Tuesday, October 27, 2009
10:45 AM

ExamIIv2

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PLEASE print your name and section number at the top of each page!!!!
Failure to do so will result in a 2 point deduction per page.

Exam pages will be unstapled and graded problem by problem, so **if more space is required to answer the question PLEASE continue your answer on the back of the SAME page.** No credit will be given to problems answered on other pages.

There are 5 problems worth 100 points:

- Problem #1 – 20 pts
- Problem #2 – 25 pts
- Problem #3 – 30 pts
- Problem #4 – 15 pts
- Problem #5 – 10 pts

All answers should be in terms of the variables explicitly listed by the statement of the problem.

Integral tables and physical constants are provided on the next two pages if you require them.

You have 1 hour and 15 minutes to finish the exam.

Good luck!

Useful Data

M_e	Mass of the earth	5.98×10^{24} kg	
R_e	Radius of the earth	6.37×10^{6} m	
g	Free-fall acceleration on earth	$9.80\ m/s^2$	
G	Gravitational constant	$6.67 \times 10^{-11}\ N\ m^2/kg^2$	
k_B	Boltzmann's constant	1.38×10^{-23} J/K	
R	Gas constant	8.31 J/mol K	
N_A	Avogadro's number	6.02×10^{23} particles/mol	
T_0	Absolute zero	−273°C	
σ	Stefan-Boltzmann constant	$5.67 \times 10^{-8}\ W/m^2\ K^4$	
p_{atm}	Standard atmosphere	101,300 Pa	
v_{sound}	Speed of sound in air at 20°C	343 m/s	
m_p	Mass of the proton (and the neutron)	1.67×10^{-27} kg	
m_e	Mass of the electron	9.11×10^{-31} kg	
K	Coulomb's law constant ($1/4\pi\epsilon_0$)	$8.99 \times 10^{9}\ N\ m^2/C^2$	
ϵ_0	Permittivity constant	$8.85 \times 10^{-12}\ C^2/N\ m^2$	
μ_0	Permeability constant	1.26×10^{-6} Tm/A	
e	Fundamental unit of charge	1.60×10^{-19} C	
c	Speed of light in vacuum	3.00×10^{8} m/s	
h	Planck's constant	6.63×10^{-34} J s	4.14×10^{-15} eV s
$\hbar$	Planck's constant	1.05×10^{-34} J s	6.58×10^{-16} eV s
a_B	Bohr radius	5.29×10^{-11} m	

Common Prefixes

Prefix	Meaning
femto-	10^{-15}
pico-	10^{-12}
nano-	10^{-9}
micro-	10^{-6}
milli-	10^{-3}
centi-	10^{-2}
kilo-	10^{3}
mega-	10^{6}
giga-	10^{9}
terra-	10^{12}

Conversion Factors

Length
1 in = 2.54 cm
1 mi = 1.609 km
1 m = 39.37 in
1 km = 0.621 mi

Velocity
1 mph = 0.447 m/s
1 m/s = 2.24 mph = 3.28 ft/s

Mass and energy
1 u = 1.661×10^{-27} kg
1 cal = 4.19 J
1 eV = 1.60×10^{-19} J

Time
1 day = 86,400 s
1 year = 3.16×10^{7} s

Pressure
1 atm = 101.3 kPa = 760 mm of Hg
1 atm = 14.7 lb/in^2

Rotation
1 rad = $180°/\pi$ = 57.3°
1 rev = 360° = 2π rad
1 rev/s = 60 rpm

Cos(60°) = 1/2	Cos(30°) = $\sqrt{3}/2$	Cos(45°) = $\sqrt{2}/2$
Sin(60°) = $\sqrt{3}/2$	Sin(30°) = 1/2	Sin(45°) = $\sqrt{2}/2$
Tan(60°) = $\sqrt{3}$	Tan(30°) = $1/\sqrt{3}$	Tan(45°) = 1

Derivatives

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(ax) = a$$

$$\frac{d}{dx}\left(\frac{a}{x}\right) = -\frac{a}{x^2}$$

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(\ln(ax)) = \frac{1}{x}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

Integrals

$$\int x\,dx = \frac{1}{2}x^2$$

$$\int x^2\,dx = \frac{1}{3}x^3$$

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{a+x} = \ln(a+x)$$

$$\int \frac{x\,dx}{a+x} = x - a\ln(a+x)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) + \frac{x}{2a^2(x^2 + a^2)}$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x\,dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{1}{a^2}e^{ax}(ax - 1)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

NAME / Section #:	Exam II Problem #1 Phys270
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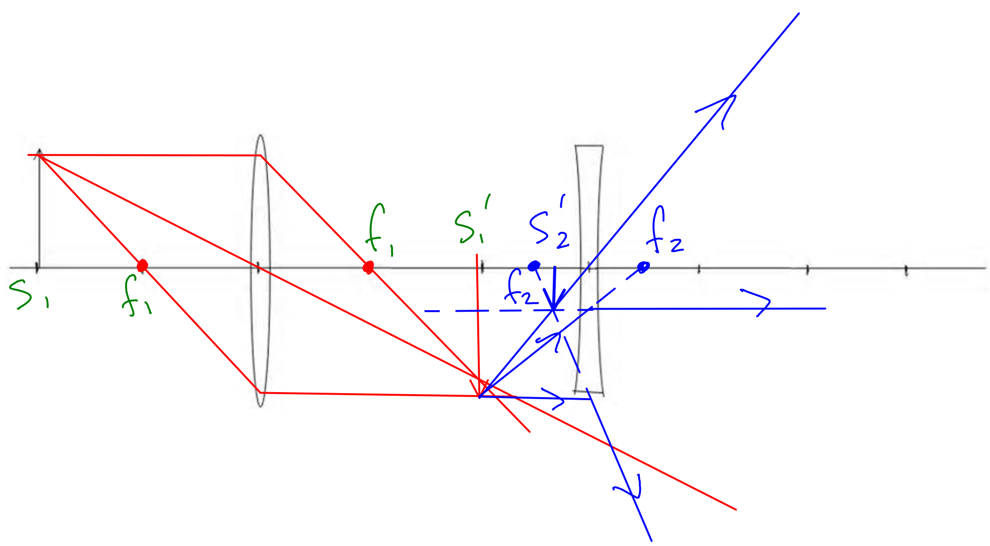
1. [20 pts] A 10.0-cm-tall object is 20 cm to the left of a converging lens with a focal length of 10 cm. A second diverging lens with a focal length of -5.0 cm is 30 cm to the right of the first lens.

a. [10 pts] Use your formula card as a straight edge to draw the ray diagram on the schematic below. Each tick mark represents 10 cm. Make sure to label the focal points, the image produced by the first lens, and the image produced by the second lens.



b. [10 pts] Calculate the image position and height.

$S_1' = \frac{S f_1}{S - f_1} = \frac{20 \cdot 10}{20 - 10} = 20$ cm, $M_1 = \frac{h'}{h} = \frac{-S_1'}{S_1} = -1$

$S_2' = \frac{S_2 f_2}{S_2 - f_2}$, $S_2 = 30 - 20 = 10$ cm $\Rightarrow S_2' = \frac{10(-5)}{10+5}$

$\Rightarrow S_2' = \frac{-10}{3} = -3\frac{1}{3}$ cm, $M_2 = \frac{-S_2'}{S_2} = \frac{10/3}{10} = \frac{1}{3}$

$\therefore M = M_1 M_2 = -\frac{1}{3}$ $\therefore$ $h = \frac{10\text{ cm}}{3} = 3\frac{1}{3}$ cm tall, upside down

Image is $3\frac{1}{3}$ cm to left of diverging lens

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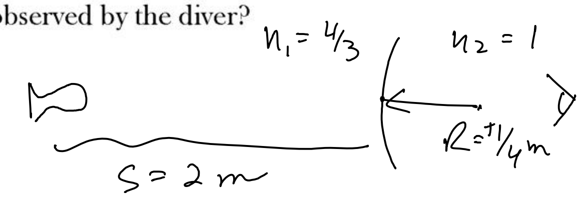
2. [25 pts] A deep sea diver wearing a spherical plastic helmet of diameter 0.50 m looks directly at a fish, and the fish looks directly back. The fish is 2.00 m from the helmet, and the the diver's eye is 0.20 m from the helmet. Ignore any refraction effects from the plastic helmet itself (that is, assume that the plastic helmet is infinitely thin). The index of refraction of water is 1.33.

a) [12 pts] How far does the fish appear to be from the surface of the helmet as observed by the diver?

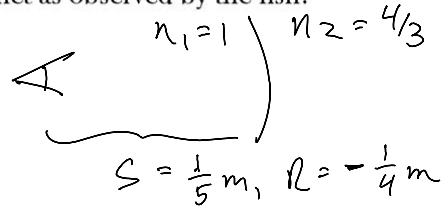


$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{n_2}{s'} = \frac{n_2 - n_1}{R} - \frac{n_1}{s} = \frac{(n_2 - n_1)s - n_1 R}{Rs}$$

$$\Rightarrow s' = \frac{n_2 R s}{(n_2 - n_1)s - n_1 R} = \frac{\frac{1}{4} \cdot 2}{-\frac{1}{3} \cdot 2 - \frac{4}{3} \cdot \frac{1}{4}} = \frac{1/2}{-1} = -\frac{1}{2} \text{ m}$$

Image is $\frac{1}{2}$ m outside of Helmet

b) [13 pts] How far does the diver's eye appear to be from the surface of the helmet as observed by the fish?



$$s' = \frac{n_2 R s}{(n_2 - n_1)s - n_1 R} = \frac{\frac{4}{3}\left(-\frac{1}{4}\right)\frac{1}{5}}{\frac{1}{3} \cdot \frac{1}{5} + \frac{1}{4}} = \frac{-1}{1 + 15/4} = \frac{-4}{19}$$

$\Rightarrow s' = -.21$ m, or .21 m inside Helmet

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3. [30 pts] Light of wavelength λ impinges upon two parallel slits whose width is 'a' and separation is 'd'. The diffraction pattern is projected onto a screen a distance 'L' away. Assume that the slit separation d>a, the distance to the screen is much larger than the slit spacing L>>d, 'a' is small enough to observe diffraction effects, and the entire apparatus is in air.

interference fringes

a) [8 pts] List all the ways one can cause the diffraction pattern maxima on the screen to move closer together. Make sure to explain your reasoning.

$I_{avg} = I_0 \cos^2\left(\frac{\pi y d}{\lambda L}\right) = I_0 \cos^2(k_2' y)$, $k_2' = \frac{\pi d}{\lambda L}$

k_2' increases ⇒ pattern moves closer together

⇒ d increases, λ decreases, L decreases

b) [7 pts] What would happen to the diffraction pattern if the entire apparatus were to be submerged in an oil of index n? Explain your reasoning.

This effectively reduces the wavelength 7 $\lambda \to \frac{\lambda}{n}$ where $n > 1$.

As in part a), decreasing λ will cause diffraction maxima to move closer together

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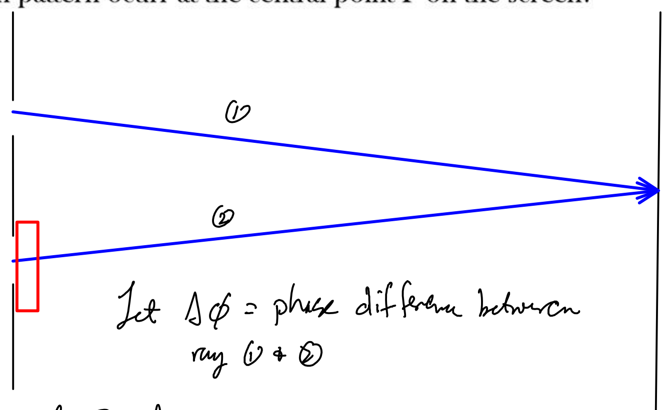
Problem #3 Continued ----

c) [15 pts] The aparatus is in air. Consider the middle of the central maximum on the screen (where the path length is the same for light emanating from both slits) and call it point P. A small glass plate of index n_g is then inserted directly behind ***one*** of the slits. Assume the thickness of the glass is very small compared to L (that is, assume that the rays of interest enter and leave the glass plate at approximately normal incidence). For what thickness of the glass plate does a minimum in the diffraction pattern ocurr at the central point P on the screen?



Let $\Delta\phi$ = phase difference between ray ① + ②

$\Delta\phi = k_2\delta - k_1\delta = (2m+1)\pi$ for destructive interference

$k_1 = \frac{2\pi}{\lambda_1}$, $\delta = t$, thickness of glass,

$k_2 = \frac{2\pi}{\lambda_2}$, $\lambda_2 = \frac{\lambda_0}{n_g}$, $\lambda_1 = \lambda$, λ wavelength in air

$\Rightarrow 2\pi\left(\frac{n_g}{\lambda} - \frac{1}{\lambda}\right)t = (2m+1)\pi$

$\Rightarrow t = \frac{(2m+1)}{2}\frac{\lambda}{(n_g-1)}$

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4. [15 pts]

a. [5 pts] What are the energies of the first two energy energy levels of an electron confined in a one-dimensional box of length 0.50 nm?

$$E_n = E_1 n^2, \quad E_1 = \frac{h^2}{8 m_e L^2} = \frac{[6.6 \cdot 10^{-34}]^2}{8 \cdot (9.1 \cdot 10^{-31} kg)[5 \cdot 10^{-10}]^2}$$

$$\Rightarrow E_1 = \frac{6.6^2}{8 \cdot 9.1 \cdot 5^2} \cdot 10^{-17} J = \boxed{2.4 \cdot 10^{-19} J}$$

$10^{51} \cdot 10^{-68} = 10^{-17}$

$$\boxed{E_2 = 9.6 \cdot 10^{-19} J}$$

b. [5 pts] How much energy must the electron lose to move from the n=2 energy level to the lowest n=1 energy level?

$$\Delta E = E_1(4-1) = 3E_1 = \boxed{7.2 \cdot 10^{-19} J}$$

c. [5 pts] Suppose that an electron can move from the n=2 level to the n=1 level by emitting a photon of light. If energy is conserved, what must the photon's wavelength be?

$$E = \hbar\omega = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

$$\Rightarrow \lambda = \frac{(6.6 \cdot 10^{-34})(3 \cdot 10^8)}{7.2 \cdot 10^{-19}}$$

$-26+19=-7$

$$= 275 \, nm$$

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5. [10 pts] Electrons pass through a 1.0-μm-wide slit with a speed of 4.0×10^6 m/s. What is the width of the electron diffraction pattern on a detector 2.0m behind the slit?

"width of Central Maximum"

$a = 1.0\,\mu m$, $L = 2.0\,m$

$$p = \hbar k = \frac{h}{\lambda} = m_e v \Rightarrow \lambda = \frac{h}{m_e v}$$

Central Maxima width:

$$\sin(\Delta\theta) = \frac{\lambda}{a} \approx \frac{w/2}{L} \Rightarrow w = \frac{2\lambda L}{a}$$

$$\text{or } w = \frac{2h}{m_e}\left(\frac{L}{av}\right) = 2\left(\frac{6.6 \cdot 10^{-34}}{9.1 \cdot 10^{-31}}\right)\underbrace{\left[\frac{2}{10^{-6}(4 \cdot 10^{6})}\right]}_{1/2}$$

$$w = \frac{6.6}{9.1} \cdot 10^{-3}\,m = .73\,mm$$

$$= 730\,\mu m$$