outside this region.



(a) [4 pts] Determine the value of the constant c, as defined in the figure.

$$\text{Area} = 1 = \int_{-\infty}^{\infty} |\psi(x)|^2 dx = \int_{-4}^{4}\left(\frac{c}{4}x\right)^2 dx$$

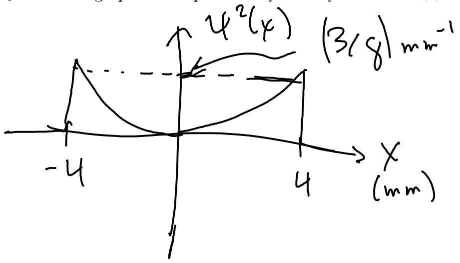
$$\Rightarrow \frac{c^2}{16}\int_{-4}^{4} x^2 dx = \frac{c^2}{16}\,\frac{x^3}{3}\Big|^{4} = \frac{c^2}{3}\,2\cdot\frac{4^3}{4^2} \Rightarrow \frac{8}{3}c^2 = 1 \Rightarrow c = \sqrt{\frac{3}{8}} = .612\; mm^{-1/2}$$

(b) [3 pts] Draw the graph of the probability density function P(x).

$\psi(x) = \frac{c}{4} x \Rightarrow \psi^2(x) = \frac{c^2}{16} x^2$

$\Rightarrow \psi^2(x) = \frac{3}{128} x^2$

$|\psi(4)|^2 = 3/8 \text{ mm}^{-1}$



(c) [3 pts] Calculate the probability of finding the particle in the interval x>=0 and x<=2.0 mm.

$P = \int_0^2 |\psi(x)|^2 \, dx =$

$\left(\frac{c}{4}\right)^2 \frac{x^3}{3}\Big|_0^2 = \frac{c^2}{6} = \left(\frac{3}{8}\right) \cdot \frac{1}{6} = .063$

Pasted from <file:///C:\Greg\Teaching\Phys270\Sync\Quizzes\ExamIIIbv2.doc>