Exam I - quick review

Review ALL homework problems and quizzes

Review your book notes

Make up your 6”x4” card

PLEASE print your name and section number at the top of each page!!!!
Failure to do so will result in a 2 point deduction per page.

Exam pages will be unstapled and graded problem by problem, so **if more space is required to answer the question PLEASE continue your answer on the back of the SAME page.**

There are 6 problems worth 100 points:
Problem #1 - 10 pts
Problem #2 - 20 pts
Problem #3 - 30 pts
Problem #4 - 15 pts
Problem #5 - 10 pts
Problem #6 - 15 pts

All answers should be in terms of the variables explicitly listed by the statement of the problem.

Integral tables and physical constants are provided on the next two pages if you require them.

You have 1 hour and 15 minutes to finish the exam.

Good luck!

Useful Data

M_e	Mass of the earth	5.98×10^{24} kg	
R_e	Radius of the earth	6.37×10^{6} m	
g	Free-fall acceleration on earth	$9.80\ m/s^2$	
G	Gravitational constant	$6.67 \times 10^{-11}\ N\ m^2/kg^2$	
k_B	Boltzmann's constant	1.38×10^{-23} J/K	
R	Gas constant	8.31 J/mol K	
N_A	Avogadro's number	6.02×10^{23} particles/mol	
T_0	Absolute zero	−273°C	
σ	Stefan-Boltzmann constant	$5.67 \times 10^{-8}\ W/m^2\ K^4$	
p_{atm}	Standard atmosphere	101,300 Pa	
v_{sound}	Speed of sound in air at 20°C	343 m/s	
m_p	Mass of the proton (and the neutron)	1.67×10^{-27} kg	
m_e	Mass of the electron	9.11×10^{-31} kg	
K	Coulomb's law constant ($1/4\pi\epsilon_0$)	$8.99 \times 10^{9}\ N\ m^2/C^2$	
ϵ_0	Permittivity constant	$8.85 \times 10^{-12}\ C^2/N\ m^2$	
μ_0	Permeability constant	1.26×10^{-6} Tm/A	
e	Fundamental unit of charge	1.60×10^{-19} C	
c	Speed of light in vacuum	3.00×10^{8} m/s	
h	Planck's constant	6.63×10^{-34} J s	4.14×10^{-15} eV s
$\hbar$	Planck's constant	1.05×10^{-34} J s	6.58×10^{-16} eV s
a_B	Bohr radius	5.29×10^{-11} m	

Common Prefixes

Prefix	Meaning
femto-	10^{-15}
pico-	10^{-12}
nano-	10^{-9}
micro-	10^{-6}
milli-	10^{-3}
centi-	10^{-2}
kilo-	10^{3}
mega-	10^{6}
giga-	10^{9}
terra-	10^{12}

Conversion Factors

Length
1 in = 2.54 cm
1 mi = 1.609 km
1 m = 39.37 in
1 km = 0.621 mi

Velocity
1 mph = 0.447 m/s
1 m/s = 2.24 mph = 3.28 ft/s

Mass and energy
1 u = 1.661×10^{-27} kg
1 cal = 4.19 J
1 eV = 1.60×10^{-19} J

Time
1 day = 86,400 s
1 year = 3.16×10^{7} s

Pressure
1 atm = 101.3 kPa = 760 mm of Hg
1 atm = 14.7 lb/in^2

Rotation
1 rad = $180°/\pi$ = 57.3°
1 rev = 360° = 2π rad
1 rev/s = 60 rpm

Cos(60°) = 1/2	Cos(30°) = $\sqrt{3}/2$	Cos(45°) = $\sqrt{2}/2$
Sin(60°) = $\sqrt{3}/2$	Sin(30°) = 1/2	Sin(45°) = $\sqrt{2}/2$
Tan(60°) = $\sqrt{3}$	Tan(30°) = $1/\sqrt{3}$	Tan(45°) = 1

Derivatives

$$\frac{d}{dx}(a) = 0$$

$$\frac{d}{dx}(ax) = a$$

$$\frac{d}{dx}\left(\frac{a}{x}\right) = -\frac{a}{x^2}$$

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

$$\frac{d}{dx}(\ln(ax)) = \frac{1}{x}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$$

Integrals

$$\int x\,dx = \frac{1}{2}x^2$$

$$\int x^2\,dx = \frac{1}{3}x^3$$

$$\int \frac{1}{x^2}\,dx = -\frac{1}{x}$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad n \neq -1$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \frac{dx}{a+x} = \ln(a+x)$$

$$\int \frac{x\,dx}{a+x} = x - a\ln(a+x)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{(x^2 + a^2)^2} = \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right) + \frac{x}{2a^2(x^2 + a^2)}$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$$

$$\int \frac{x\,dx}{(x^2 \pm a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 \pm a^2}}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{1}{a^2}e^{ax}(ax - 1)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax)\,dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$

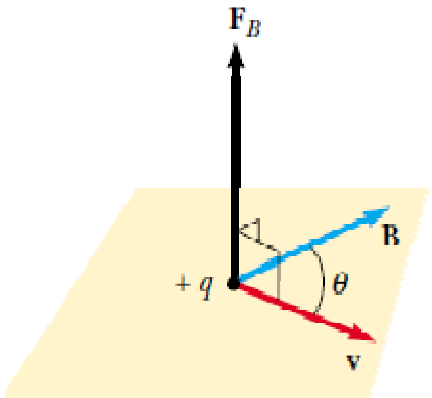
$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

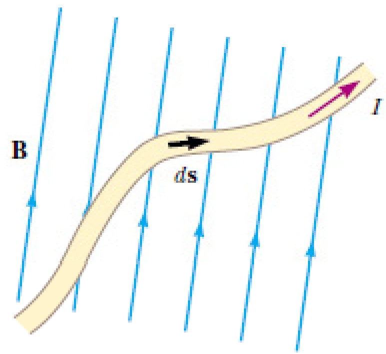
Last time:

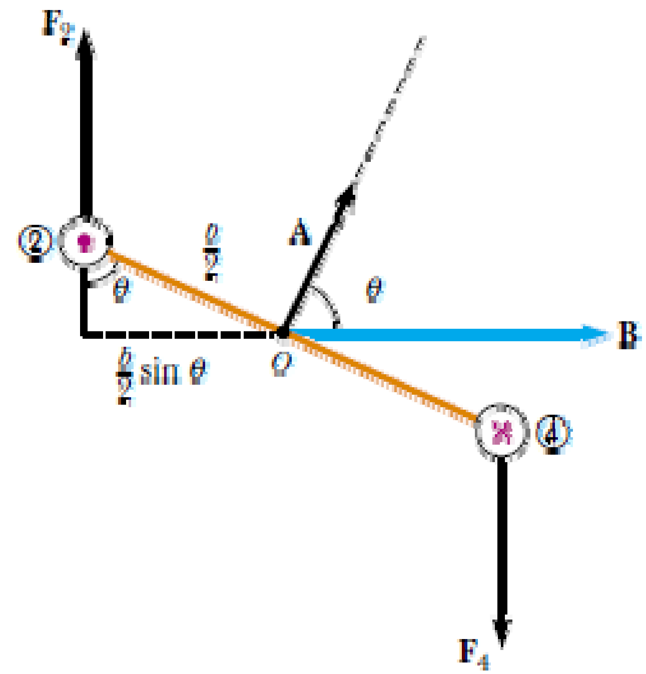
-Moving charges (and currents) create magnetic fields

-Moving charges (and currents) feel a force in magnetic fields

- $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$ Defines the B-field; Examples - cyclotron frequency, velocity selector, mass spectrometer







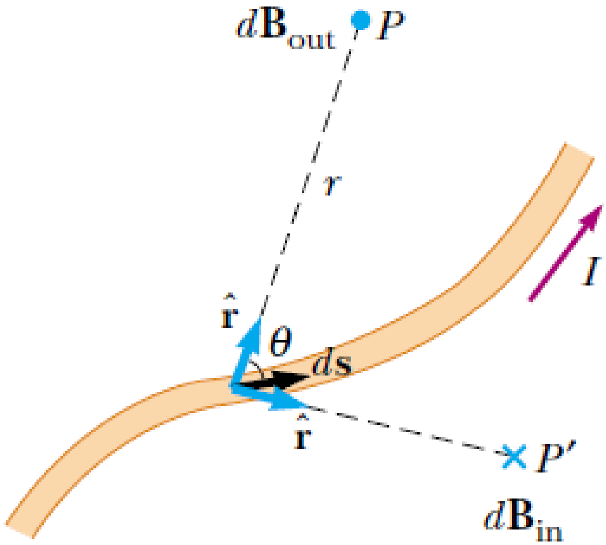
- Force on a wire segment: $$\mathbf{F}_B = I\int_a^b d\mathbf{s} \times \mathbf{B}$$

- Force on a straight wire segment in uniform B-field: $$\mathbf{F}_B = I\mathbf{L} \times \mathbf{B}$$

- Torque on current loop in uniform magnetic field: $$\boldsymbol{\mu} = I\mathbf{A} \qquad \boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

- Potential energy of magnetic dipole (current loop) in magnetic field $$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

The Source of the Magnetic Field: Moving Charges



Biot–Savart law

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ toward P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude ds of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

Biot-Savart Law examples:

Found B-fields for

1. Center of current arc → Circle,
2. On axis of current loop

$$\vec{B}_{center} = \frac{\mu_0 I}{2a}$$

Current loops are "magnetic dipoles"

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{x^3} \quad \text{on axis} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{x^3}$$

$$\vec{\tau}_B = \vec{\mu} \times \vec{B} \qquad \vec{\tau}_E = \vec{p} \times \vec{E}$$

$$U_B = -\vec{\mu} \cdot \vec{B} \qquad U_E = -\vec{p} \cdot \vec{E}$$

Ampere's Law:

"derived" Ampere's Law for 2-D loops and infinite current carrying wire

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{through}$$

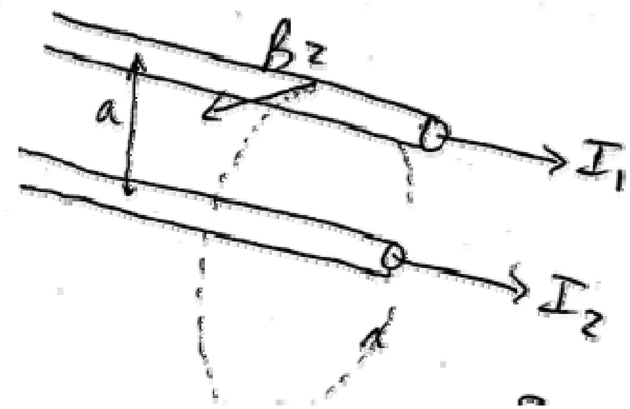
Used Ampere's Law to find B-fields for:

1. Infinite straight wire (inside and outside of wire)
2. Long solenoid – B uniform inside
3. Torus

Force between two straight wires:

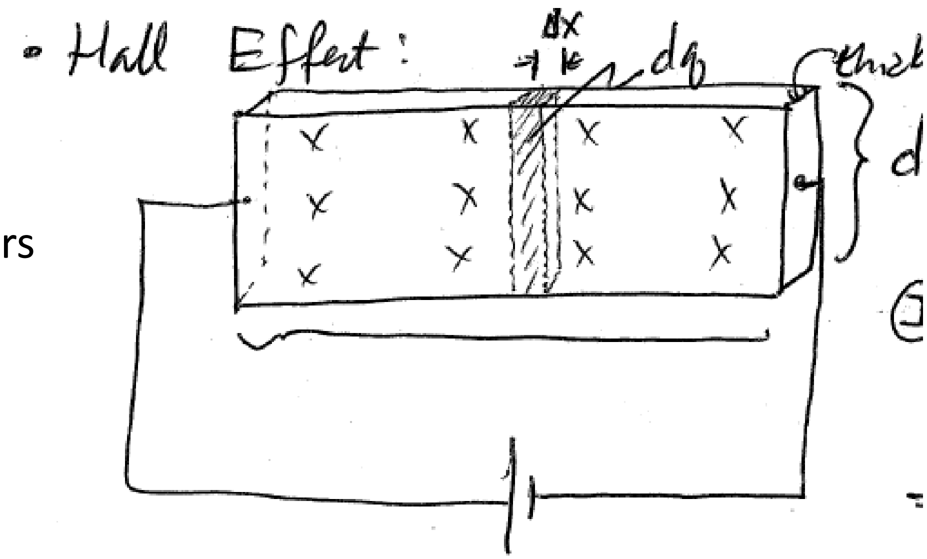
current in from an infinite wire produces a B-field, and the second wire with a current in it feels a force when placed inside the B-field

$$F_1 = I_1 l_1 \frac{\mu_0 I_2}{2\pi a} = \boxed{\frac{\mu_0}{2\pi a} I_1 I_2}$$



Parallel currents attract, Opposite currents repel.

DC Hall Effect



If charges positive, "+" charges on top

If charges negative, "-"charges on top

Hall voltage gives sign and density of charge carriers

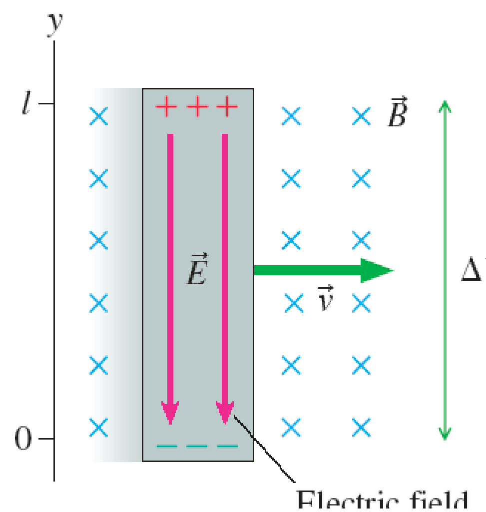
$$\Delta V_H = B \cdot d \left[\frac{I}{n q_0 d z}\right] = \boxed{\frac{IB}{n q_0 z}}$$

Motional emf:

Example in uniform field:

translation

rotation



$F_B = F_E \Rightarrow qvB = qE$ or

$E = vB$

For uniform E-field:

$|\Delta V| = \int \vec{E} \cdot d\vec{s} = E\ell$

or $\Delta V = vB\ell$

Magnetic flux:

$$\Phi_B = \int_{\text{surface}} \vec{B} \cdot d\vec{A}$$

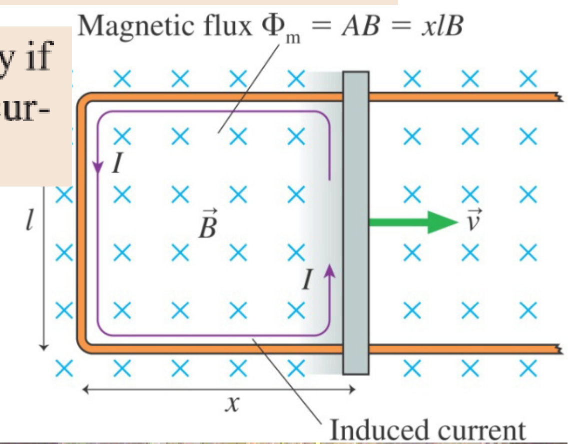
Example: Magnetic flux from the current in a long straight wire

Faraday's law An emf $\mathcal{E}$ is induced around a closed loop if the magnetic flux through the loop changes. The magnitude of the emf is

$$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| \qquad (34.14)$$

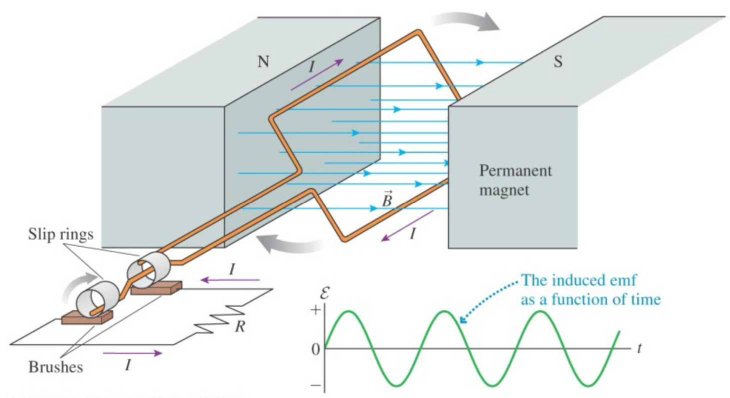
and the direction of the emf is such as to drive an induced current in the direction given by Lenz's law.

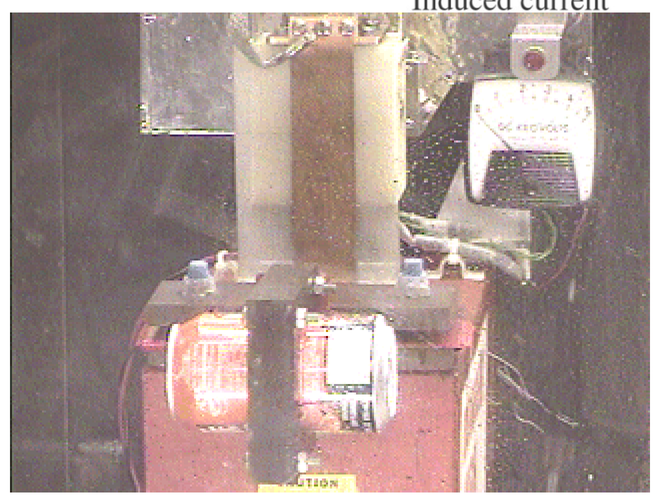
Lenz's law There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.



Can change the flux through a loop three ways:

Example 1: Change the size of the loop -
Example 2:Change the orientation of the loop
Example 3: Change the strength of magnetic field





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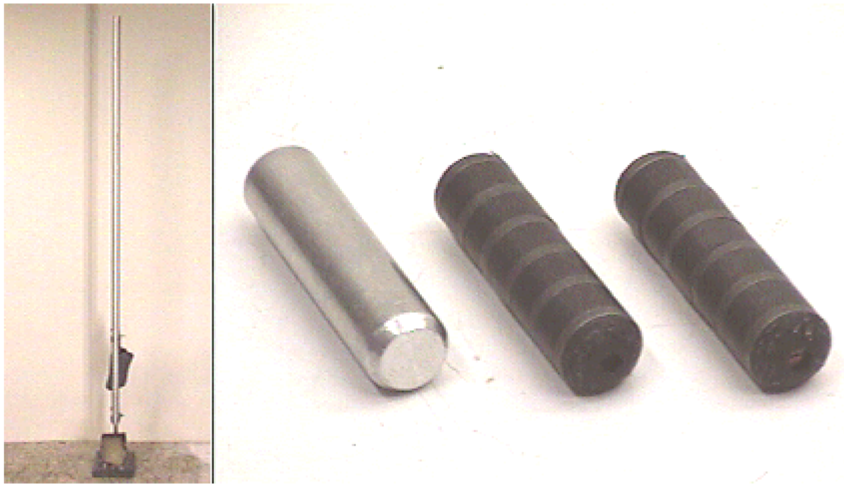
Lenz's law There is an induced current in a closed, conducting loop if and only if the magnetic flux through the loop is changing. The direction of the induced current is such that the induced magnetic field opposes the *change* in the flux.

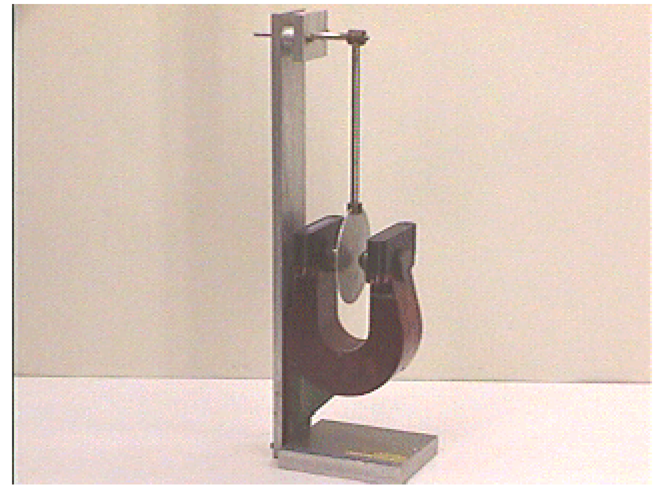
Changing flux creates E-fields:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

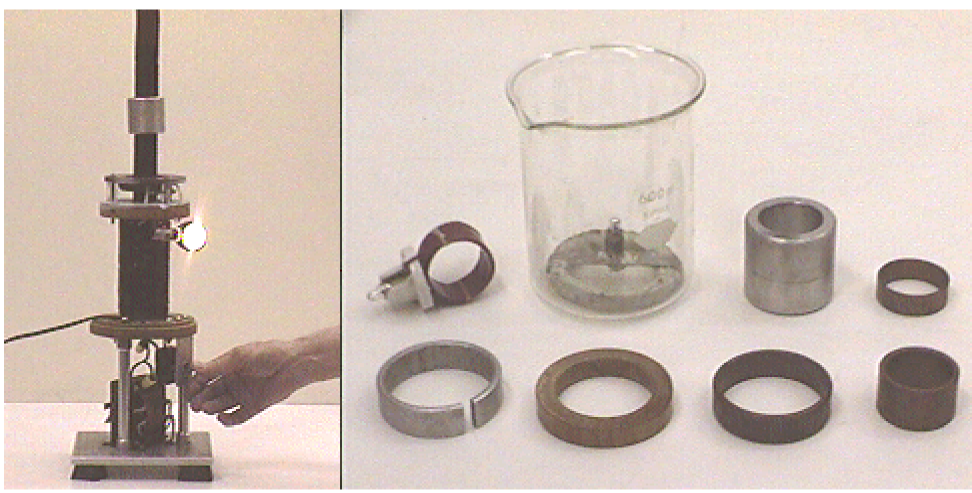
Lenz's Law



K2-42: LENZ'S LAW - MAGNET IN ALUMINUM TUBE



K2-44: EDDY CURRENT PENDULUM



K2-61: THOMSON'S COIL

$$\therefore |\mathcal{E}_{back}| \propto \frac{dI}{dt}$$

Define the self inductance L as the proportionality constant:

$$\Delta V_L = -L \frac{dI}{dt}$$

Back – emf, Inductance:

examples:

Infinite solenoid (inductor!)

$$L = A \mu_0 \frac{N^2}{\ell}$$

Energy in B-fields: $u_B = \frac{PE}{\text{Volume}} = \frac{1}{2\mu_0} B^2$

Recall for ||-plate capacitor: $u_E = \frac{\varepsilon_0}{2} E^2$

Inductor applications: Transformers, LR and LC circuits

E-fields and B-fields transform into one another with relative motion!

E-fields and B-fields transform into one another with relative velocity between inertial reference Frames.

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B} \qquad \vec{B}' = \vec{B} - \frac{1}{c^2}\vec{V} \times \vec{E}$$

or

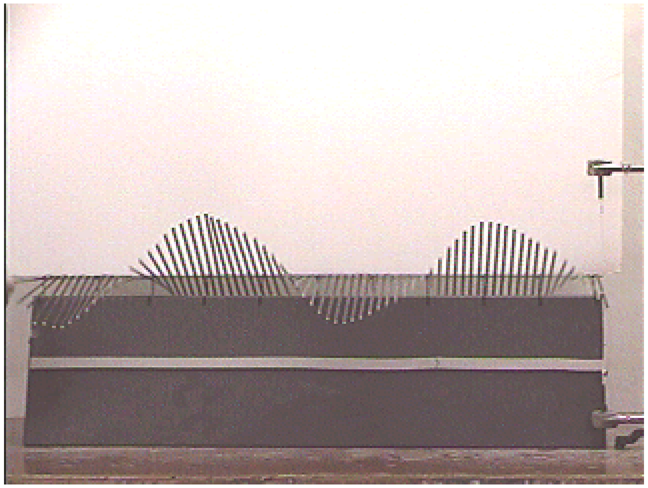
$$\vec{E} = \vec{E}' - \vec{V} \times \vec{B}' \qquad \vec{B} = \vec{B}' + \frac{1}{c^2}\vec{V} \times \vec{E}'$$

Maxwell's Equations

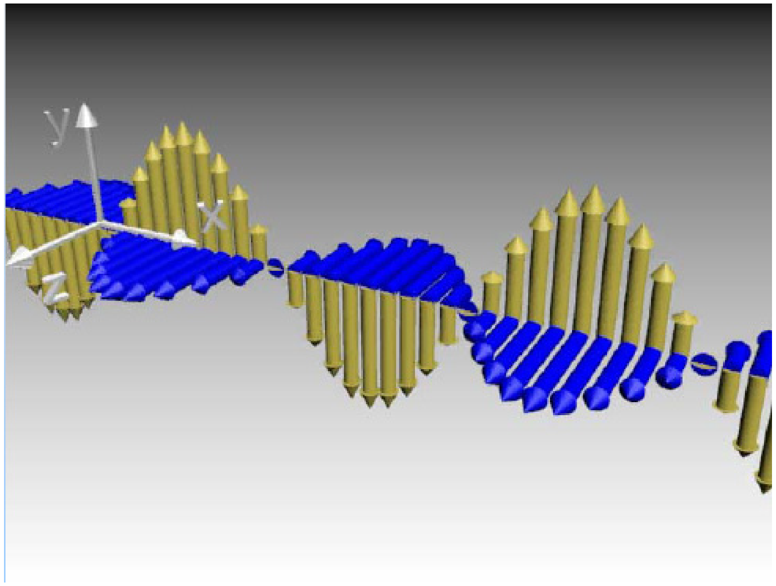
$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$	(Gauss's Law)
$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$	(Faraday's Law)
$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	(Magnetic Gauss's Law)
$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	(Ampere-Maxwell Law)

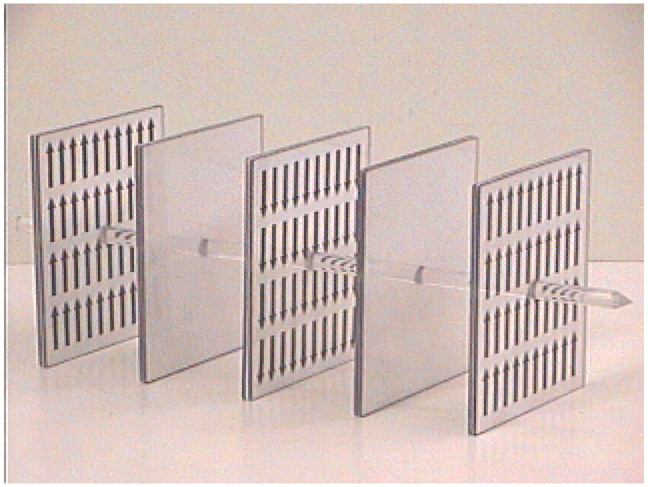
$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$ (Lorentz force Law)

Review of traveling waves



E&M Plane waves:





E&M plane waves: Wave equation, B-fields relate to E-fields

Both E & B travel like waves:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

But there are strict relations between them:

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \qquad \frac{\partial B_z}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$$

$$E_y = E_0 \cos(kx - \omega t), \quad B_z = B_0 \cos(kx - \omega t)$$

$$B_0 = E_0 / c, \quad c = 1/\sqrt{\mu_0 \varepsilon_0}$$

Same amount of energy carried by E-field and B-field

Poynting vector and intensity:

Intensity = Power/Area

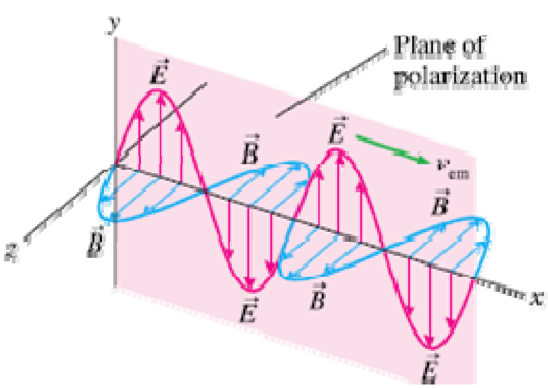
$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}, \quad \vec{S}_{avg} = \frac{1}{2}\frac{1}{\mu_0}\vec{E}_0\times\vec{B}_0$$

Radiation pressure:

$P = \frac{I}{c}$ for perfect absorber

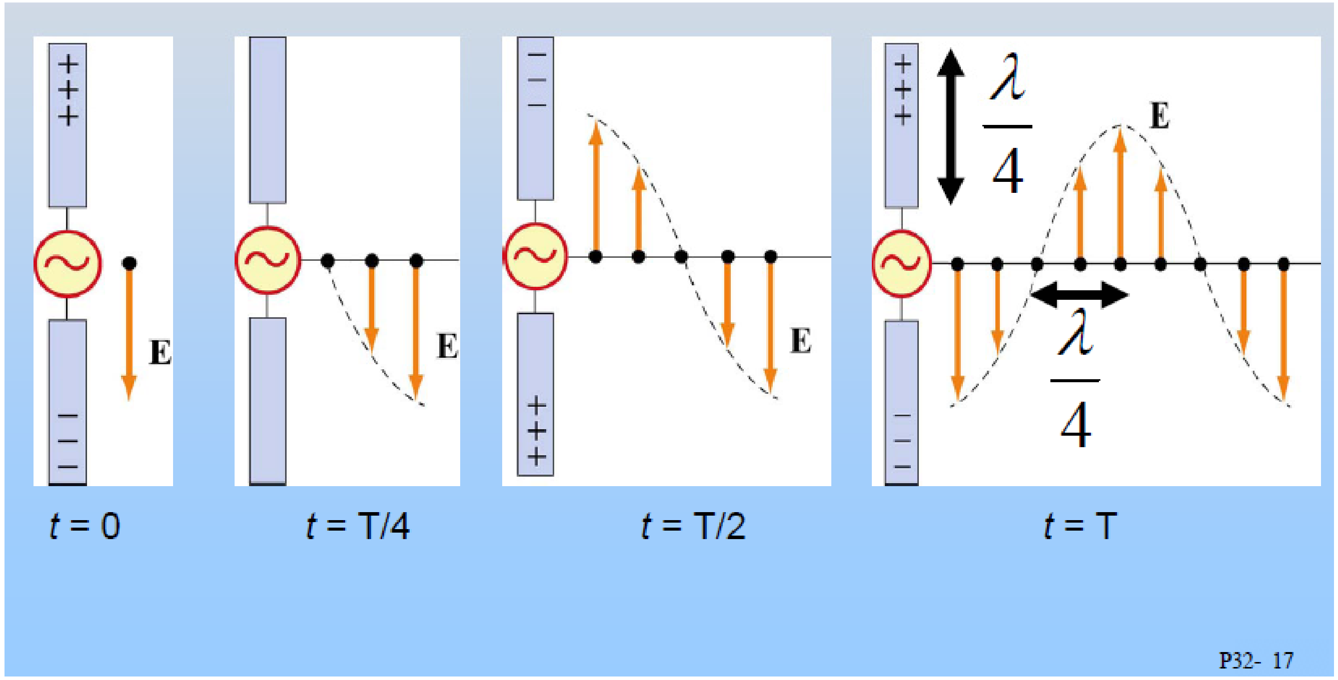
$P = \frac{2I}{c}$ for perfect reflector

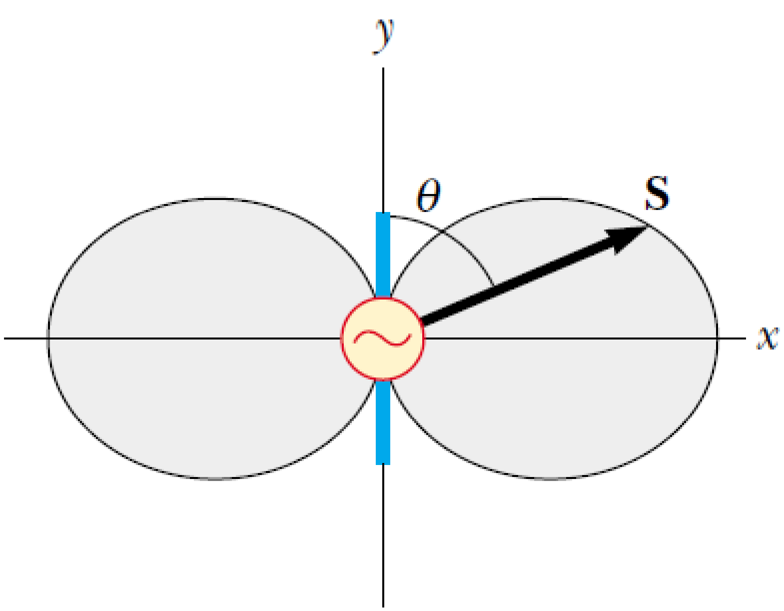
Polarization:



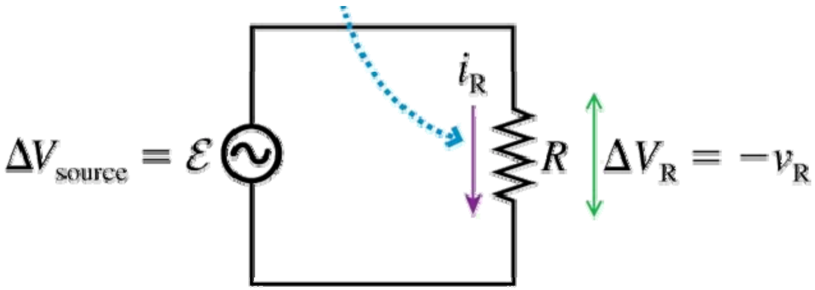
Malus's Law:

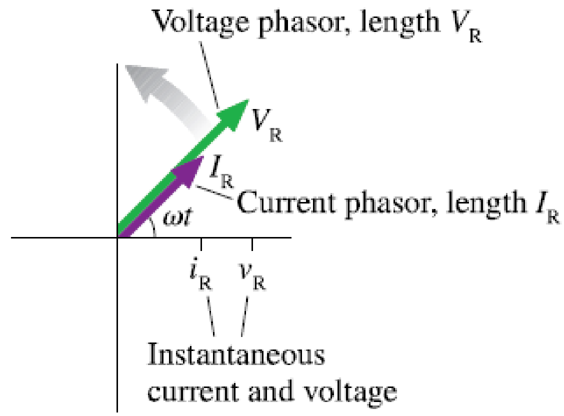
$$I_{\text{transmitted}} = I_0 \cos^2\theta$$





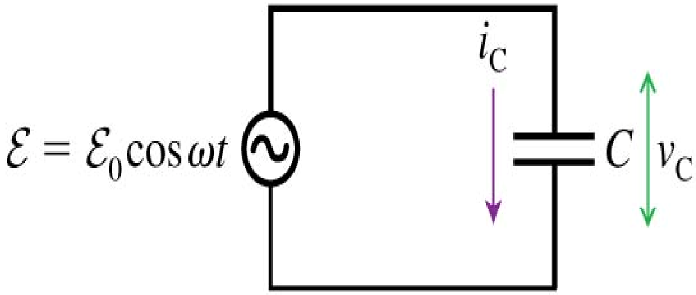
$$I \propto \frac{\sin^2\theta}{r^2}$$

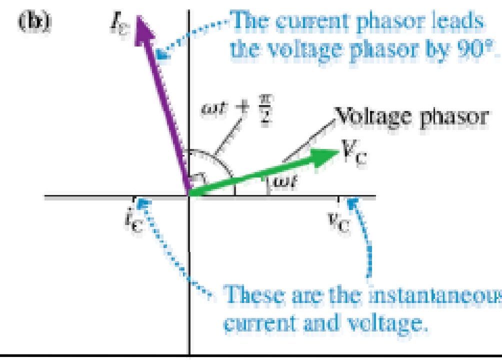




$v_r = i R$

$V_R = I R$

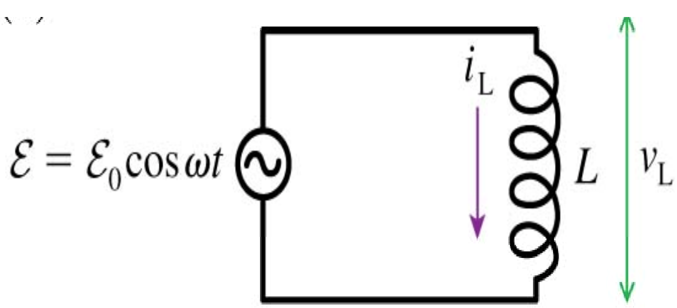


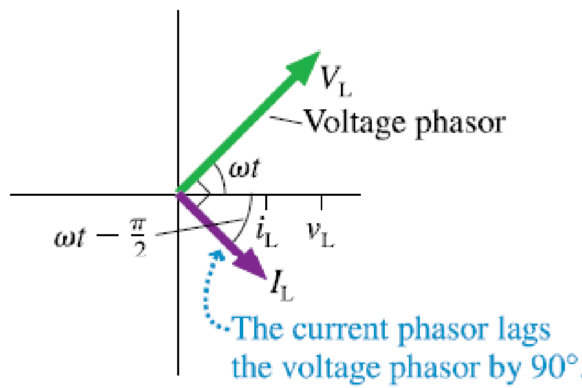


$v_c \neq i X_c$

$V_c = I X_c$

$X_c = \frac{1}{\omega C}$





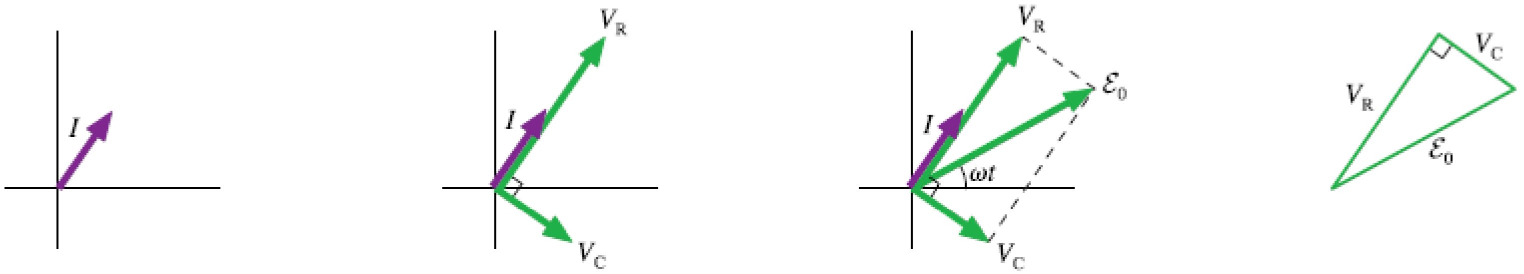
$v_L \neq i X_L$

$V_L = I X_L$

$X_L = \omega L$

RC Filters – Analysis

Analyzing an *RC* circuit



$$\mathcal{E}_0^2 = V_R^2 + V_C^2 = (IR)^2 + (IX_C)^2 = (R^2 + X_C^2)I^2$$
$$= (R^2 + 1/\omega^2 C^2)I^2$$

Consequently, the peak current in the *RC* circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

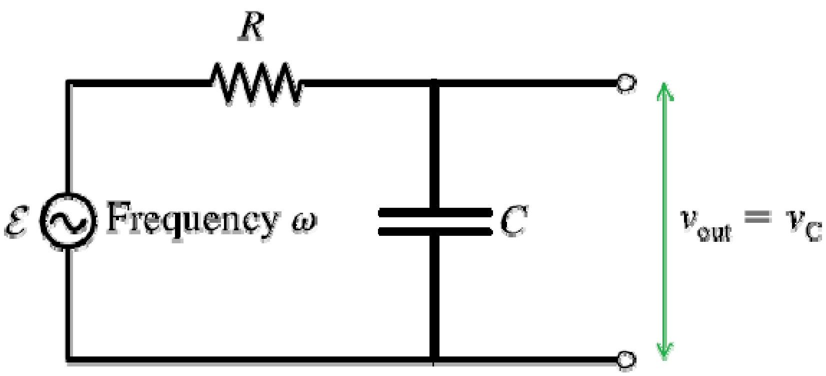
Knowing I gives us the two peak voltages:

$$V_R = IR = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

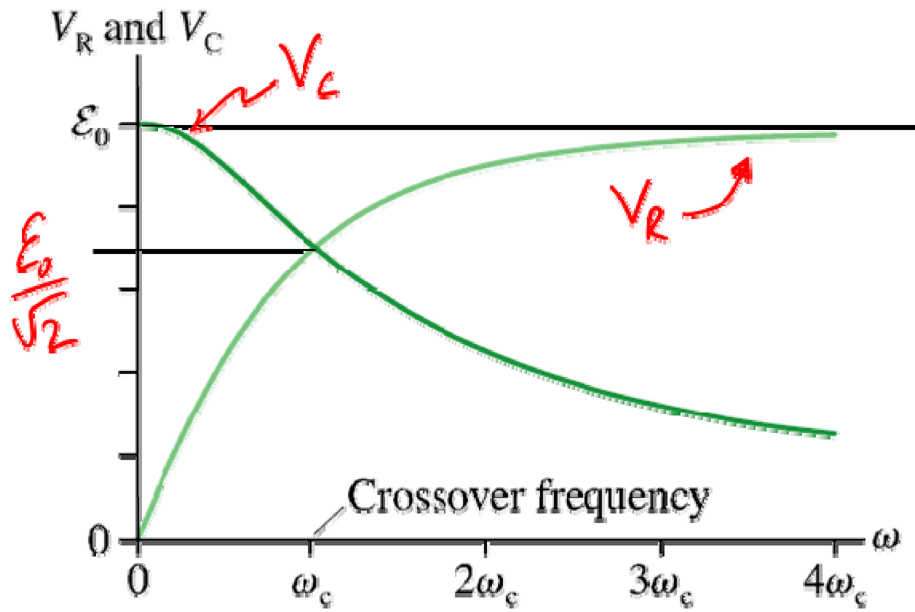
$$V_C = IX_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0/\omega C}{\sqrt{R^2 + 1/\omega^2 C^2}}$$

RC Filters – Analysis

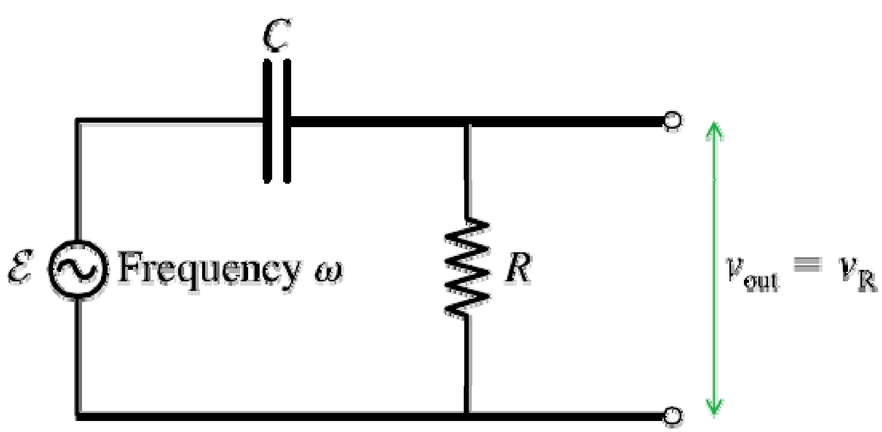
(a) Low-pass filter



Transmits frequencies $\omega < \omega_c$ and blocks frequencies $\omega > \omega_c$.



(b) High-pass filter



Transmits frequencies $\omega > \omega_c$ and blocks frequencies $\omega < \omega_c$.

- **Capacitor like a short at high frequencies since:**

$$X_C = \frac{1}{\omega C} \longrightarrow 0 \text{ at High } \omega$$

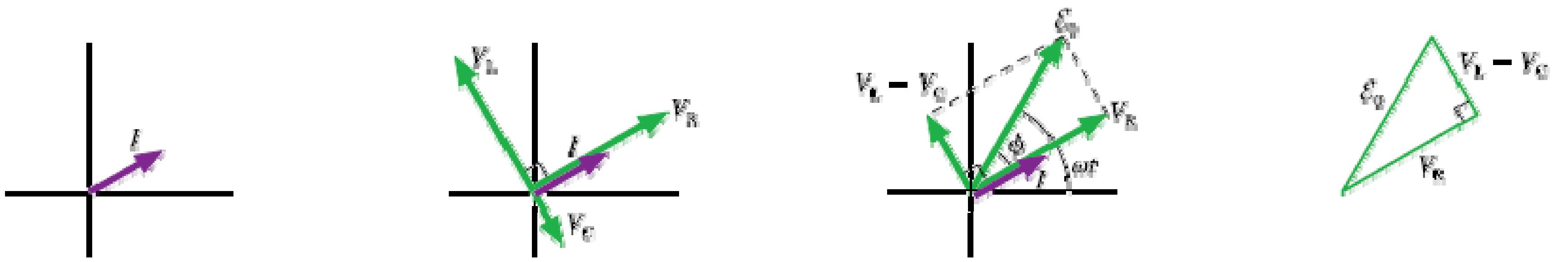
- **Voltage across Capacitor dominates at low frequencies since:**

$$X_C = \frac{1}{\omega C} \Rightarrow X_C >> R \text{ as } \omega \rightarrow 0$$

- **If you input music, voltage across resistor would be like treble and voltage across capacitor would be like bass. Build your own speaker cross-over for woofer and tweeter.**

LRC Filters – Analysis

Analyzing an *RLC* circuit



$$\mathcal{E}_0^2 = V_R^2 + (V_L - V_C)^2 = [R^2 + (X_L - X_C)^2]I^2 \qquad (36.23)$$

where we wrote each of the peak voltages in terms of the peak current I and a resistance or a reactance. Consequently, the peak current in the *RLC* circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \qquad (36.24)$$

The three peak voltages, if you need them, are then found from $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$.

Want an "Ohm's Law" form, so let:

$I = \frac{\mathcal{E}_0}{Z} \Rightarrow Z$, impedance $= \sqrt{R^2 + (X_L - X_C)^2}$

where $X_L = \omega L$, $X_C = \frac{1}{\omega C}$

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$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

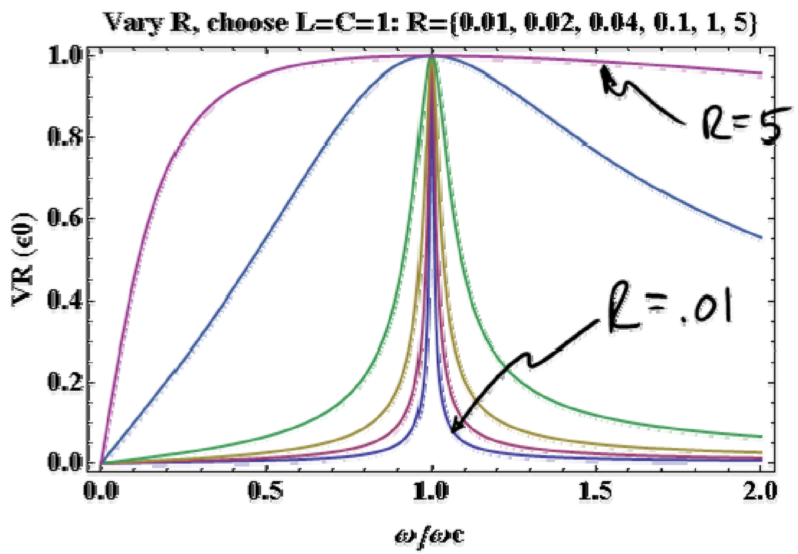
Consider voltage across resistor:

$$V_R = IR = (\mathcal{E}_0 R)\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

V_R is max when $\omega L = \frac{1}{\omega C}$, or $\omega = \sqrt{\frac{1}{LC}} \equiv \omega_c$

$\Rightarrow V_{max} = \mathcal{E}_0$

V_R decreases by increasing or decreasing ω away from ω_c



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$$V_R = (\mathcal{E}_0 R)\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$