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Summary of Links:

The following was copied from the following website:

http://www.upscale.utoronto.ca/GeneralInterest/QM.html

The subpages used are the following:

Many of the listings are roughly in the order in which these topics might be taught.

Topic	Description	Author	Format
Wave-Particle Duality	A brief summary of wave-particle duality, from a first year physics course that uses minimal mathematics; the entire set of materials from the course is available by clicking here. (14k)	Anthony W. Key	html
Quantum Interference	A brief summary of quantum interference and the uncertainty principle, from a first year physics course that uses minimal mathematics; the entire set of materials from the course is available by clicking here. (39k)	Anthony W. Key	html
Double Slit: html pdf	A discussion of the "Feynman double slit," which forms the basis of many discussions of Quantum Mechanics. The topic is quite subtle, but the document is equally accessible to students at all levels. (183k/216k)	David M. Harrison	html and pdf
Schrödinger's Cat html pdf	A very brief introduction, originally designed for upper-year liberal arts students. (31k/34k)	David M. Harrison	html and pdf
Quantum Mechanics: a Poor Person's Guide	An overview of quantum mechanics, from a first year physics course that uses minimal mathematics; the entire set of materials from the course is available by clicking here. (13k)	Anthony W. Key	html
Quantum Mechanics: Interpretation	An overview of quantum mechanics, from a first year physics course that uses minimal mathematics; the entire set of materials from the course is available by clicking here. (10k)	Anthony W. Key	html
Locality and Quantum Mechanics html pdf	A brief introduction to the conflict between local cause and effect and Quantum Mechanics. Based on a discussion in an upper year liberal arts course in physics without mathematics. (24k/39k)	David M. Harrison	html and pdf

Complementarity & Copenhagen Interpretation html pdf	A discussion of Bohr's Principle of Complementarity and its extension to the Copenhagen Interpreation of Quantum Mechnics. Based on a discussion for an upper-year liberal arts course in modern physics without mathematics. (89k/115k)	David M. Harrison	html and pdf
The Development of Quantum Mechanics html pdf	A brief survey of the development of Quantum Mechanics in the 1920's by Schrödinger and Heisenberg. Some of the material is non-traditional. Based on a discussion in an upper year liberal arts course in physics without mathematics. (13k/26k)	David M. Harrison	html and pdf
Stern-Gerlach Experiment html pdf	This classic experiment introduces the notion of quantum spin; it is a vital introduction to many treatments of the "Einstein-Podolsky-Rosen" paradox and to Bell's theorem. This document is equally accessible to students at all levels. (76k/106k)	David M. Harrison	html and pdf
Bell's Theorem html pdf	A derivation of the theorem and a discussion of the consequences. A somewhat subtle topic, but here it is treated in a non-technical fashion. It assumes knowledge of wave-particle duality such as can be found in the *Double Slit* or the *Wave-Particle Duality* documents; also assumed is considerable knowledge of the *Stern-Gerlach Experiment*, for which there is also a document here. (150k/151k)	David M. Harrison	html and pdf
Two analogies to Bell's Theorem html pdf	Two analogies to Bell's Theorem. They are both somwhat simpler than the previous document on Bell's Theorem on this list.(62k/74k)	Daivd M. Harrison	html and pdf
Quantum Teleportation	A discussion of Quantum Teleportation, Information, and Cryptography. Based on a presentation to an upper-year course in modern physics without mathematics. (41k)	David M. Harrison	html
Deterministic Quantum Teleportation	A report of a measurement of quantum teleportation. One of the authors, D.F.V. James, is now at the University of Toronto.	M. Reibe et al.	pdf

This page was last revised (m/d/y) on 09/09/07

The Feynman Double Slit

http://www.upscale.utoronto.ca/GeneralInterest/Harrison/DoubleSlit/DoubleSlit.html

Here we discuss one of the two major paradoxes that we use to introduce Quantum Mechanics. It is the double slit experiment for bullets, water waves and electrons. Although many people have experimented with the systems to be discussed and written about them, Richard Feynman's treatment is so clear that physicists often call it the "Feynman" double slit. At the end, 2 references are given so you may read the "master" on this topic.

With one exception noted below, each section of this "page" depends on the previous sections. Nonetheless, for review purposes you may jump directly to any section by clicking on it in the following Table of Contents:

If you are reading this document on-line, there are a couple of links to Flash animations. To see them requires the Flash player, which is free and available from http://www.macromedia.com/.

Operational Definitions for "Particles" and "Waves"

An "operational definition" is just a well-defined repeatable experimental procedure whose result defines a word or words. For example, one may have wide-ranging discussions of the meaning of the word *intelligence*. An operational definition of intelligence which side-steps these discussions could be:

I administer the Stanford-Binet IQ test to a person and score the result. The person's intelligence is the score on the test.

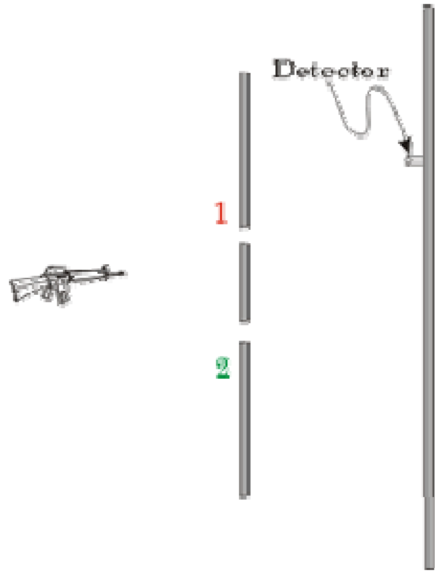
Here we build operational definitions for the words "particles" and "waves."

First we discuss "particles" and will take as our prototype bullets from a machine gun. We have the machine gun, a piece of armor-plate in which two small slits have been cut, labeled"1" and "2", a detector and a solid armor-plate backstop. The detector is quite simple: it is a can in which we have placed some sand. We will turn the gunner loose for, say, a 1 minute burst, and then see how many bullets arrive in the can. We empty the can, and then move it to a different position on the backstop, turn the gunner loose for another 1 minute burst, and see how many bullets have arrived at the new position. By repeating the procedure, we can determine the distribution of bullets arriving at different positions on the backstop.

It turns out the the machine gunner is drunk, so that he is spraying the bullets randomly in all directions.

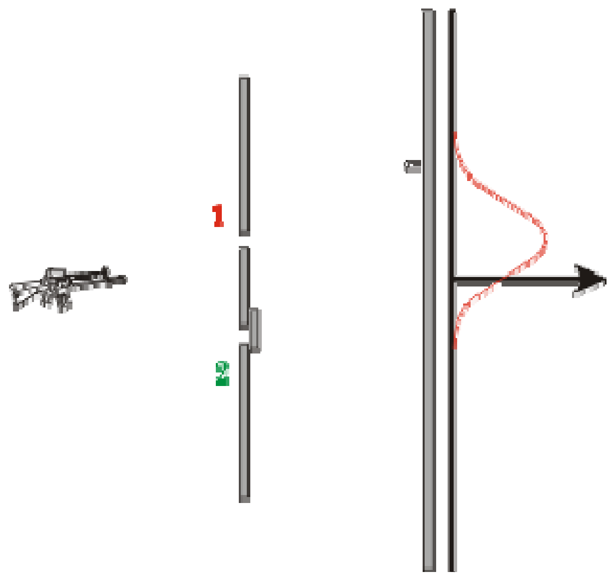
The apparatus is shown to the right.

We will do three different "experiments" with this apparatus.

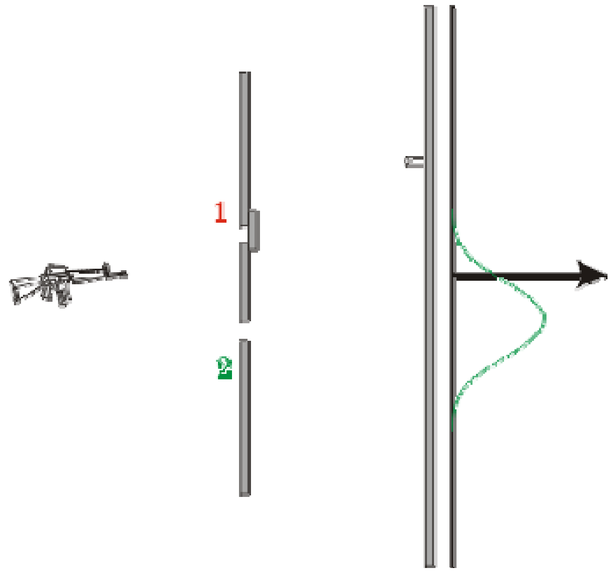


First we close up the lower slit and measure the distribution of bullets arriving at the backstop from the upper slit.

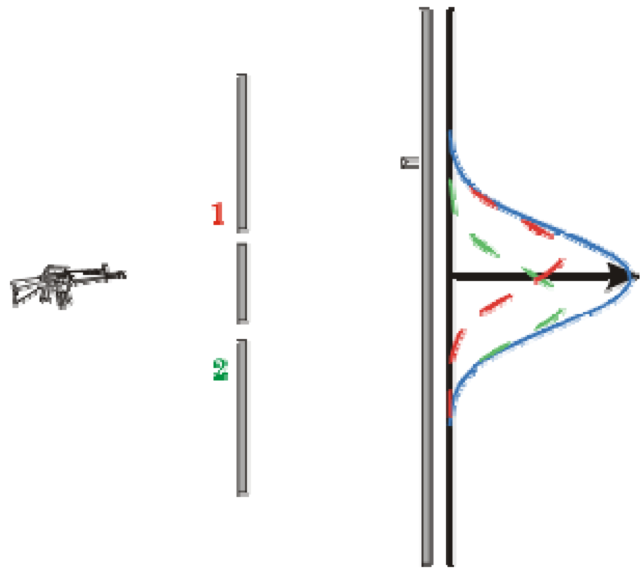
For some bullet sizes and slit widths, although many bullets will go straight through the slit a significant fraction will ricochet off the armor plate. So the distribution of bullets looks as shown by the curve to the right.



Next we close up the upper slit, and measure the distribution of bullets arriving at the backstop from the lower slit. The shape, shown as the curve to the right, is the same as the previous one, but has been shifted down.



Finally, we leave both slits open and measure the distribution of bullets arriving at the backstop from both slits. The result is the solid curve shown to the right. Also shown as dashed lines are the results we just got for bullets from the upper slit and bullets from the lower slit.

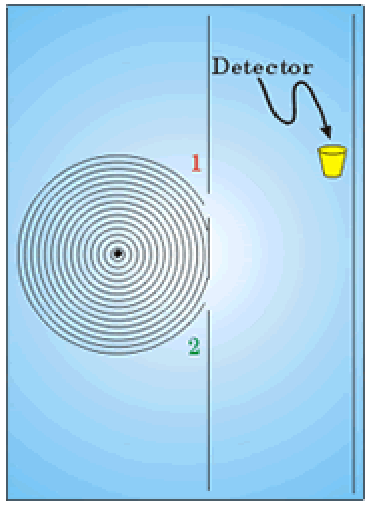


The result is just what you probably have predicted: the number of bullets arriving from both slits is just the sum of the bullets from the upper slit and the bullets from the lower slit.

It will be useful later for you to realize that since the path of a single bullet is random, the distributions we were measuring above are essentially measuring the *probability* that a given bullet will arrive at a particular position at the backstop.

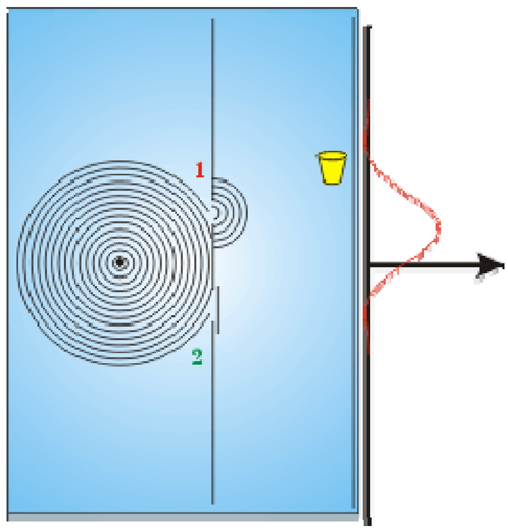
Now we turn our attention to waves. My high school physics teacher had a device called "ripple tank" which is just a tank made of plexiglass which could be filled with water. Various devices would tap the surface of the water, causing water waves to spread out from the device. One may insert slits and other objects in the path of the waves. The whole apparatus was mounted on an overhead projector, so could be used as a class demonstration. My teacher absolutely *loved* his ripple tank, so physics class was basically water-play. I don't know quite why he was so enamored with the device or what he expected us to learn from it, but to this day when I think of a prototype wave I think of water waves in a ripple tank. So we will repeat the double slit experiments we just did in a ripple tank.

First we show the apparatus. The thing that is tapping the surface of the water is the little black circle in the middle of all the concentric circles. The concentric circles are the water waves spreading out away from the source. Just as before we have two slits and a backstop. Just in front of the backstop is our "detector", which is just a cork floating on the surface of the water. Se we measure how much the cork bobs up and down and determine the amount of wave energy arriving at that position at the backstop. Moving the cork to other positions will allow us the determine the distribution of wave energy at the backstop.

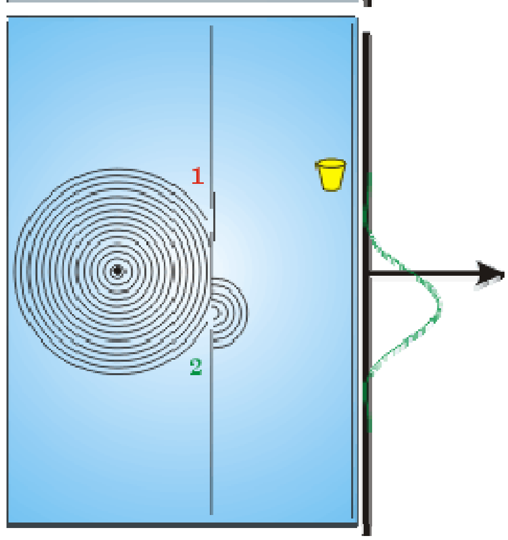


Now we close up the lower slit, and measure the distribution of wave energy arriving at the backstop just from the upper slit. For some combinations of slit width and wavelength, there will be significant spreading of the wave after it passes through the slit. If you have ever observed surf coming in through a relatively small slit in a seawall, you may have observed this.

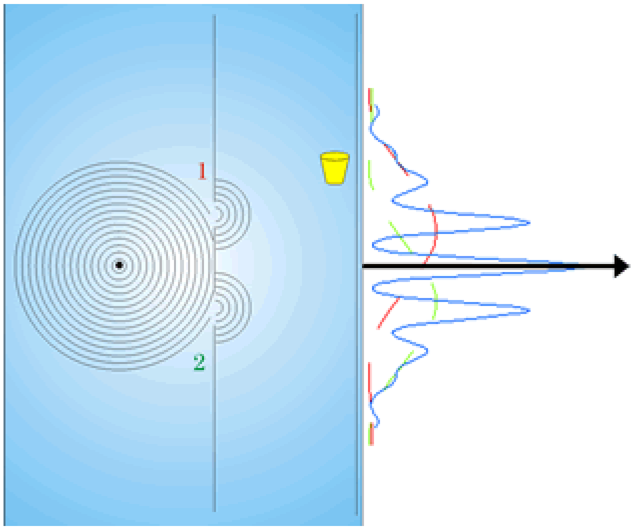
The distribution is shown by the curve to the right. Note that it is very similar to the distribution of bullets from a single slit.



Now we close the upper slit and measure the distribution of wave energy arriving from the lower slit, as shown to the right.



Finally, we leave both slits open and measure the distribution. The result is shown to the right. As we did for the bullets, the dashed lines show the results we just obtained for the distribution from the upper and lower slits alone, while the solid line is the result for both slits open.



This looks nothing like the result for bullets. There are places where the total wave energy is much greater than the sum from the two slits, and other places where the energy is almost zero.

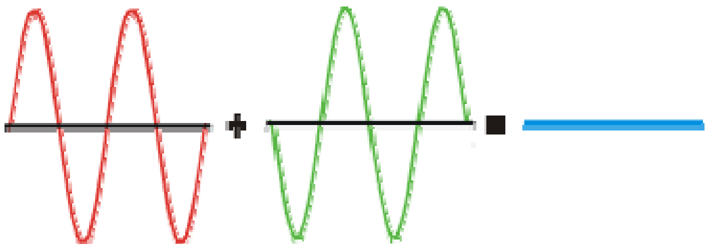
Such a distribution is called an *interference pattern*.

This completes the "operational definition" that we need to define waves and particles. In the two slit experiment, a **particle** does not show an interference pattern and the probability of a particle arriving at a location at the backstop with both slits open is just the sum of the probability of it arriving through the upper slit plus the probability of it arriving through the lower slit. A **wave** shows an interference pattern.

If you think about conservation of energy, you may worry a bit about the interference pattern for waves. There is no problem. The total energy in the interference pattern is equal to the energy arriving from the upper slit plus the energy arriving from the lower slit: the interference pattern re-arranges the energy but conserves the total amount of energy.

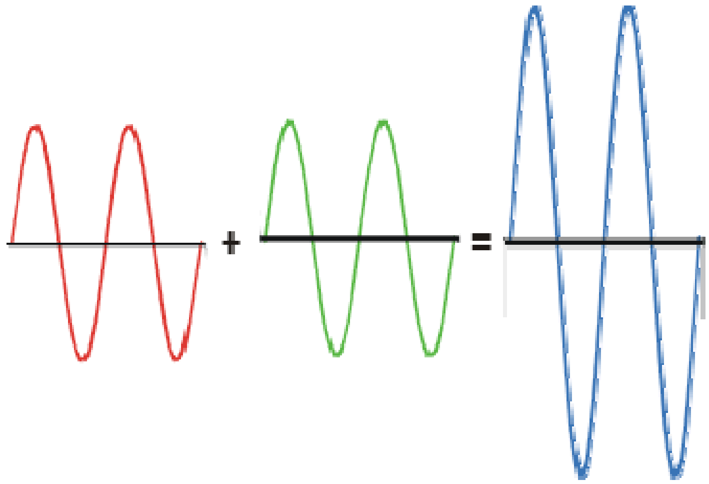
We can explain the interference pattern for waves. When the two waves from the two slits arrive at some position at the backstop, except for right in the middle they will have traveled different distances from the slits. This means that their "waving" may not be in sync.

The figure to the right shows two waves totally "out of phase" with each other. Their sum is always zero.



This is basically what is happening at the *minima* in the interference pattern.

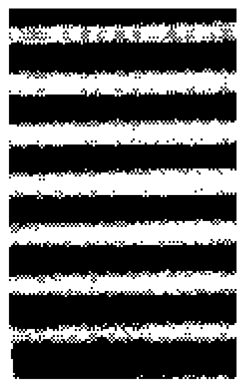
The figure to the right shows the two waves in phase. The total wave is the sum of the two. This is what is occurring at the *maxima* in the interference pattern.



The Two Slit Experiment for Light

In ancient Greece there was a controversy about the nature of light. Euclid, Ptolemy and others thought that "light" was some sort of ray that travels from the eye to the observed object. The atomists and Aristotle assumed the reverse. Nearly 800 years after Ptolemy, circa 965 CE, in Basra in what is now Iraq, Abu Ali al-Hasan Ibn al-Haytham (Alhazen) settled the controversy with a clever argument. He said that if you look at the Sun for a long time you will burn your eyes: this is only possible if the light is coming from the Sun to our eyes, not vice versa.

In 1672 another controversy erupted over the nature of light: Newton argued that light was some sort of a particle, so that light from the sun reaches the earth because these particles could travel through the vacuum. Hooke and Huygens argued that light was some sort of wave. In 1801 Thomas Young put the matter to experimental test by doing a double slit experiment for light. The result was an interference pattern. Thus, Newton was wrong: light is a wave. The figure shows an actual result from the double slit experiment for light.

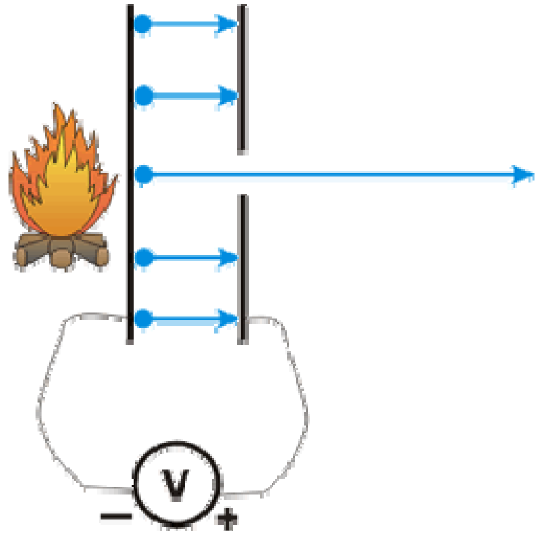


Of course, we haven't said anything about what is "waving" or in what medium it is waving. But, in terms of our operational definition it is clear that light is a wave of something.

Electron Guns

An electron gun, such as in a television picture tube, generates a beam of electrons. In this section we discuss how it works. These details are not important for our primary purpose here, so you may jump to the next section by clicking here.

A diagram of an electron gun appears to the right. There are two vertical metal plates; the right hand plate has a small hole cut in it. A voltage source, indicated by **V**, maintains a voltage across the plates, with the left hand plate negative and the right hand plate positive.



When a metal plate is heated, a process called *thermionic emission* literally boils electrons off the surface of the metal. Normally the electrons only make it a fraction of a millimeter away; this is because when the electron boiled off the surface of the metal, it left that part of the plate with a net positive electric charge which pulls the electron right back into the plate.

In the figure, we are heating up the left hand plate so thermionic electrons will be boiled off the surface. But because of the voltage difference being maintained across the plate, electrons that boil off between the two plates do not fall back into the plate, but instead are attracted to the right hand positive plate. Most of the electrons crash into the positive plate, as shown. However, the electron in the middle would have crashed into the plate except that we have cut a hole in that part of it. So we get a beam of electrons out of this "electron gun."

In real electron guns, such as at the back of a TV picture tube, the negative plate is not heated with a campfire as in our figure. Instead, a small filament of wire has a current passed through it. The filament heats up, glows red, and heats up the negative plate. You may have seen that red glow in the back of a TV picture tube.

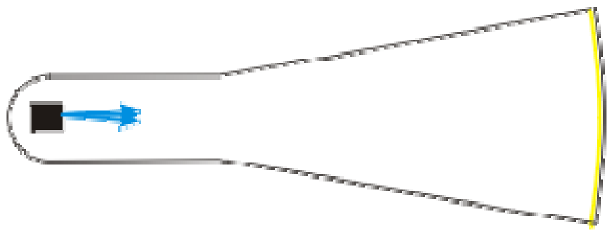
We control the speed of the electrons in the beam with the voltage, and the number of electrons by how hot we make the negatively charged plate.

One more small point. Because the hole in the right hand plate is not of zero size, electrons can emerge in directions slightly away from perfectly horizontal. Thus, the beam of electrons will tend to "spray" somewhat.

From now on we will put the electron gun in a black box, and represent the electron beam coming from it as shown to the right.

The Two Slit Experiment for Electrons

In the previous section we discussed how to produce a beam of electrons from an electron gun. Here we place the electron gun inside a glass tube that has had all the air evacuated. The right hand glass screen has its inside coated with a phosphor that will produce a small burst of light when an electron strikes it. In a TV picture tube, for example, fields direct the beam of electrons to the desired location, the intensities of the electrons are varied depending on where we are steering the beam, and our minds and/or eyes interpret the flashes as the image we are seeing on the television.

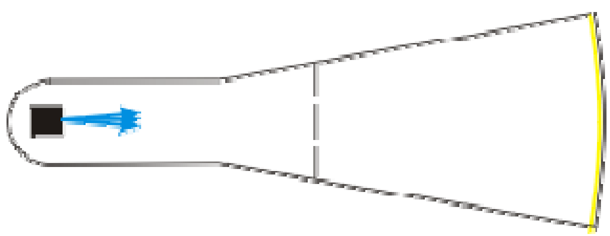


Now, "everybody knows" that electrons are particles. They have a well defined mass, electric charge, etc. Some of those properties are listed to the right. Waves do not have well defined masses etc.

Property	Value
Mass	9.11×10^{-31} kg
Electric Charge	1.60×10^{-19} Coulombs
Spin angular momentum	5.28×10^{-35} Joule-seconds

When an electron leaves the electron gun, a fraction of a second later a flash of light appears on the screen indicating where it landed. A wave behaves differently: when a wave leaves the source, it spreads out distributing its energy in a pattern as discussed at the beginning of this document.

Except, when we place two slits in the path of the electrons, as shown, on the screen we see an interference pattern! In fact, what we see on the screen looks identical to the double slit interference pattern for light that we saw earlier.



If this seems very mysterious, you are not alone. Understanding what is going on here is in some sense equivalent to understanding Quantum Mechanics. I do not understand Quantum Mechanics. Feynman admitted that he never understood Quantum Mechanics. It may be true that *nobody* can understand Quantum Mechanics in the usual meaning of the word "understand."

We will now extend our understanding of our lack of understanding. One possibility about the origins of the interference pattern is that the electrons going through the upper slit are somehow interacting with the electrons going through the lower slit. Note that we have no idea what such a mechanism could be, but are a little desperate to understand what is going on here. We can explore this idea by slowing down the rate of electrons from the gun so that only one electron at a time is in the system. What we do is fire an electron, see where the flash of light occurs on the phosphor screen, wait a while for everything to settle down, then fire another electron, noting where it lands on the screen.

After we have fired a large number of electrons, we will discover that the distribution of electrons is still the interference pattern.

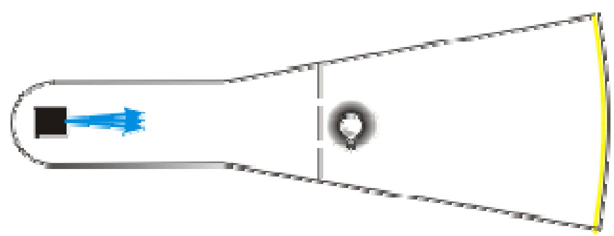
I have prepared a small Flash animation that simulates this result. You may access the animation by clicking on the red button to the right. The file size is 6.4k. You may get the Flash player free from http://www.macromedia.com/; our animation is for Version 5 or later of the player.

You may wish to know that in the animation, the position of the electron is generated randomly using a *Monte Carlo* technique. Thus, if you "Rewind" the animation to start it over, the build-up of the histogram is almost certain to not be identical to the previous "trial."

We conclude that whatever is going on to cause the interference pattern does not involve two or more electrons interacting with each other. And yet, with one electron at a time in the system, with both slits open there are places on the screen where the electrons do not go, although with only one slit open some electrons do end up at that position.

Now, to get an interference pattern we take a wave, split it up into two parts, send each part through one of the slits, and then recombine the waves. Does this mean that a single electron is somehow going through both slits at once? This too is amenable to experimental test.

The result of doing the test turns out to be independent of the details of how the experiment is done, so we shall imagine a very simple arrangement: we place a light bulb behind the slits and look to see what is going on. Note that in a real experiment, the light bulb would have to be smaller than in the figure and tucked in more tightly behind the slits so that the electrons don't collide with it.

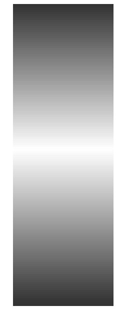


We will see a small flash of light when an electron passes through the slits.

What we see is that every electron is acting completely "normal": one-half the electrons are going through the upper slit, one-half are going through the lower slit, and which is going to be the case for a given electron appears to be random. A small (24k) gif animation of what we might see in this experiment may be seen here.

But meanwhile, we have a colleague watching the flashes of light on the phosphor coated screen who says "Hey, the interference pattern has just gone away!" And in fact the distribution of electrons on the screen is now exactly the same as the distribution of machine gun bullets that we saw above.

The figure to the right is what our colleague sees on the screen.



Evidently, when we look at what is going on at the slits we cause a qualitative and irreversible change in the behavior of the electrons. This is usually called the "Heisenberg Uncertainty Principle."

Everyone has always known that doing any measurement on any system causes a disturbance in the system. The classical paradigm has been that at least in principle the disturbance can be minimised to the point that it is negligible.

Is it possible to minimise the disturbance being caused by the light bulb? We can turn down the intensity of the light it is emitting. However, if we try it, just at the point that the light is getting so faint that we are missing some of the electrons, the interference pattern starts to come back! In fact, if the light intensity is, say, such that we are missing one-half of the electrons, we have one-half an interference pattern and one-half a particle distribution. So this attempt to minimise the disturbance didn't work out: we still don't know what is going on at the slits when we see the interference pattern.

There is yet another way to minimise the disturbance. The light contains energy, and it turns out that if we increase the wavelength of the light, towards the infrared, the energy of each part of the light goes down. Perhaps if we decrease the energy in the light we won't be scattering it off the electrons so violently. So, we start increasing the wavelength of the light emitted by the light bulb. We continue to see all the electrons, and at first we always see that one-half of them are going through the upper slit and one-half are going through the lower slit.

However, our ability to resolve two positions in space by looking depends on the wavelength of the light that we are seeing with. And just at the point that the wavelength of the light from the lightbulb gets so large that although we can see the electrons we can't tell which slit they went through, the interference pattern comes back.

A student once remarked that we should do a "better" experiment. The Heisenberg Uncertainty Principle says that such a better experiment does not exist. Einstein in particular devoted a lot of time trying to devise such a better measurement; all his attempts failed.

The conclusion of all this is that there is **no** experiment that can tell us what the electrons are doing at the slits that does not also destroy the interference pattern. This seems to imply that there is **no** answer to the question of what is going on at the slits when we see the interference pattern. The path of the electron from the electron gun to the screen is not knowable when we see the interference pattern. As Heisenberg said, "The path [of the electron] comes into existence only when we observe it."

We will be discussing interpretations of what all this may mean in great detail later. For now I will briefly mention a "standard" if incomplete interpretation. If we think that the probability of where the electron is in space is a wave, then when we don't look the probability wave has two pieces at the slits, representing the fact that there is a 50% chance the electron went through the upper slit and a 50% chance it went through the lower slit. These two probability waves from the two slits, then, recombine at the screen and cause the interference pattern.

When we look, we "collapse the state" in a 100% chance it went through one slit and a 0% chance it went through the other. And in this circumstance the two probability waves for the two slits cannot then recombine at the screen to cause an interference pattern: for each electron there is only one non-zero probability wave.

Finally, then, we have two contradictory yet complementary models of the two-slit experiment for electrons. In one model the electron is a particle that somehow exhibits an interference pattern. In the other model, the electron is a wave that somehow manifests as a particle whenever we look at it.

A Flash animation of these two models, both incomplete, may be accessed by clicking the red button to the right. The file size is 23k and will appear in a separate window.

Click Me

References

- Richard Feynman, **The Character of Physical Law** (MIT, 1965), Chapter 6
- Richard P. Feynman, Robert B. Leighton and Matthew Sands, **The Feynman Lectures on Physics** (Addison-Wesley, 1963), Vol III, Chapter 1

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Schrödinger's Cat

http://www.upscale.utoronto.ca/GeneralInterest/Harrison/SchrodCat/SchrodCat.html

Radioactive Decay

Some elements are unstable, decaying via the weak interaction into other elements. Such substances are called *radioactive*. For example, Nitrogen-13 decays into Carbon-13 plus an electron plus an anti-neutrino.

There are two factors that determine that rate at which a sample of radioactive atoms decays:

1. How many atoms are there? Twice as many atoms will have a total decay rate that is double.
2. What is the tendency of a particular atom to decay.

The tendency of an element to decay is expressed by its *half-life*.

The half-life of Nitrogen-13 is almost exactly 10 minutes. What this number means is that if we have a large "pot" of Nitrogen-13 and wait 10 minutes one half of the Nitrogen-13 atoms will have decayed and one half will not have decayed. If we wait a further 10 minutes one half of the remaining sample of atoms will have decayed and one half will not. After a further 10 minutes one half of that remaining sample will have decayed.

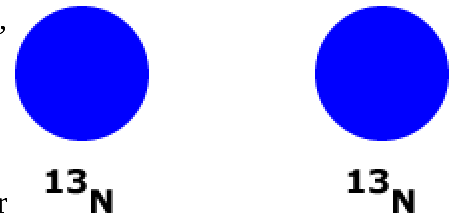
The concept of half-live is sort of reminiscent of a famous paradox by Xeno, who argued that we can never walk out of a room. If we head towards the door, eventually we get half way towards it. If we keep walking we will have gotten half way through the remaining distance to the door. And then if we keep walking we get half way through that remaining distance. So we keep getting half way towards the door and never actually get there.

Here are some half-lives:

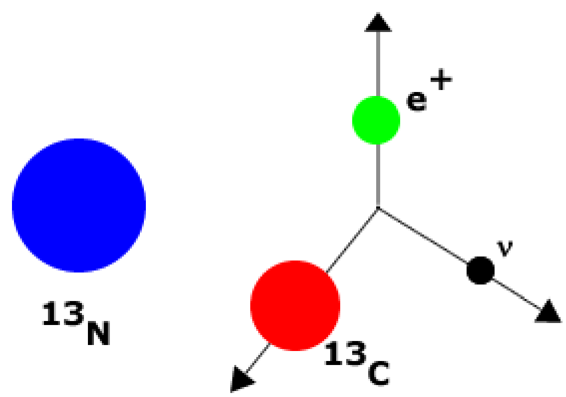
Atom	Half-Life
protons	> 10^{32} years (i.e. consistent with being stable
Carbon-14	5,730 years
Cobalt-60	1,925 days
Nitrogen-13	10 minutes

You may access a Flash animation of 500 radioactive atoms of the fictitious element *Balonuim* by clicking here.

For radioactive substances, one crucial factor in discussing the half-life is that we were discussing a *large* collection of atoms. What if we only have two such Nitrogen-13 atoms? Then if we wait 10 minutes, one-half life, there is a 50% chance that one of the atoms will have decayed. So this is sort of similar to flipping two coins. Whether a particular coin comes up heads is about a 50% chance. For flipping two coins we can get both heads, one head and one tail, or both tails. Similarly for two radioactive atoms we could end up with both decaying, one decaying and the other not, or both not decaying.



Imagine that after 10 minutes the atom on the right decayed and the one on the left did not decay.

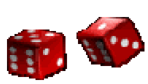


We ask a basic question: **What is the difference between the two Nitrogen-13 atoms?**

The answer to this is trivially easy: one atom decayed and the other did not.

A more interesting question is: **What *was* the difference between the two Nitrogen-13 atoms before we waited 10 minutes?**

The answer to this better question is sort of hard. According to Quantum Mechanics there was no difference between the two atoms: we had two completely identical atoms but one decayed and the other did not.



Einstein never accepted Quantum Mechanics, and this part of the theory is one of the reasons. He summarised his objections by saying "God does not play at dice with the universe." Bohr responded "Quit telling God what to do!"

Einstein's God may not play at dice, but there are other views of divinity. For example, in the **Bhagavad Gita** Krishna says:

"I am the game of dice. I am the *self* centered in the heart of all beings."

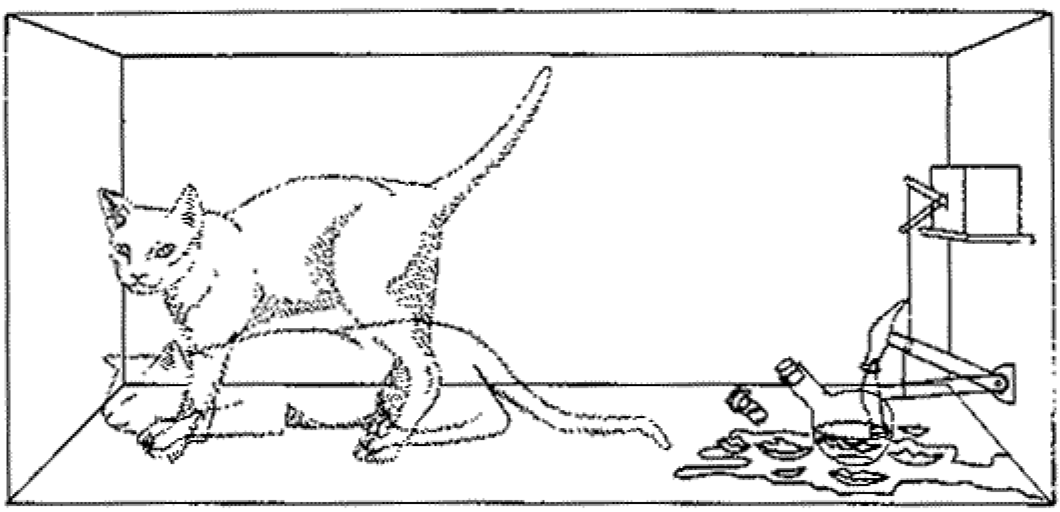
If, with Einstein, we reject the idea that completely identical initial states can evolve to different outcomes, then we conclude that initially there must have been some difference, some variable, that distinguishes the two Nitrogen-13 atoms. To date all attempts to discover what that variable is have failed; thus we would say that there is some *hidden variable* inside the atoms. In Quantum Mechanics there are no such variables.

The Cat Paradox

In the early 1930's Erwin Schrödinger published a way of thinking about the circumstance of radioactive decay that is still useful. We imagine an apparatus containing just one Nitrogen-13 atom and a detector that will respond when the atom decays. Connected to the detector is a relay connected to a hammer, and when the atom decays the relay releases the hammer which then falls on a glass vial containing poison gas. We take the entire apparatus and put it in a box. We also place a cat in the box, close the lid, and wait 10 minutes.

We then ask: **Is the cat alive or dead?**

The answer according to quantum mechanics is that it is 50% dead and 50% alive.



Quantum Mechanics describes the world in terms of a *wave function*. DeWitt wrote about the cat that "at the end of [one half-life] the total wave function for the system will have a form in which the living cat and dead cat are mixed in equal portions." (Reference: B.S. DeWitt and N. Graham, eds., **The Many-Worlds Interpretation of Quantum Mechanics** (Princeton, 1973), pg. 156.)

When we open the box, we "collapse the wave function" or "collapse the state" and have either a live cat or a dead cat.

Of course, this is just a thought experiment. So far as I know nobody has actually every done this experiment.

In a sense the cat is a "red herring" [sorry!]. The paradox is just an illuminating way of thinking about the consequences of radioactive decay being totally random.

Imagine we have a friend waiting outside when we open the box. For us the wave function collapses and we have, say, a live cat. But our friend's wave function does not collapse until he comes into the room. This leads to a strong solipsism, since our friend can they say that we owe our objective existence to his kind intervention in coming into the room and collapsing our state.

As Heisenberg said, then, "The wave function represents partly a fact and partly our knowledge of a fact."

Our friend needn't have come into the room to collapse his wave function: if we have a cell phone we can call him and tell him the result of the experiment. Of course, this assumes that we don't lie to him and tell him the cat is dead when it is alive.

Unexplained but apparently true is the fact that when a state collapses, it collapses into the same state for everybody. If we see a live cat everybody sees a live cat (unless they or us are hallucinating).

As de Beauregard commented: "Finally, the need for consistency of the whole scheme leads me to think of the world we are living in as a Leibnitzian world, where cats are rather high in the hierarchy of monads." Reference: Foundations of Physics **6**, 539 (1976).

Paradoxes of Quantum Mechanics

There are two major paradoxes of Quantum Mechanics, each illustrating different aspects of the quantum mystery. Schrödinger's Cat is one of them, and the other is the *Double Slit*. Notes on the double slit may be accessed either in html or pdf by clicking on the links to the left.

Each paradox shows different aspects of the "collapse of the state."

Double Slit

> **Shows that the collapse of the state is real, irreversible, and causes a qualitative change in the later time evolution of the system.**

Schrödingers Cat

> **Shows that our consciousness and knowledge are somehow mixed up in this process.**
>
> **GJ addition: This last statement is not worded in a way that I agree with. Consciousness and knowledge really have nothing to do with quantum mechanics. The entanglement of the quantum system you are probing (in this case, 13N) with the many quantum states of your probe (in this case, the 13N detector) causes the collapse of the wave function.**

Author

This document was written by David M. Harrison, Department of Physics, University of Toronto, harrison@physics.utoronto.ca in 1999.

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A POOR PERSON'S QUANTUM THEORY

The Wave Function

The observation of the world lets us know that something very odd is going on. The Double Slit Experiment is the prototypical experiment of Quantum Physics. Guided by this experiment and others like it, a mathematical theory, called Quantum Theory has been developed to give results which are in agreement with our observations; the "weirdness" we find in Nature is reflected in the way in which the Quantum Theory is constructed. It goes a bit like this.

The state of a physical system is described by a "wave function", usually denoted by the symbol **Ψ.**. In particular cases, we will know more or less exactly what function this is - e.g a sine or a cosine, a quadratic expression, etc. This wave function can depend on time, spatial coordinates, etc. Quantum Theory tells us that to make calculations about real measurements that could be made on the system, we must **take the square of the wave function.** The value so obtained will give us the **probability** of obtaining, through measurement on the system, a particular value of the quantity we are interested in. We would expect a theory from Classical Physics to give us an **exact** value of the quantity we were interested in; here, however, the best we can do is calculate a probability of obtaining the value. The wave function is also called a Probability Amplitude, for this reason.

For example, if, on the basis of our knowledge of conditions in which a particle might find itself (in a box, with a magnetic field, for example) we knew how to write down the particle's wave function; and let's say this wave function depended on its position (call that x) and the time measured from some starting time (call that t). In that case, we would write its wave function as $\Psi(x,t)$. x could take any values of position that the particle could reach. If then, we wanted to know the chance, or probability, of finding the particle at a particular value of x, say $x = 45$ cm, at a particular time of, say 7 seconds, Quantum Theory tells us that the answer is $\Psi(45,7)^2$. Note that this is very different from Classical Physics; there, we might know that the "equation of motion" of the particle was, e.g. $x = 6t$; then the answer to our question would be that the position of the particle at time 7 seconds, would be the exactly $x = 42$ cm.

Now let us look at the odd way in which Quantum Theory does its calculations about the world. Suppose we have an experiment about a physical process which can happen in more than one way, and we know the Probability Amplitude (or wave function) for each way. To calculate what results we would expect in an experiment which does not distinguish which way actually happens, we have to **first** add the Probability Amplitudes; **then** we square the result of this addition to get the answer to compare to measurement. If, on the other hand, the experiment does distinguish which way actually happens, we **square** the Probability Amplitudes **before** adding them. To see how this works, let's look at the Double Slit Experiment for electrons.

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The Double Slit Experiment Again

Suppose that Ψ_1 is the Probability Amplitude for the electron's going through one slit, and Ψ_2 is the Probability Amplitude for its going through the other slit; then the Probability Amplitude to calculate the results of an experiment which does not determine through which slit the electron goes (call it Experiment **I**) is written as $\Psi_I = \Psi_1 + \Psi_2$. [This is called a "(linear) superposition of probable states"]. Now, if we want to make a theoretical calculation of the results of a real experiment we might carry out (e.g. the distribution of the electrons on the detecting screen), we have to take the **square** of this total Probability Amplitude, i.e. $\Psi_I^2 = \{\Psi_1 + \Psi_2\}^2$. Multiplying out, this result can be written as $\Psi_I^2 = \Psi_1^2 + \Psi_2^2 + 2\Psi_1\Psi_2$. (In this not-quite-correct formulation, Ψ_1 can equal $-\Psi_2$).

Suppose we have a set-up which has equal size slits, located at the same distance from the source of electrons, then the probability that the electron goes through slit number 1 is equal to the probability that it goes through slit number 2. We express this fact by writing $\Psi_1^2 = \Psi_2^2 = 0.5$ (or 50%).

Then : EITHER $\Psi_1 = +\Psi_2$ and the result is **1** (or 100%);
OR $\Psi_1 = -\Psi_2$, and the result is **0** (or 0%).

For the Double Slit Experiment, this is obviously (??) a calculation of the interference pattern, with its maxima (**1**, in some arbitrary units) and minima (**0**) which we observe. However, if our experiment has some means for detecting, even in principle, which hole the electron goes through (Experiment **II**), the result of **this** experiment must be written as $\Psi_{II}^2 = \Psi_1^2 + \Psi_2^2$. This is clearly (??) the case in which **no** interference is observed.

Thus the Quantum Theory has managed to come up with a recipe to give calculations which agree with the observations we make on this weird world in which we live.

What can we say about the wave function (Probability Amplitude) of the electron after it has gone through the slit system, but **just before** we look at it to decide which slit it went through? In this case, Nature tells us we must write its wave function as $\Psi = \Psi_1 + \Psi_2$, as explained above. But if we make a measurement to determine which slit the electron did go through, we know we must get the result Ψ_1 (if it went through slit number 1) OR Ψ_2 (if it went through slit number 2). Then we say that the wavefunction has **collapsed** on to its final value.

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What is an Electron?

According to Schrödinger, the electron can be represented by a wave-function, which contains all the information we can know about the particle. If an electron looks like anything we are familiar with (and it doesn't!!), it comes closest to a small "packet" of waves confined to a region of space Δx. This wavefunction obeys a wave equation first written down by Schrödinger. The **square** of the wavefunct ion gives the probability of finding it at a given place (and time).

(MATH NOTE: To represent such a function, we need a superposition of many wave forms, with a "spread" of wavelengths. Since $\mathbf{p} = \mathbf{h}/\lambda$ this implies a corresponding spread in momentum; this can be calculated to be $\Delta \mathbf{p} = \mathbf{h}/\Delta \mathbf{x}$ - as we might have expected from Heisenberg's Uncertainty Principle).

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Schrödinger's Cat (or Is the Moon There when Nobody Looks?)

By analogy, in the case of Schrödinger's cat, the state of the cat **before we open the box** is : $\Psi_{cat} = \Psi_{alive} + \Psi_{dead}$. If we have designed the experiment so that there is equal probability for finding the cat alive or dead, we must have $\Psi_{alive}^2 = \Psi_{dead}^2$. When we open the box, since the cat must be alive OR dead, the total wave function, Ψ_{cat} must be EITHER = Ψ_{alive} OR = Ψ_{dead}; we don't know which before we open the box. However, it appears that **just before** we open the box, the cat is NEITHER alive OR dead, but a superpostition of the two states! Just try telling that to your grand mother!

The Implications of the Quantum

Quantum Physics forces us to the conclusion that:

a. there are no certainties, only probabilities - and the future is unpredictable.

b.Physical properties have no objective reality independent of the act of observation OR the act of measurement can, in principle, act *instantaneously* over enormous distances (i.e. non- local interactions exist). (*Bell's Theorem and the experiments of Aspect et al.*)

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INTERPRETATIONS OF THE QUANTUM WORLD

Albert Einstein - Einstein objected to the Quantum Theory on several grounds. Firstly it does not seem to give objective reality to individual events; he believed that an objective world exists, independent of any observer or observing process. Yet Quantum theory seems to imply that our method of observation determines what we will see. Secondly it does not seem to be a complete theory; it is essentially statistical in its predictions and cannot completely describe individual quantum events. His other objections were formalised in the Einstein-Podolsky-Rosen paper, and concerned what he called "spooky actions at a distance". (**NOTE:** in this area at least, Einstein seems to have been wrong. Bell's Theorem and the experiments of Aspect et al have proven conclusively that EITHER there is no objective reality OR that these "spooky" non-local interactions exist).

Neils Bohr - *the Copenhagen collapse*. Bohr believed that the wave function represents our knowledge of the physical phenomena we are studying, not the phenomena itself. In this sense, it is a potential which is realised only when we make an observation; this observation causes the wave function to "collapse" into the actual manifestation of the route taken.

David Bohm - *A Higher Multi-Dimensional Order*. In his book "Wholeness and the Implicate Order" Bohm suggests that the strange effects of the Quantum world may imply the existence of a a deeper, non-local level of reality. At this level - called the implicate order - all things are interconnected in an unbroken whole; "everything interpenetrates everything". Our observational world - which Bohm calls the explicate order - has access to this underlying reality in only a partial and incomplete fashion. Bohm's view has been likened to the suggestion that the Universe is a multi-dimensional hologram; any little piece of the hologram will recover the image, but not the full reality. We are reminded of Blake's wish - "to see the world in a grain of sand".

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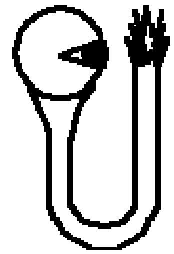
Eugene Wigner - *Human consciousness*. Wigner goes even further than Bohm by claiming that it is the entry of human consciousness into the picture that causes the wave function to collapse.

The Cartesian mind-body dualism is re-established and the influence of the mind on the physical world is explicit. Wigner believes that the Newtonian concept of action-reaction and quantum physics both are evidence for this belief.



John Wheeler - *The Participatory Universe*. The renowned mathematician, John von Neumann was also an adherent to this view, which claims that the universe does not exist until a human mind is there to observe it. In this view, the universe is a self-observing system; the early stages of the universe can be promoted to concrete reality through its later observation by conscious ness, which itself depends on that reality (!!)

Hugh Everett and Bryce de Witt - *The Many Worlds Interpretation*. Far-fetched though this sounds it provides one of the cleanest explanations of the wave function collapse. The idea is that at each observation of the world ALL possibilities allowed by the wave function of the system are actually realised. The universe splits into branches, each corresponding to one of the possibilities available to it. Each branch is completely independent of the others, and no communication can take place between branches.

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Locality and Quantum Mechanics

Author

This document was written in March 2000 by David M. Harrison, Department of Physics, University of Toronto, mailto:harrison@physics.utoronto.ca. This is version 1.4, date (m/d/y) 03/12/02.

Introduction

You may be aware of the fact that Einstein never accepted Quantum Mechanics. He explained his objections by discussing two particular aspects of the theory. One was the fact that the theory is probabilistic, and seems to imply that the future is random. Einstein repeatedly said "God does not play dice with the universe," to which Bohr responded "Quit telling God what to do!" The probabilistic nature of cause and effect in Quantum Mechanics is pinpointed particularly well by the Schrödinger's Cat paradox; a document on this topic is available here.

The second aspect of Quantum Mechanics that greatly bothered Einstein is what he called a "spooky action at a distance" implied by the theory. Often this aspect of the theory is characterised as *non-locality*.

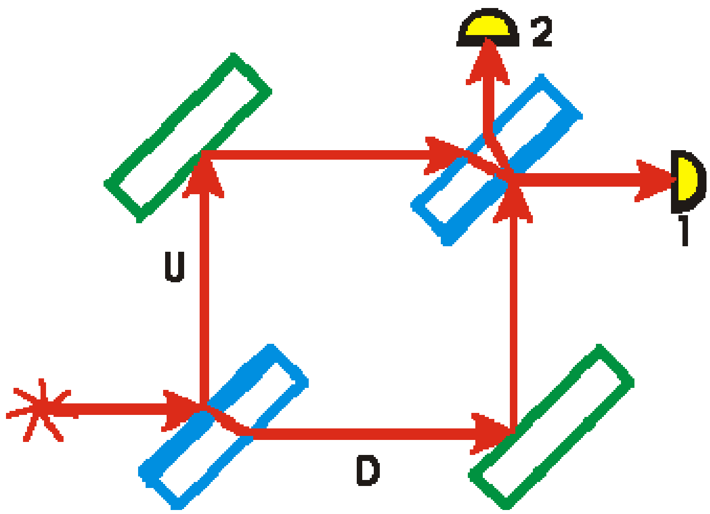
Locality means the reasonable assumption that no signal can travel faster than the speed of light. This imposes constraints on cause and effect. Thus, if we send a signal travelling at light speed to Alpha Centarui, which is 4.5 light years away from us, that signal will have no effect on Alpha Centauri for 4.5 years: locality says it is impossible to cause some effect on Alpha Centauri any faster than this. If we send a signal at light speed to the other side of a room which is about 10 meters away, the signal can have an effect in about 3 billionths of a second: the other side of the room is more local than Alpha Centauri.

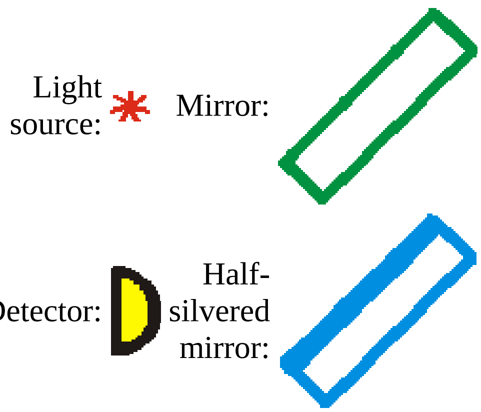
As we shall see, locality is in some conflict with Quantum Mechanics.To illustrate the conflict we will only need the fact that we can view phenomena such as light as both a wave and as a particle.

The following discussion is at the level of an upper year liberal arts course in modern physics without mathematics that is given at the University of Toronto.

Mach-Zehnder Interferometer

We shall use the *Mach-Zehnder Interferometer* to illustrate non-locality. The device, co-invented by Ernst Mach, the "grandfather" of the Theories of Relativity, is shown to the right:





The legend for the figure is shown to the left.

Recall that a "half-silvered mirror" is a mirror that only reflects one-half of the light incident on it; the other half is transmitted through the mirror. In the figure, the reflecting surface is drawn as the thick one.

Light leaves the source and travels to the first half-silvered mirror. One half of the light is reflected as the upper *U* beam, which is reflected by the upper-left mirror, and travels to the upper-right half-silvered mirror. There, one half of the beam is transmitted to Detector *1*, and the other half is reflected into Detector *2*.

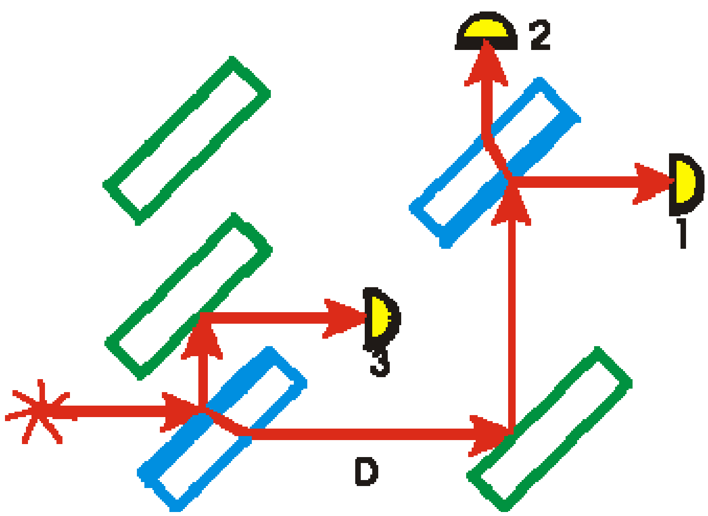
From the lower-left half-silvered mirror, the lower *D* beam is reflected by the lower-right mirror, and travels to the upper-right half-silvered mirror. There, one half of the beam is reflected to Detector *1*, and the other half is transmitted into Detector *2*

It turns out that, despite contrary appearances in the figure, all of the light that leaves the source ends up in Detector *1*; no light enters Detector *2*. What is happening is that the two beams, *U* and *D*, constructively interfere at Detector *1* and destructively interfere at Detector *2*. The details of why this is so are sort of complex and not important for our purpose here. Those details are related to the fact that when light goes from one medium to another various "phase changes" occur; a document going through these complexities is available here.

This type of interferometer is still in regular use in laboratories around the world. It turns out that the balance of constructive and destructive interference at the detectors is extremely sensitive to any phase changes in the two beams of light. Thus, by inserting, say, a gas sample into the path of one of the beams, the additional phase shift caused by the gas allows the deduction of information on the density, pressure and temperature of the gas by observing the changes in intensity of the signals arriving at the two detectors.

Another Interferometer Arrangement

Now consider the arrangement to the right. A third mirror deflects all of the upper *U* beam to a third detector. Now there can not be any interference effects at detectors 1 and 2 because there is only one beam reaching the upper-right half-silvered mirror. This beam is made up of one-half of the light from the source; the other half is reflected by the lower-left half-silvered mirror and ends up in detector 3.



In summary, for this arrangement, the percentages of the light leaving the source that arrive at the detectors are:

Detector	**Percentage of Light from Source**
1	25%
2	25%
3	50%

Now we begin to think of the light in the interferometer as *photons*, its particulate aspect. In the arrangement discussed in this section, 25% of the photons that leave the source end up in detector 2. Think for a moment about one of those photons. It travels along the lower path *D* and ends up in the detector. It can only end up in that detector if the third mirror is deflecting the *U* beam. But how did that photon "know" whether or not the *U* beam was being deflected? It was never anywhere near the third mirror.

It is here that we see a hint of non-locality. The existence of a third mirror that is deflecting the *U* beam has an immediate non-local effect on photons that were never near that mirror.

This non-locality is consistent with Quantum Mechanics, and can be demonstrated in other circumstances. For example, we consider the double slit experiment for electrons; a document on this topic is available here. There are positions at the observing screen where electrons will not go when both slits are open, the *minima* in the interference pattern. But with only one slit open, some electrons do go to that position on the screen. So if an electron goes through, say, the upper slit it seems to "know" whether or not the lower slit is open, so it knows whether or not it can go to one of the positions of the minima in the interference pattern.

Conclusion

The potential conflict between locality and Quantum Mechanics has been known since at least the early 1930's, and was the focus of a famous paper by Einstein, Pololsky and Rosen (EPR) in 1934. In that paper, they concluded that Quantum Mechanics must be at least incomplete.

The possible non-local and/or probabilistic nature of cause and effect is explored more deeply by Bell's Theorem of 1964 and its subsequent experimental tests. A document on Bell's Theorem is available here

Both for the material discussed here and especially in commentary about Bell's Theorem, one sometimes sees statements that according to Quantum Mechanics one may transmit *information* at speeds greater than the speed of light. I have never seen such an argument that I believe is correct. Whatever is being transmitted at superluminal speeds is somewhat less than information; d'Espagnat uses the word *influence*.

If we have some influence or even information being transmitted at superluminal speed from *A* to *B*, then according to the Special Theory of Relativity there are reference frames where the influence is travelling from B to A; the influence is still travelling faster than the speed of light with respect to all observers. The conclusion is that any superluminal influence has to be viewed as a *connection* between A and B, and identifying which is the cause and which is the effect is problematic.

A JPU200Y student recently made a startling suggestion that the *influence* can be travelling at superluminal speeds via a mini-blackhole wormhole connection, ie. through the *quantum foam* that we have seen pervades spacetime at very small distances.

Complementarity and the Copenhagen Interpretation of Quantum Mechanics

Click here to go to the *UPSCALE* home page.
Click here to go to the Physics Virtual Bookshelf home page.

Introduction

Neils Bohr (1885 - 1962) was one of the giants in the development of Quantum Mechanics. He is best known for:

1. The development of the *Bohr Model of the Atom* in 1913. A small document on this topic is available here.
2. The principle of *Complementarity*, the "heart" of Bohr's search for the significance of the quantum idea. This principle led him to:
3. The *Copenhagen Interpretation of Quantum Mechanics*.

In this document we discuss Complementarity and then the Copenhagen Interpretation.

But first we shall briefly discuss the general issue of interpretations of Quantum Mechanics, and briefly describe two interpretations. The discussion assumes some knowledge of the *Feynman Double Slit*, such as is discussed here; it also assumes some knowledge of *Schrödinger's Cat*, such as is discussed here. Finally, further discussion of interpretations of Quantum Mechanics can be meaningfully given with some knowledge of *Bell's Theorem*; a document on that topic is here.

The level of discussion in what follows is based on an upper-year liberal arts course in modern physics without mathematics given at the University of Toronto. In that context, the discussion of Bell's Theorem mentioned in the previous paragraph is deferred until later.

A recommended reference on the material discussed below is:

F. David Peat, **Einstein's Moon** (Contemporary Books, 1990), ISBN 0-8092-4512-4 (cloth), 0-8092-3965-5 (paper).

Interpretations of Quantum Mechanics

Although the basic mathematical formalism of Quantum Mechanics was developed independently by Heisenberg and Schrödinger in 1926, a full and accepted interpretation of what that mathematics means still eludes us. If we found such an interpretation, then in some sense we could claim to understand Quantum Mechanics.

Richard Feynman stated that he never understood Quantum Mechanics. Certainly the author of this document does not understand Quantum Mechanics. This may be because Quantum Mechanics is not understandable, at least in the usual sense of the meaning of the word *understandable*.

Regrettably, some physicists claim that it is not important whether or not we understand Quantum Mechanics: what is important is that we know how to manipulate the mathematical formalism to get answers to our quantitative questions.

Here are some statements by physicists that take the opposite position on understanding:

- "Never make a calculation until you know the answer." -- Wheeler, **Spacetime Physics**, pg 60.
- "Our mathematical procedures seem to obscure our intuitive and imaginative understanding." -- Bohm, Foundations of Physics **5**, 93 (1975).
- I feel that we do not have definite physical concepts at all if we just apply working mathematical rules; that's not what the physicist should be satisfied with." -- Dirac, **Physicist's Conception of Nature**, pg 11.

In any case, the typical education of a physicist tends to ignore the issue of interpretations.

To the extent that the usual course in Quantum Mechanics for physics students discusses interpretations at all, it usually presents only a simple probability view. Quantum Mechanics describes the world in terms of a "wave function" or "state function." When we see, say, electrons in a two slit experiment forming an interference pattern, we say that a wave has split up, gone through the two slits, and then re-combined. This is the normal way of explaining two slit interference of any type of wave. The Quantum Mechanical wave, the wave function, is interpreted as being the amplitude of the probability of finding the electron at some position in space. Thus, when we don't look at what is happening at the slits, there is a 50% chance a given electron went through the upper slit and a 50% chance it went through the lower slit. Thus the wave function has an amplitude at both slits, and then when later the wave functions re-combine we get interference. If we set up an experiment at the slits to see what the electrons are doing, we see each electron going through either the upper slit or through the lower slit, never through both slits at once. But the process of doing this measurement "collapses the wave function" so that it has a non-zero amplitude only at the slit where we see the electron. And it is this collapse that destroys the interference pattern.

Similarly, before we look the Quantum Mechanical description of Schrödinger's Cat states that after one half-life the cat is 50% alive and 50% dead. When we open the box and look we similarly "collapse the state."

Deeper consideration of this "interpretation" will quickly lead to the conclusion that it is at least incomplete.

Another interpretation of Quantum Mechanics was devised by Hugh Everett, III as a PhD thesis when he was a graduate student of John Wheeler at Princeton in 1957. The thesis itself was nine pages in length, which is about typical for the length of the *bibliography* of a typical PhD thesis in physics.

In Everett's interpretation when we, say, look in the box and find that Schrödinger's Cat is alive, that measurement has created a parallel universe where we found that the cat is dead. And similarly, every conscious act of perception bifurcates the universe.

In this view, then, when we find, say, a live cat the apparent fixed outcome is *illusion*, because we have created another parallel universe where we found a dead cat. And the totality is both universes, one with a live cat and the other with a dead one.

Borges' **A Garden of Forking Paths** evokes images reminiscent of the many-worlds interpretation:

" ... a picture, incomplete yet not false, of the universe as Ts'ui Pen conceived it to be. Differing from Newton and Schopenhauer, ... [he] did not think of time as absolute and uniform. He believed in an infinite series of times, in a dizzily growing, every spreading network of diverging, converging and parallel times. This web of time - the strands of which approach one another, bifurcate, intersect or ignore each other through the centuries - embraces *every* possibility. We do not exist in most of them. In some you exist and not I, while in others I do, and you do not, and in yet others both of us exist. In this one, in which chance has favored me, you have come to my gate. In another, you, crossing the garden, have found me dead. In yet another, I say these very same words, but am an error, a phantom."

Complementarity

Here is a favorite statement of Bohr's Principle of Complementarity, based on so-called *wave-particle duality* for light:

"But what is light really? Is it a wave or a shower of photons? There seems no likelihood for forming a consistent description of the phenomena of light by a choice of only one of the two languages. It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do." -- Albert Einstein and Leopold Infeld, **The Evolution of Physics**, pg. 262-263.

Incidentally, I have been told that Infeld wrote the entire book **The Evolution of Physics** in 1938, but was experiencing difficulty in getting anyone to publish it. Once Einstein put his name on it, all such difficulties disappeared.

John Wheeler, with his usual insight and striking prose, neatly summarises the status of the principle:

"Bohr's principle of complementarity is the most revolutionary scientific concept of this century and the heart of his fifty-year search for the full significance of the quantum idea." -- Physics Today **16**, (Jan 1963), pg. 30.

A nice analogy is *Figure-Ground* studies such as the one shown to the right. Looked at one way, it is a drawing of a vase; looked at another way it is two faces.

We can switch back and forth between the two viewpoints. But we can not see both at once. But the figure *is* both at once.

Similarly, we can think of an electron as a wave or we can think of an electron as a particle, but we can not think of it as both at once. But in some sense the electron *is* both at once. Being able to think of these two viewpoints at once is in some sense being able to understand Quantum Mechanics.

I do not believe that Quantum Mechanics is understandable, at least for the usual meaning of the word *understand*.

Thus when we think of an electron in a Hydrogen atom, we can imagine it as a particle in orbit around the central proton. We can also imagine it as the *wave function*, its wave aspect; it turns out that the wave function for the electron in the Hydrogen atom is spherically symmetric with maximum density at the center of the atom.

A Flash animation of these two viewpoints of an electron in a Hydrogen atom may be accessed by clicking on the red button to the right. It will appear in a separate window, and has a file size of 9.6k In order to view it, you need to have the Flash player of Version 5 or later installed on your computer; the Flash player is available free from http://www.macromedia.com/

We can illustrate the Principle of Complementarity with some examples by Bohr himself:

1. "The opposite of a true statement is a false statement, but the opposite of a profound truth is usually another profound truth."
2. Life: a form through which matter streams.
   Life: a collection of matter.

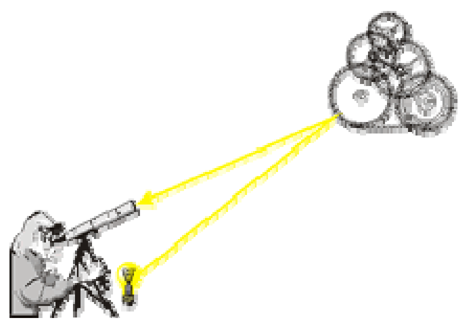
3. Justice and love.

References to the above examples are:

1. Ken Wilbur, **Spectrum of Consciousness**, pg. 34.
2. Heisenberg, **Physics and Beyond**, pg. 105.
3. From the Chinese Taoist text **I Ching**, as reported in Reference 2, pg. 15.

Copenhagen Interpretation of Quantum Mechanics

If the Complementarity Principle is subtle, complex, and difficult to understand, then its extension into Bohr's interpretation of Quantum Mechanics will share these characteristics with some added subtlety. We begin by considering an apparatus making a measurement on some system:



The *apparatus* consists of a light source and a detector, which is the telescope and observer. The apparatus is making measurements on a *system* which in this example consists of some gears. The light source emits *photons*, the particulate aspect of light. The photons are "reflected" by the system, and enter the detector. The reflection of the photons off the gears necessarily disturbs the system we are attempting to measure.

If we attempt to reduce the disturbance on the system due to our measurements on it, we eventually reach a point where we have reached an irreducible minimum: this is when the interaction involves the exchange of a single quantum of energy, emitted by the light source, reflected off the system, and detected by the telescope and observer

The situation is actually even more complex than this. In fact, the photon from the light bulb is absorbed by the gears, which then emits another photon which ends up in the detector. If the "observer" were some sort of detector capable of detecting a single photon that would be the end of the story. But for the human observer shown, that single photon would have to enter his eye, be absorbed by the retina, which in turn causes an electrical impulse to go up the optic nerve to the brain where in principle the brain would process it; for real humans the minimum light level that is perceptible corresponds to a few photons, not a single one. So for a human to participate in this minimal observation, we would require a detector capable of registering a single photon and sending a larger signal, such as a flashing light, which a human can perceive.

The fact that the interaction cannot be reduced beyond a minimum amount, the interchange of a single photon, is the heart of the *Heisenberg Uncertainty Principle*. However, Bohr realised that it means even more than this. At this level we can not divide the quantum of energy into a contribution from the apparatus and a contribution from the system: the process is inseparable. Thus it is *holistic*.

This in turn means that at this level it is not meaningful to talk about the system at all separate from the apparatus observing it. As Bohr repeatedly said, "The quantum world does not exist."

Wheeler made a similar conclusion when he suggested that we should drop the word *observer* from our vocabulary, replacing it with the word *participator*.

In fact, the separation between the observer and the observed is always more-or-less arbitrary, although we customarily ignore that fact. An example by Bohr may clarify:

We customarily think of the outside world as separate from ourselves, and the boundary between the two is the surface of our skin. However, think of a blind person who gets around with the assistance of a cane. In time that person will probably treat the cane as part of his or her body, and will think of the outside world as beginning just at the tip of the cane. Now imagine the blind man's sense of touch extending out of the tip of the cane and into the roadway itself. Imagine it extending further, down the block, into the countryside, to the whole world. There is no point where the blind man ends and the world begins. Similarly, we can not say which is the system and which is us observing it.

This is the heart of the Copenhagen Interpretation of Quantum Mechanics.

We conclude this section with a further subtlety. The energy of a photon, the particulate aspect of light, is related to the wavelength of the wave aspect of the same light. We can reduce the energy of an individual photon by increasing the wavelength. Thus, we can reduce the disturbance on the system we are attempting to observe by using light of a larger wavelength. However, our ability to see the details of an object also depends on the wavelength of the light: we can not see details that are smaller than the wavelength. In the usual case we don't notice this because the wavelength of visible light is so small compared to everyday distances.

However, in the case of a quantum measurement we are typically investigating systems that are very small. Thus there is a meaningful maximum wavelength for the light we are using if we wish to see the system. So the minimum interaction between the apparatus and the system involves a single photon with a maximum wavelength, i.e. a single photon with a minimum energy. This minimum energy of the photon further constrains the minimum amount of disturbance we introduce by doing a measurement.

Technical note: the energy of the photon equals $h\ c\ /\ wavelength$, where h is Planck's constant and c is the speed of light.

Author

This document was written by David M. Harrison, Dept. of Physics, University of Toronto, harrison@physics.utoronto.ca in March 2000. This is version 1.6, date (m/d/y) 03/27/06.

The Stern-Gerlach Experiment, Electron Spin, and Correlation Experiments

Click here to go to the *UPSCALE* home page.
Click here to go to the Physics Virtual Bookshelf.

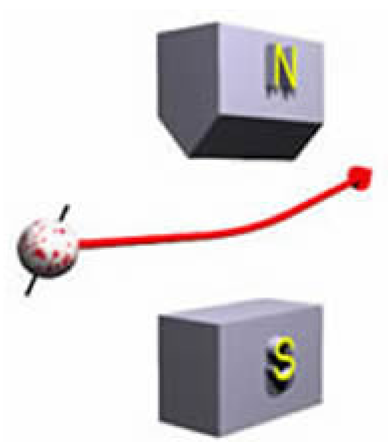
Introduction

This page summarises summarises the classic Stern-Gerlach experiment on "spin" and extends the treatment to a discussion of correlation experiments. As is often the case, I build up maximum complexity as I examine the experimental details, and then hide them in a 'box'. This time the box will turn out to be literal.

Here we concentrate on electrons, which have only two spin-states. We also mention photons, which also have two spin-states. The approach is largely based on one by Feynman which he used for objects with three spin states: see R.P. Feynman, R.B. Leighton and M. Sands, **The Feynman Lectures on Physics**, Vol III, Chapter 5 for this discussion.

Classical Charged Spinning Objects

We begin by considering a macroscopic charged ball that is thrown between the poles of a magnet. If the ball is not spinning, a "knuckleball" to a baseball fan, it will not be deflected. However, if it is spinning it will be deflected as shown:



We ignore:

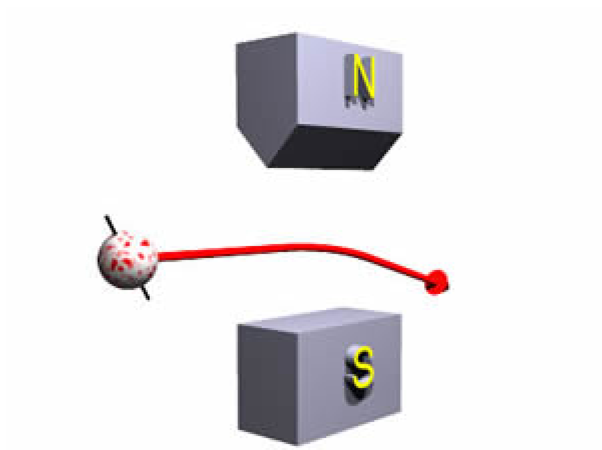
- The weird shape of the magnet pole pieces.
- The fact that there will be horizontal deflections. These can be cancelled by putting an electric field perpendicular to the plane of the ball's trajectory.

A Flash animation of this case has been prepared. It may be viewed by clicking here.
For the case shown above, the figure to the right shows the spin of the charge.

We shall call this orientation "spin up" since it is deflected up by the magnets.

The total amount of deflection is a function of

- The total amount and distribution of electric charge on the ball.
- The orientation and rate of spin. As the rate of spin increases, so does the deflection. As the axis of the spin becomes more vertical, that amount of deflection also increases.



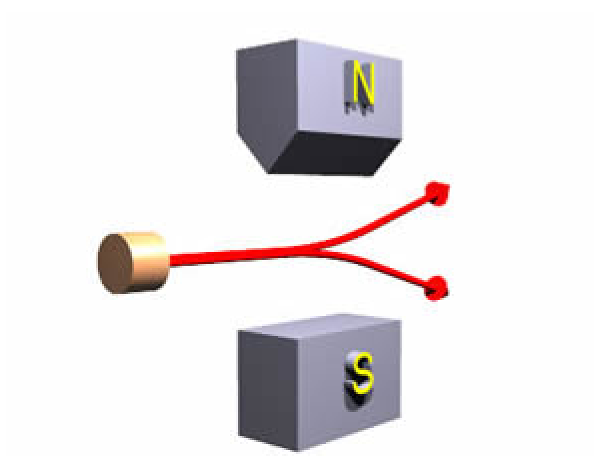
By contrast a "spin down" electron would have its spin oriented as shown to the left:

Such an object is deflected down by the magnets.

All of the above is just classical 19th century electricity and magnetism.

The Spin of the Electron

An "electron gun" produces a beam of electrons. Further information may be found here.



If the beam from the electron gun is directed to the magnets, as shown to the right, the beam is split into two parts. One half of the electrons in the beam are deflected up, the other half were deflected down. The amount of deflection up or down is exactly the same magnitude. Whether an individual electron is deflected up or down appears to be random.

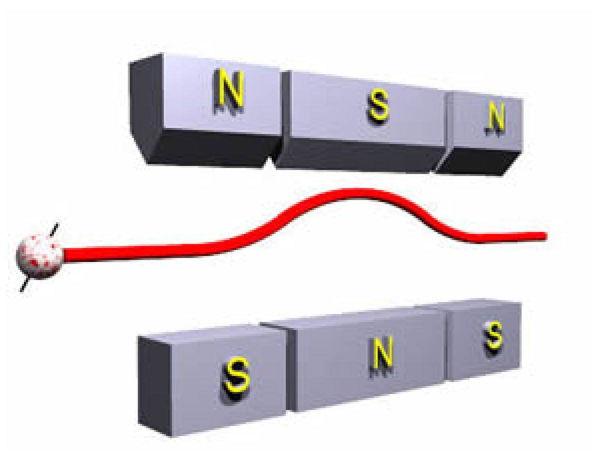
Stern and Gerlach did a version of this experiment in 1922.

This is very mysterious. It seems that the "spin" of electrons comes in only two states. If we assume, correctly, that the rate of spin, total charge, and charge distribution of all electrons is the same, then evidently the *magnitude* of the angle the spin axis makes with the horizontal is the same for all electrons. For some electrons, the spin axis is what we are calling "spin up", for others "spin down".

You should beware of the term "spin." If one uses the "classical radius of the electron" and the known total angular momentum of the electron, it is easy to calculate that a point on the equator of the electron is moving at about 137 times the speed of light! Thus, although we will continue to use the word "spin" it is really a shorthand for "intrinsic angular momentum."

Building a Spin Filter

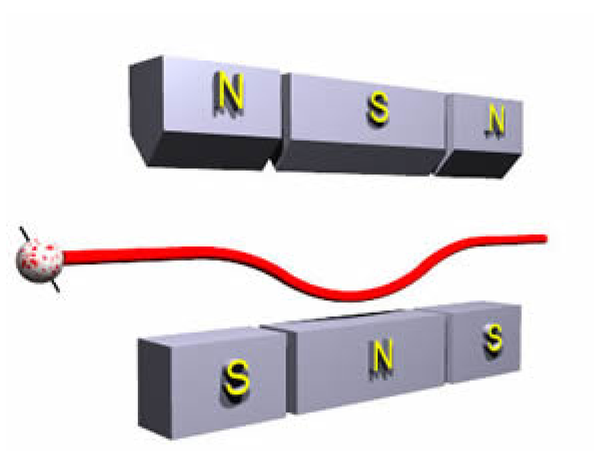
As promised at the beginning, we now make the situation a bit more complex. Consider the arrangement shown to the right:



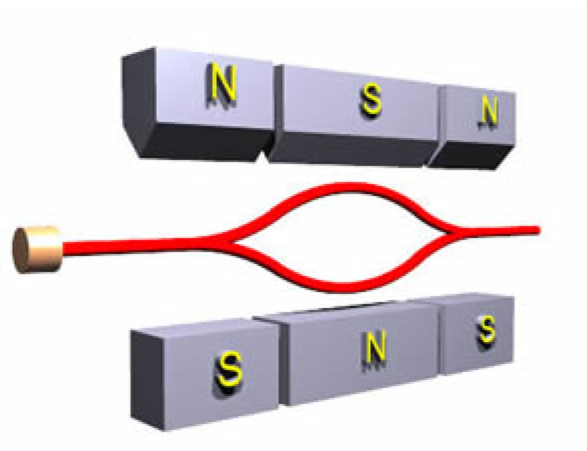
Note that the polarity of the middle longer magnet is reversed from the other two. We have also drawn the path of a "spin up" object. When the object emerges from the magnets it is going the same direction as before it entered them with the same speed.

A Flash animation of this case may be viewed by clicking here.

The path of a "spin down" object is:

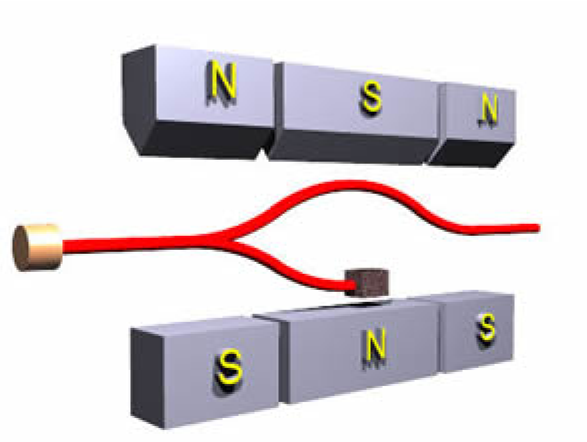


For a beam of electrons, one-half will go follow the upper path while and other half will follow the lower path:



Finally, we imagine putting a small block of lead in the path of the "spin down" electrons.

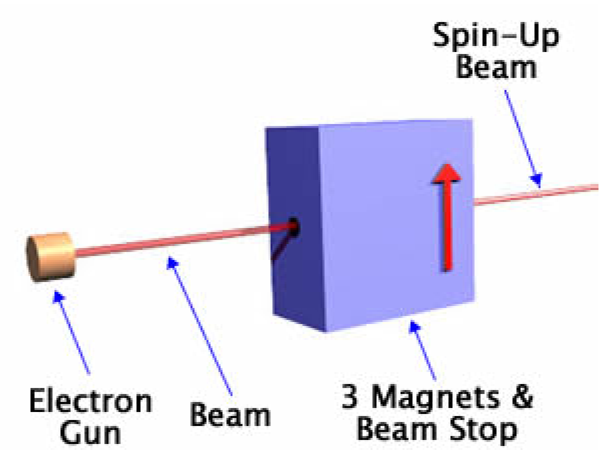
Here, one-half of the incident beam, the spin-down electrons, will be stopped inside the apparatus, while all the spin-up electrons will emerge in the same direction as before they entered the magnets and at the same speed. Thus this is a "filter" that selects spin-up electrons.



Now, again as promised, we simplify by taking all three magnets and the beam stopper and put it in a box. In the figure we also have included an electron gun firing a beam of electrons at the box.

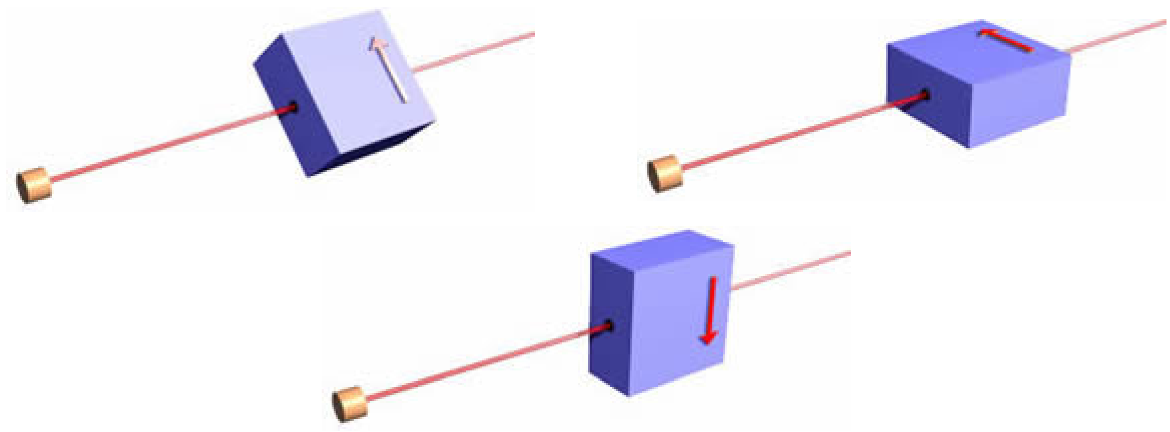
So one-half of the incident beam of electrons will emerge.

It will be important to notice that we have painted an arrow on the front side of the box to indicate what direction is "up." You can't see it yet, but there is also an arrow pointing in the same direction on the back of the box.

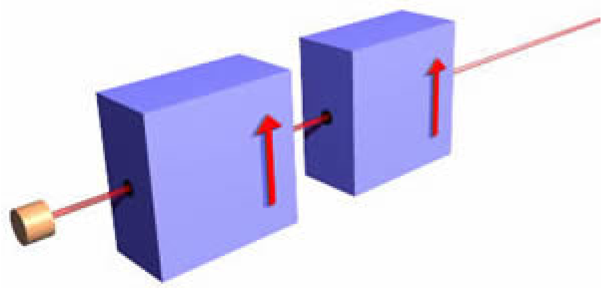


Using the Spin Filter

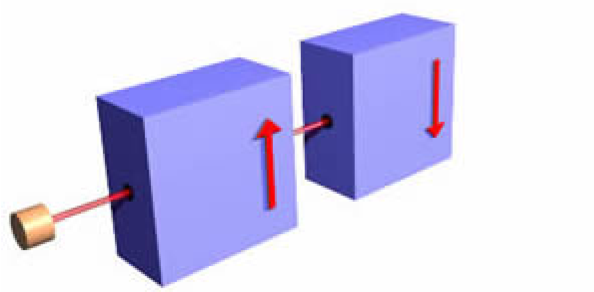
Note that one-half of the incident beam of electrons on the filter emerge from the box, while the other half do not. This is independent of the orientation of the filter; in all the orientations shown below one-half of the incident electrons emerge, while the other half do not.



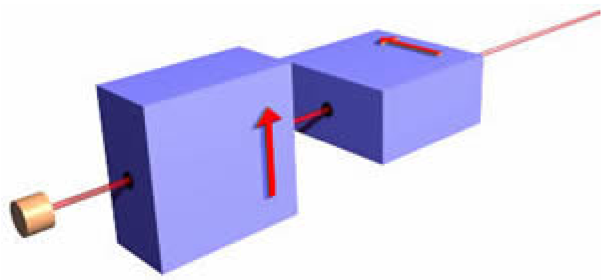
Evidently the direction of "up" is defined by the orientation of the filter doing the measurement. This is sometimes called *spatial quantisation*, a term I do not like.



We now put a second filter behind the first with the same orientation. The second filter has no effect. Half of the electrons from the electron gun emerge from the first box, and **all** of those electrons pass through the second filter. So, once "up" is defined by the first filter, it is the same as the "up" defined by the second.



Now we put the second filter behind the first and upside down relative to the first. As always, half of the beam of electrons from the electron gun emerge from the first filter, and **none** of those electrons emerge from the second filter. So, evidently once the first filter defines "up" that definition is the second filter's definition of "down."



Here is another orientation for the second filter, this time oriented at 90° relative to the first one.

To repeat once again, half of the beam of electrons from the electron gun emerge from the first filter. It turns out that one-half of those electrons pass through the second filter. So if we have two definitions of "up" from two filters at right angles to each other, one half of the electrons will satisfy both definitions.

If we slowly rotate the orientation of the second filter with respect to the first one from zero degrees to 180 degrees, the fraction of the electrons that passed the first filter that get through the second filter goes continuously from 100% to 0%.

Technical note: if the relative angle is *A*, the percentage is $100 \cos^2(A/2)$.

All of the above may remind you of polaroid filters for light. One half of a beam of light from, say, an incandescent lamp will pass through such a filter. If a second filter is placed behind the first one with the same orientation, all the light from the first filter passes through the second (at least in the case of perfect polaroid filters). A brief summary of light polarisation appears here.

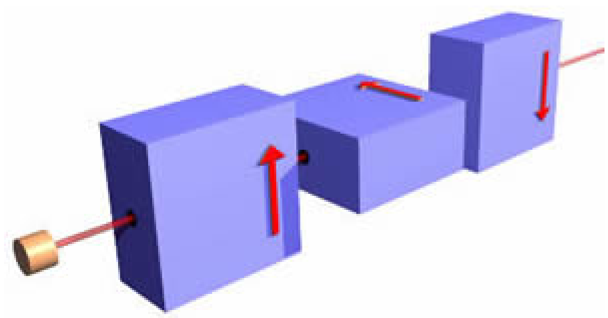
If the relative orientation of the two polaroid filters for light is 90°, then **no** light emerges from the second filter. This corresponds to the case above for electron filters when the relative orientation is 180°.

If the relative orientation of the two polaroid filters for light is 45°, one half of the light from the first filter will emerge from the second. This corresponds to the case above for electron filters when the relative orientation is 90°.

We conclude that the only difference between electron and light filters is a factor of 2 in the relative orientations. Thus, often we call the electron filters "polarisers."

Here is a final example of combining electron filters.

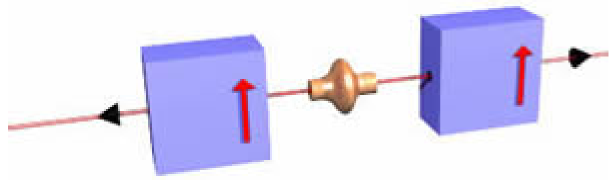
One-half of the beam from the electron gun emerges from the first polariser; one-half of those electrons emerge from the second filter. And one-half of those electrons will make it through the third upside-down filter! Note that if the second filter were not present, **no** electrons will emerge from the upside-down filter. So we see that the middle filter actually changes the definition of "up" for the electrons. This is yet another manifestation of the Heisenberg Uncertainty Principle.



A Flash animation of up to 3 of these Stern-Gerlach filters has been prepared. It requires Flash 7, and has a file size of 130k. It will appear in a separate window. To access the animation, click here.

Correlation Measurements

We imagine a radioactive substance that emits a pair of electrons in each decay. These two electrons go in opposite directions, and are emitted nearly simultaneously. When another nucleus in the sample decays, another pair of electrons are emitted nearly simultaneously and in opposite directions. So we can have a sample emitting these pairs of electrons. To the right we show such a sample, enclosed in a copper colored device, and electron filters measuring the spin of each member of the pair:

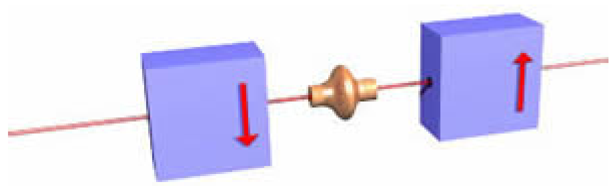


For the radioactive substance we will be considering here, one-half of the electrons incident on the right hand filter emerge and one-half do not. Similarly, one-half of the electrons incident on the left hand filter emerge and one-half do not.

But if we look at the *correlation* between these electrons, we find that if, say, the right hand electron does pass through the filter, then its left hand companion does not pass its filters. Similarly, if the right hand electron does not pass through the filter, then its left hand companion always emerges from its filter.

We say that each radioactive decay has a total spin of zero: if one electron is spin up its companion is spin down. Of course, this is provided that both filters have the same definition of up.

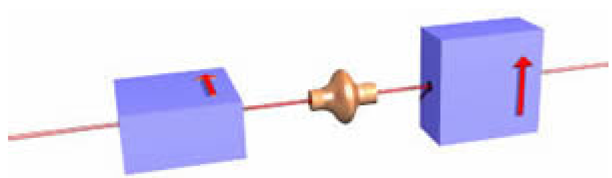
To the right is a case where the two filters have opposite definitions of up.



Again, one-half of the right hand electrons pass through their filter and one-half of the left hand electrons pass through their filter. But this time if a particular right hand electron passes its filter, then its companion left hand electron always passes its filter. Similarly, if the right hand electron does not pass its filter, its companion electron doesn't pass through its filter either.

Now we consider yet another example.

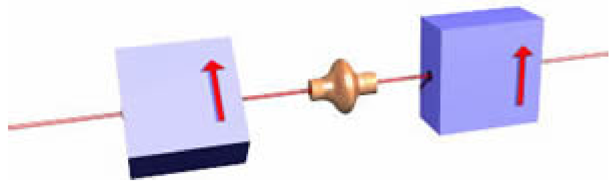
The two filters define "up" to be in perpendicular directions to each other. If you are still following this business with electron filters, you will not be surprised to learn that:



1. One-half of the right hand electrons emerge from their filter.
2. One-half of the left hand electrons emerge

from their filter.
3. If a particular right hand electron passes its filter, one-half of the time its companion left hand electron will emerge from its filter, one-half of the time it will not.

These sorts of measurements are called *correlation experiments*. We show an arbitrary relative orientation of the two filters.



We summarise all of the above by saying that when the two filters have the same orientation, the correlation is zero: if the right hand electron passes its companion does not. When the two filters have opposite orientations, the correlation is 100%: if the right hand electron passes, so does its companion, while if the right hand electron does not pass, neither does its companion. When the two filters have perpendicular orientations, the correlation is 50%. It turns out that the correlation goes smoothly from zero to 100% as the relative orientation goes from 0° to 180°. For the mathphilic student, the actual formula is that the correlation is sin(*a*/2) squared, where *a* is the relative angle between the filters.

There are radioactive substances that emits pairs of photons similar the the pairs of electrons we have been consider so far. Some such substances have similar correlations to the electron source we have been considering, except that there is a difference of a factor of two in the relative orientations of the polarisers. If the light polarisers have the same orientation, the correlation is zero; this is the same as for electrons.

If the light polarisers have a relative orientation of 90°, the correlation is 100%: if the right hand photon passes through its polariser it companion photon will pass its polarisers, while if the right hand photon does not pass, neither does its companion. This corresponds to the case for electrons where the relative orientation of the filters was 180°.

Similarly, if you are still following all this, the correlation when the relative orientation of the light polarisers is 45° is 50%, just the correlation for electron with relative filter orientations of 90°.

As we shall see these correlation experiments, both for electrons and photons, have been performed and turn out to give us important information about the way the world is put together. This is the thrust of *Bell's Theorem,* also sometimes known as the *Einstein-Podolsky-Rosen (EPR)* paradox.

Author

Bell's Theorem

Click here to go to the Physics Virtual Bookshelf

INTRODUCTION

In 1975 Stapp called Bell's Theorem "the most profound discovery of science." Note that he says *science*, not *physics*. I agree with him.

In this document, we shall explore the theorem. We assume some familiarity with the concept of wave-particle duality; a document on this may be found here. We also assume considerable familiarity with the Stern-Gerlach experiment and the concept of a correlation experiment; a document on these may be found here.

A much simpler introduction to the theorem, with some loss of completeness, has been prepared. You may access an html or pdf version with the links to the right.

html

pdf

The origins of this topic is a famous paper by Einstein, Rosen and Podolsky (EPR) in 1935; its title was *Can Quantum-Mechanical Description of Physical Reality be Considered Complete?* They considered what Einstein called the "spooky action-at-a-distance" that seems to be part of Quantum Mechanics, and concluded that the theory must be incomplete if not outright wrong. As you probably already know, Einstein never did accept Quantum Mechanics. One of his objections was that "God does not play at dice with the universe." Bohr responded: "Quit telling God what to do!"

In the early 1950's David Bohm (not "Bohr") was a young Physics professor at Princeton University. He was assigned to teach Quantum Mechanics and, as is common, decided to write a textbook on the topic; the book is still a classic. Einstein was at Princeton at this time, and as Bohm finished each chapter of the book Einstein would critique it. By the time Bohm had finished the book Einstein had convinced him that Quantum Mechanics was at least incomplete. Bohm then spent many years in search of *hidden variables*, unobserved factors inside, say, a radioactive atom that determines when it is going to decay. In a hidden variable theory, the time for the decay to occur is not random, although the variable controlling the process is hidden from us. We will discuss Bohm's work extensively later in this document.

In 1964 J.S. Bell published his theorem. It was cast in terms of a hidden variable theory. Since then, other proofs have appeared by d'Espagnat, Stapp, and others that are not in terms of hidden variables. Below we shall do a variation on d'Espagnat's proof that I devised; it was originally published in the American Journal of Physics **50**, 811 - 816 (1982).

PROVING BELL'S INEQUALITY

We shall be *slightly* mathematical. The details of the math are not important, but there are a couple of pieces of the proof that will be important. The result of the proof will be that for any collection of objects with three different parameters, *A*, *B* and *C*:

The number of objects which have parameter A but not parameter B plus the number of objects which have parameter B but not parameter C is greater than or equal to the number of objects which have parameter A but not parameter C.

We can write this more compactly as:

Number(A, not B) + Number(B, not C) greater than or equal to Number(A, not C)

The relationship is called *Bell's inequality*.

In class I often make the students the collection of objects and choose the parameters to be:

A: male **B**: height over 5' 8" (173 cm) **C**: blue eyes

Then the inequality becomes that the number of men students who do not have a height over 5' 8" plus the number of students, male and female, with a height over 5' 8" but who do not have blue eyes is greater than or equal to the number of men students who do not have blue eyes. I absolutely guarantee that for any collection of people this will turn out to be true.

It is important to stress that we are not making any statistical assumption: the class can be big, small or even zero size. Also, we are not assuming that the parameters are independent: note that there tends to be a correlation between gender and height.

Sometimes people have trouble with the theorem because we will be doing a variation of a technique called *proof by negation*. For example, here is a syllogism:

All spiders have six legs. All six legged creatures have wings. Therefore all spiders have wings

If we ever observe a spider that does not have wings, then we know that at least one and possibly both of the assumptions of the syllogism are incorrect. Similarly, we will derive the inequality and then show an experimental circumstance where it is not true. Thus we will know that at least one of the assumptions we used in the derivation is wrong.

Also, we will see that the proof and its experimental tests have absolutely nothing to do with Quantum Mechanics.

Now we are ready for the proof itself. First, I assert that:

Number(A, not B, C) + Number(not A, B, not C) must be either 0 or a positive integer

or equivalently:

Number(A, not B, C) + Number(not A, B, not C) greater than or equal to 0

This should be pretty obvious, since either no members of the group have these combinations of properties or some members do.

Now we add **Number(A, not B, not C) + Number(A, B, not C)** to the above expression. The left hand side is:

Number(A, not B, C) + Number(A, not B, not C) + Number(not A, B, not C) + Number(A, B, not C)

and the right hand side is:

0 + Number(A, not B, not C) + Number(A, B, not C)

But this right hand side is just:

Number(A, not C)

since for all members either **B** or **not B** must be true. In the classroom example above, when we counted the number of men without blue eyes we include both those whose height was over 5' 8" and those whose height was not over 5' 8".

Above we wrote "since for all members either **B** or **not B** must be true." This will turn out to be important.

We can similarly collect terms and write the left hand side as:

Number(A, not B) + Number(B, not C)

Since we started the proof by asserting that the left hand side is greater than or equal to the right hand side, we have proved the inequality, which I re-state:

Number(A, not B) + Number(B, not C) greater than or equal to Number(A, not C)

We have made two assumptions in the proof. These are:

- *Logic is a valid way to reason.* The whole proof is an exercise in logic, at about the level of the "Fun With Numbers" puzzles one sometimes sees in newspapers and magazines.
- *Parameters exist whether they are measured or not.* For example, when we collected the terms **Number(A, not B, not C) + Number(A, B, not C)** to get **Number(A, not C)**, we assumed that either **not B** or **B** is true for every member.

Bell's Inequality second proof:

http://www.ipod.org.uk/reality/reality_entangled.asp

In 1964, John Bell devised an ingenious test for the existence of hidden variables. Bell's theorem (which is commonly called *Bell's Inequality*) has been called *"the most profound discovery of science"* (see here).

Bell showed that for a group of objects with fixed properties *A*, *B* and *C*, the number of objects which have property *A* but not property *B* plus the number of objects which have property *B* but not property *C* is greater than or equal to the number of objects which have property *A* but not property *C*.

This can be written more compactly as:

Number(A, not B) + Number(B, not C) >= Number(A, not C)

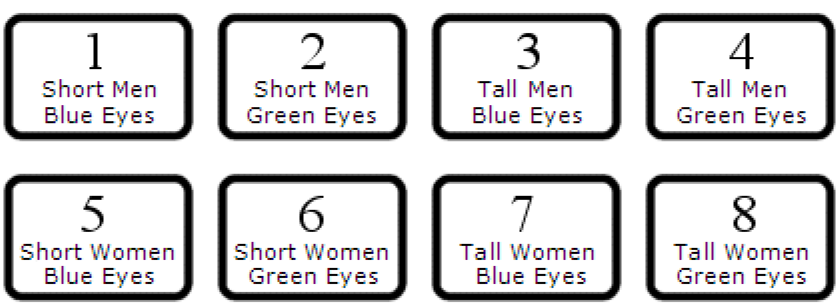
An easy-to-understand version of this inequality is provided by David M. Harrison of the University of Toronto (see here). Let's consider our collection of objects with fixed properties to be a collection of people. And let their fixed properties be the following:

- **A: Sex** ("Male" or "Female")
- **B: Height** (over 5' 6" ("Tall") or under 5' 6" ("Short" - don't be offended!))
- **C: Eye colour** ("Blue" or "Green")

Then, no matter which group of people you are dealing with, you are always able to issue the following statement (inequality): *"The number of short males plus the number of tall people, male and female, with green eyes will always be greater than or equal to the number of males with green eyes. I absolutely guarantee that for any collection of people this will turn out to be true."*

That's always true. Isn't that amazing? That's a bit of quantum mechanics you can try out at your next party!

It's relatively simple to prove this. Note that every person can be classified into one of the following eight groups:



Referring to this diagram, Bell's Inequality is saying that:

(Group 1 + **Group 2**) + (**Group 4** + Group 8) >= (**Group 2** + **Group 4**)

Which, if you study it, is clearly always going to be true.

APPLYING BELL'S INEQUALITY TO ELECTRON SPIN

Consider a beam of electrons from an electron gun. Let us set the following assignments for the three parameters of Bell's inequality:

A: electrons are "spin-up" for an "up" being defined as straight up, which we will call an angle of zero degrees. **B**: electrons are "spin-up" for an orientation of 45 degrees. **C**: electrons are "spin-up" for an orientation of 90 degrees.

Then Bell's inequality will read:

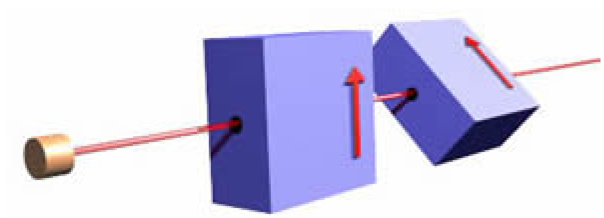
Number(spin-up zero degrees, not spin-up 45 degrees) + Number(spin-up 45 degrees, not spin-up 90 degrees) greater than or equal to Number(spin-up zero degrees, not spin-up 90 degrees)

But consider trying to measure, say, **Number(A, not B)**. This is the number of electrons that are spin-up for zero degrees, but are not spin-up for 45 degrees. Being "not spin-up for 45 degrees" is, of course, being spin-*down* for 45 degrees.

We know that if we measure the electrons from the gun, one-half of them will be spin-up and one-half will be spin-down for an orientation of 0 degrees, and which will be the case for an individual electron is random. Similarly, if measure the electrons with the filter oriented at 45 degrees, one-half will be spin-down and one-half will be spin-up.

But if we try to measure the spin at both 0 degrees and 45 degrees we have a problem.

The figure to the right shows a measurement first at 0 degrees and then at 45 degrees. Of the electrons that emerge from the first filter, 85% will pass the second filter, not 50%. Thus for electrons that are measured to be spin-up for 0 degrees, 15% are spin-down for 45 degrees.

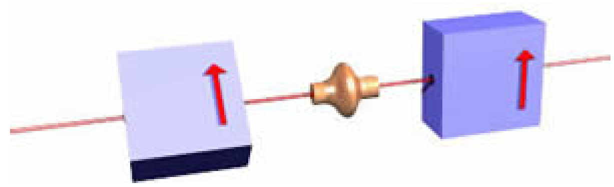


Thus measuring the spin of an electron at an angle of zero degrees irrevocably changes the number of electrons which are spin-down for an orientation of 45 degrees. If we measure at 45 degrees first, we change whether or not it is spin-up for zero degrees. Similarly for the other two terms in this application of the inequality. This is a consequence of the Heisenberg Uncertainty Principle. So this inequality is not experimentally testable.

In our classroom example, the analogy would be that determining the gender of the students would change their height. Pretty weird, but true for measuring electron spin.

However, recall the correlation experiments that we discussed earlier. Imagine that the electron pairs that are emitted by the radioactive substance have a total spin of zero. By this we mean that if the right hand electron is spin-up its companion electron is guaranteed to be spin-down provided *the two filters have the same orientation*.

Say in the illustrated experiment the left hand filter is oriented at 45 degrees and the right hand one is at zero degrees. If the left hand electron passes through its filter then it is spin-up for an orientation of 45 degrees. Therefore we are guaranteed that if we had measured its companion electron it would have been spin-down for an orientation of 45 degrees. We are simultaneously measuring the right-hand electron to determine if it is spin-up for zero degrees. And since no information can travel faster than the speed of light, the left hand measurement cannot disturb the right hand measurement.



So we have "beaten" the Uncertainty Principle: we have determined whether or not the electron to the right is **spin-up zero degrees, not spin-up 45 degrees** by measuring its spin at zero degrees and its companion's spin at 45 degrees.

Now we can write the Bell inequality as:

Number(right spin-up zero degrees, left spin-up 45 degrees) + Number(right spin-up 45 degrees, left spin-up 90 degrees) greater than or equal to Number(right spin-up zero degrees, left spin-up 90 degrees)

This completes our proof of Bell's Theorem.

The same theorem can be applied to measurements of the polarisation of light, which is equivalent to measuring the spin of photon pairs.

The experiments have been done. For electrons the left polarizer is set at 45 degrees and the right one at zero degrees. A beam of, say, a billion electrons is measured to determine **Number(right spin-up zero degrees, left spin-up 45 degrees)**. The polarizers are then set at 90 degrees/45 degrees, another billion electrons are measured, then the polarizers are set at 90 degrees/zero degrees for another billion electrons.

The result of the experiment is that the inequality is violated. The first published experiment was by Clauser, Horne, Shimony and Holt in 1969 using photon pairs. The experiments have been repeated many times since.

The experiments done so far have been for pairs of electrons, protons, photons and ionised atoms. It turns out that doing the experiments for photon pairs is easier, so most tests use them. Thus, in most of the remainder of this document the word "electron" is generic.

Technical note: You may recall from our discussion of the Stern-Gerlach experiment that doing a correlation experiment for electrons with the polarisers at some relative angle is equivalent to doing the experiment for photons with the polarisers at half the relative angle of the electron polarisers. Thus, when we discuss an electron measurement with the polarisers at, say, zero degrees and 45 degrees, for a photon experiment it would be zero degrees and 22.5 degrees.

In the last section we made two assumptions to derive Bell's inequality which here become:

- Logic is valid.
- Electrons have spin in a given direction even if we do not measure it.

Now we have added a third assumption in order to beat the Uncertainty Principle:

- No information can travel faster than the speed of light.

We will state these a little more succinctly as:

1. Logic is valid.
2. There is a reality separate from its observation
3. Locality.

You will recall the we discussed proofs by negation. The fact that our final form of Bell's inequality is experimentally violated indicates that at least one of the three assumptions we have made have been shown to be wrong.

You will also recall that earlier we pointed out that the theorem and its experimental tests have nothing to do with Quantum Mechanics. However, the fact that Quantum Mechanics correctly predicts the correlations that are experimentally observed indicates that the theory too violates at least one of the three assumptions.

Finally, as we stated, Bell's original proof was in terms of hidden variable theories. His assumptions were:

1. Logic is valid.
2. Hidden variables exist.
3. Hidden variables are local.

Most people, including me, view the assumption of local hidden variables as very similar to the assumption of a local reality.

WHAT NOW?

As can be easily imagined, many people have tried to wiggle out of this profound result. Some attempts have critiqued the experimental tests. One argument is that since we set the two polarizers at some set of angles and then collect data for, say, a billion electrons there is plenty of time for the polarizers to "know" each other's orientation, although not by any known mechanism. More recent tests set the orientation of the the polarizers randomly *after* the electrons have left the source. The results of these tests are the same as the previous experiments: Bell's inequality is violated and the predicted Quantum correlations are confirmed. Still other tests have set the distance between the two polarizers at 11 km, with results again confirming the Quantum correlations.

Another critique has been that since the correlated pairs emitted by the source go in all directions, only a very small fraction of them actually end up being measured by the polarizers. Another experiment using correlated Beryllium atoms measured almost all of the pairs, with results again confirmed the Quantum correlations.

There is another objection to the experimental tests that, at least so far, nobody has managed to get totally around. We measure a spin combination of, say, zero degrees and 45 degrees for a collection of electrons and then measure another spin combination, say 45 degrees and 90 degrees, for *another* collection of electrons. In our classroom example, this is sort of like measuring the number of men students whose height is not over 5' 8" in one class, and then using another class of different students to measure the number of students whose height is over 5' 8" but do not have blue eyes. The difference is that a collection of, say, a billion electrons from the source in the correlation experiments always behaves identically within small and expected statistical fluctuations with every other collection of a billion electrons from the source. Since that fact has been verified many many times for all experiments of all types, we assume it is true when we are doing these correlation experiments. This assumption is an example of inductive logic; of course we assumed the validity of logic in our derivation.

Sometimes one sees statements that Bell's Theorem says that *information* is being transmitted at speeds greater than the speed of light. So far I have not seen such an argument that I believe is correct. If we are sitting by either of the polarisers we see that one-half the electrons pass and one-half do not; which is going to be the case for an individual electron appears to be random. Thus, the behavior at our polariser does not allow us to gain any information about the orientation of the other polariser. It is only in the *correlation* of the electron spins that we see something strange. d'Espagnat uses the word *influence* to describe what may be traveling at superluminal speeds.

Imagine we take a coin and carefully saw it in half so that one piece is a "heads" and the other is a "tails." We put each half in a separate envelope and carry them to different rooms. If we open one of the envelopes and see a heads, we know that the other envelope contains a tails. This correlation "experiment" corresponds to spin measurements when both polarisers have the same orientation. It is when we have the polarisers at different orientations that we see something weird.

So far we don't know which of the assumptions we made in the proof are incorrect, so we are free to take our pick of one, two or all three. We shall close this section by briefly considering the consequences of discarding the assumption of the validity of logic and then the consequences of discarding the assumption of a reality separate from its observation. In the next section we shall explore the idea of a non-local universe.

What If Logic Is Invalid?

It has been suspected since long before Bell that Quantum Mechanics is in conflict with classical logic. For example, deductive logic is based on a number of assumptions, one of which is the Principle of the Excluded Middle: *all statements are either true or false*.

But consider the following multiple choice test question:

1. The electron is a wave.
2. The electron is a particle.
3. All of the previous.
4. None of the above.

From wave-particle duality we know that both statements 1 and 2 are both sort of true and sort of false. This seems to call into question the Principle of the Excluded Middle. Thus, some people have worked on a multi-valued logic that they hope will be more consistent with the tests of Bells' Theorem and therefore with Quantum Mechanics. Gary Zukav's **The Dancing Wu Li Masters** has a good discussion of such a quantum logic; since numerous editions of this book exist and every chapter is numbered **0**, I can't supply a more detailed reference.

Mathematics itself can be viewed as just a branch of deductive logic, so if we revise the rules of logic we will need to devise a new mathematics

You may be interested to know that deductive logic has proved that logic is incomplete. The proof was published in 1931 by Gödel; a good reference is Hofstader's **Gödel, Escher, Bach**. The key to Gödel's work is *self-reference*; we shall see an example of self-reference in the next sub-section. What he proved was that any mathematics at all, unless it is trivially limited, will contain statements that are neither true nor false but simply unprovable.

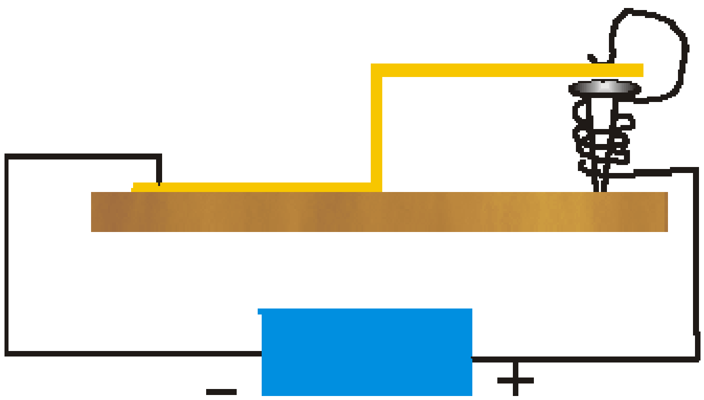
By self-reference we mean a statement or set of statements that refer to themselves. For example, consider:

This statement is false.

Note that if this statement is true, then it must be false. If the statement if false, then it must be true. So we have a chain of *True » False » True » False ....*

New Yorker, Mar 5, 2001, pg. 78.

This may remind you a bit of a simple buzzer, such as a door buzzer.



A buzzer is shown to the right. A flexible piece of metal is bent into a double *L* shape and nailed to a board. A big nail is placed just under the right hand part of the metal, and the metal is adjusted so that it does not quite touch the big nail. A battery is wired in such a fashion that when the the metal *L* is at rest, the circuit is just completed, which causes the big nail to become an electromagnet.

This of course pulls the metal down, which breaks the circuit. Thus the metal springs back up, which completes the circuit again, which pulls the metal down, and so on. Thus, if the circuit is closed, it opens, and if the circuit is open, then it is closed. Or, we say we have a chain of *Closed* » *Open* » *Closed* » *Open* .... The difference between this example and the previous self-referential statement is that here the oscillations in value are occurring in time. You may access a Flash animation of a buzzer by clicking here.

In the late nineteenth century the logician Hilbert used to say "Physics is too important to be left to the physicists." In retaliation, J.A. Wheeler has stated: "Gödel is too important to be left to the mathematicians."

Finally, although *deductive* logic is fairly well understood, nobody has succeeded in codifying iron-clad rules for *inductive* logic that work consistently. Mills tried very hard to do this, but the following story by Copi shows one problem:

"A favorite example used by critics of the Method of Agreement is the case of the Scientific Drinker, who was extremely fond of liquor and got drunk every night of the week. He was ruining his health, and his few remaining friends pleaded with him to stop. Realizing himself that he could not go on, he resolved to conduct a careful experiment to discover the exact cause of his frequent inebriations. For five nights in a row he collected instances of a given phenomenon, the antecedent circumstances being respectively scotch and soda, bourbon and soda, brandy and soda, rum and soda, and gin and soda [ugh!]. Then using the Method of Agreement he swore a solemn oath never to touch soda again!"

Reference: I. Copi, **Introduction to Logic**, 2nd ed., (Macmillan, New York, 1961), pp 394-395.

Note the "hidden variable" in the above story.

What If There Is No Reality Separate From Its Observation?

As we have seen, the title of this sub-section is very similar to asking what are the consequences of having no hidden variables. We shall concentrate on the first form of the question.

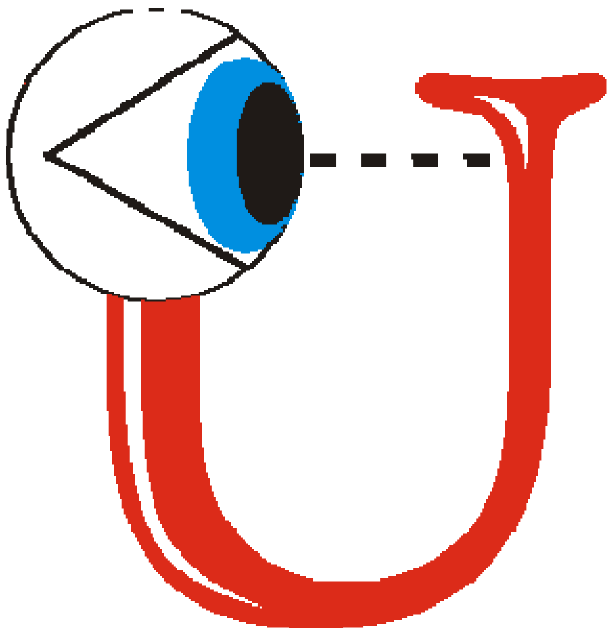
You may have already noticed that the question is a variation on the old philosophical saw regarding a tree that falls in the forest with nobody there to hear the sound.

A conflict between the assumption of reality and Quantum Mechanics has been suspected long before Bell. For example, in referring to the trajectory of the electron in, say, the double slit experiment Heisenberg stated "The path of the electron comes into existence only when we observe it."

People have long known that any measurement disturbs the thing being measured. A crucial assumption of classical sciences has been that at least in principle the disturbance can be made so small that we can ignore it. Thus, when an anthropologist is studying a primitive culture in the field, she assumes that her presence in the tribe is having a negligible effect on the behavior of the members. Sometimes we later discover that all she was measuring was the behavior of the tribe when it was being observed by the anthropologist.

Nonetheless, classically we assume a model where we, as *observers*, are behind a pane of glass where see what is going on "out there." Now we suggest that the pane of glass has been shattered. Wheeler suggests that we should drop the word *observer* entirely, and replace it with *participator*.

Wheeler has thought more deeply on the consequences of a participatory universe than anybody. He devised the figure to the right, whose caption is:



“Symbolic representation of the Universe as a self-excited system brought into being by ‘self-reference’. The universe gives birth to communicating participators. Communicating participators give meaning to the universe … With such a concept goes the endless series of receding reflections one sees in a pair of facing mirrors.”

Reference: J.A. Wheeler in Isham et al., eds, **Quantum Gravity** (Clarendon, Oxford, 1975), pg. 564-565. The colors were used by Wheeler in a colloquium in the Dept. of Physics, Univ. of Toronto some years ago.

You may have noticed a similarity between this view of Quantum Mechanics and the Idealist philosophy of Bishop Berkeley. Berkeley would likely have been very happy about Bell's Theorem. Dr. Johnson was, of course, opposed to Berkeley and used to argue against his philosophy by bellowing "I refute it thus!" while kicking a large rock. Apparently Johnson found sufficient comfort from his argument that he didn't mind hurting his foot.

d'Espagnat also tends to believe that the reality assumption is incorrect. Thus he wrote: "The doctrine that the world is made up of objects whose existence is independent of human consciousness turns out to be in conflict with quantum mechanics and with facts established by experiment."

In a participatory universe, I can argue that you owe your objective existence to my kind intervention in allowing you into my own consciousness. Thus, there is an inherent *solipsism* in this position. Wigner was one of many who was greatly troubled by this.

NON-LOCALITY AND DAVID BOHM

Recall that David Bohm set off in the early 1950's on a quest for the hidden variables. Nobody has explored the consequences of such variables being non-local more deeply than Bohm, and in the first sub-section below we shall discuss some of his work on this topic. In the next sub-section we shall discuss his later thinking about the nature of the world.

The Implicate Order

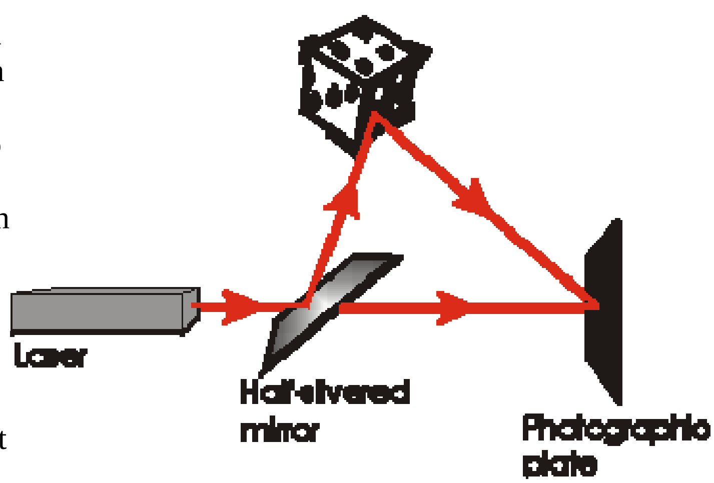
A good reference for the material of this sub-section is David Bohm, **Wholeness and the Implicate Order**. Although very deep the book is not technical except for Chapter 4, which I think should not have been included.

Bohm called our everyday world of space, time and causality the *explicate order*. He proposed that underlying this everyday world is an interconnected one which he calls the *implicate order*. He used a number of analogies and images to discuss these two orders.

In one analogy he imagined a large cylindrical glass container of glycerine mounted on a turntable. We place a spot of black ink in the glycerine. We slowly rotate the container, and the ink gradually disperses throughout the glycerine. If we slowly rotate the cylinder in the opposite direction the spot of ink gradually re-forms. When the ink is dispersed it is in an implicate state: it exists throughout the glycerine. When the ink is a spot it is explicate: it exists in one part of the glycerine but not in the other parts. If we continue rotating the cylinder in this opposite direction the spot disperses again.

We extend the image as follows. We place the spot of ink as before. We slowly rotate the cylinder one revolution, and the ink has begun to disperse. We place a second spot of ink just beside where the first spot was, and rotate for one more revolution. A third spot is placed beside where the second was, one more revolution, and we continue this for a few spots. Then we continue slowly rotating the cylinder until all the ink is fully dispersed. When we reverse the direction of rotation we see the last spot coalesce, then the next to last one right beside the last one, and so on. We could interpret what we are seeing as a single spot of ink that is moving. So in the implicate fully dispersed state we have enfolded the motion in space and time of an object throughout the glycerine. Reversing the rotation unfolds the reality back into space and time.

Another analogy is a hologram. As shown to the right, to make a hologram we split a laser beam into two pieces with a half-silvered mirror. One piece goes straight to a photographic plate, the other bounces off the object and then goes to the plate. In order to reconstruct the image of the object we shine a laser beam through the developed plate: the three-dimensional image appears. Note that in some sense the hologram on the plate is an interference pattern between the beam that has experienced the thing and the beam



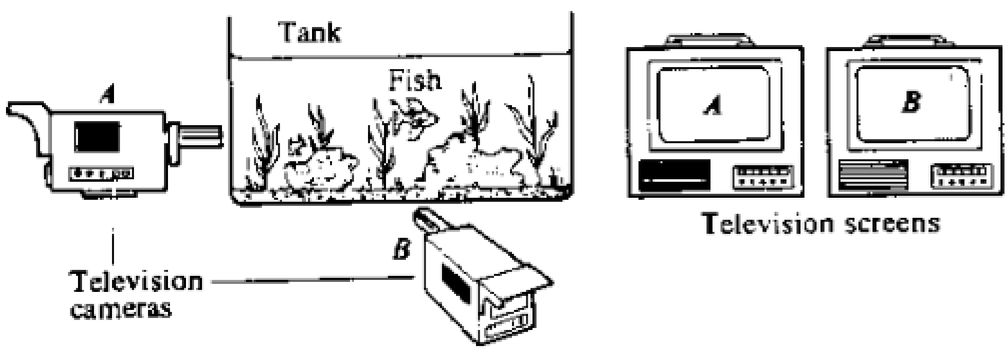
that experienced no-thing.

One characteristic of a hologram is that down to at least a few grains of the silver in the plate, each piece of the plate contains the entire image. If we cut the plate in half we do not lose half the image; instead we lose resolution and the image becomes more fuzzy. Thus each piece of the plate contains the entire space of the object in an enfolded way; this is an analogy to the implicate order. When we reconstruct the image, we have unfolded the implicate order into an explicate one.

There are "multiplexed" holograms that contain time information too. If the object is moving, we rotate the photographic plate. When we reconstruct the image if we look from different angles we see the object's motion. Here the object's time behavior is also enfolded into the totality.

We see that in the implicate order there is no spatial or time separation. Thus it is a non-local order.

Here is another image used by Bohm:



He comments: "The images on the screens are two dimensional *projections* (or facets) of a three dimensional reality. ... Yet, since these projections exist only as abstractions, the three-dimensional reality *is* neither of these. ... What is actually found [in the experimental tests of Bell's theorem] is that the behavior of the two [electrons] is correlated in a way that is rather similar to that of the two television images of the fish, as described earlier. Thus ... each electron acts as if it were a projection of a higher-dimensional reality. ... What we are proposing here is that the quantum property of a non-local, non-causal relation of distant elements may be understood through an extension of the notion described above." -- pg. 187-188.

The following table compares the explicate and implicate order:

Explicate	Implicate
parts make up the whole	whole makes up the parts

spatial separation	holographic
describable	"finger pointing to the moon"
things exist	'thing' and 'no-thing' interfere
"ten thousand things"	illusion
spacetime	spectra

Given the unbroken wholeness of the implicate order, Bohm asked why our thought is so dominated by fragmentation.

"... fragmentation is continually being brought about by the almost universal habit of taking the content of our thought for `a description of the world as it is'." -- pg. 3.

He also wrote about what to do about this:

"[Meditation] is particularly important because ... the illusion that the self and the world are broken into fragments originates in the kind of thought that goes beyond its proper measure and confuses its own product with the same independent reality. To end this illusion requires insight, not only into the world as a whole, but also into how the instrument of thought is working." -- pg. 25.

Bohm's Ontology of Quantum Mechanics

In philosophy, *epistemology* is the study of what we know and how we know it; this is as opposed to *ontology* which studies what actually exists. Most interpretations of Quantum Mechanics have been developed by people sympathetic to the idea of a participatory universe; we discussed this idea above. Therefore, these interpretations are essentially epistemology.

For Bohm, this wasn't good enough. He developed an ontology in his later years. His master work, **The Undivided Universe**, was written with his collaborator B.J. Hiley and published in 1993. It is written for physicists, and I can't really recommend it to a non-technical audience. Here we shall briefly explore some of the conclusions from this book.

Essentially, Bohm and his school re-interpreted the mathematics of Quantum Mechanics and extracted a part of the equation which they called the *quantum potential*. The quantum potential is non-local, and is responsible for all the non-local effects predicted by the theory.

The quantum potential guides, say, the path of an electron in a way similar to the way a radio beacon can guide an airplane coming in for a landing at the airport. It is the jets, ailerons, rudder, etc. on the plane that mechanically determines where the plane is going, but the beacon guides the way.

In Bohm's ontology electrons really are particles. For the case of, for example, the double slit experiment for electrons, each electron goes through either the upper slit or the lower slit; it has a definite path independent of its observation. However, the quantum potential is different depending on whether the other slit is open or closed; since this potential is non-local it can instantaneously change if the other slit is opened or closed. Thus the electron paths are different depending on whether or not the other slit is open.

You may recall that for a *chaotic* system, very small changes in initial conditions leads to radically different trajectories; you may read more about this here. It turns out that for the double slit experiment for electrons, the motion of the electron after it has passed the slits is chaotic in just this sense. Thus, even small thermal fluctuations in the electron's interaction with the slits cause the electron's future motion to be unknowable to us, even though it is strictly deterministic. Thus it seems to us that the path of the electron is random, although in reality it is not.

We call Physics before Quantum Mechanics *classical*; thus the theories of relativity are classical. Usually we characterise a classical theory as one that includes observers and strict determinism, while a non-classical theory has participators and randomness. If Bohm's interpretation is correct we need to change the way we characterise the distinction. A classical theory is local, while a non-classical one is non-local; both are strictly deterministic and have observers. Bohm had some hope that his ontology would have experimentally testable consequences, although no such experiments have yet been done.

You may wish to know that in Bohm's analysis the so-called photon is not a particle; it is an electromagnetic field whose particle-like behavior arises because of its interaction with the quantum potential.

Note that in this work, then, Bohm has finally identified the hidden variable he searched for for so many years: it is the quantum potential.

The non-locality of this potential led Bohm to invoke an image very similar to the one Wheeler used above in his discussion of the universe as a self-excited system:

"Classical physics provided a mirror that reflected only the objective structure of the human being who was the observer. There is no room in this scheme for his mental process which is thus regarded as separate or as a mere 'epiphenomenon' of the objective processes. ... [Through the] mirror [of quantum physics] the observer sees 'himself' both physically and mentally in the larger setting of the universe as a whole. ... More broadly one could say that through the human being, the universe is making a mirror to observe itself." -- Bohm and Hiley, **The Undivided Universe**, pg. 389

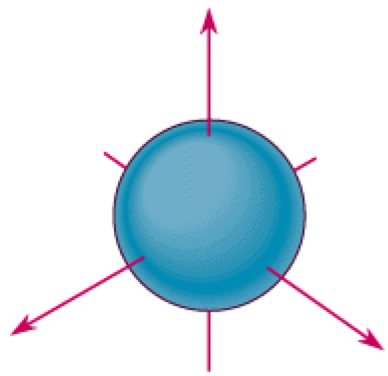
A colleague remarked to me that Bohm's heroic attempts to keep a reality separate from its observation, in this "final" form, is worse than the alternative of not having a reality. I don't know about the word *worse*, but after Bell's theorem something has to give, whether it is reality, locality and/or logic itself.

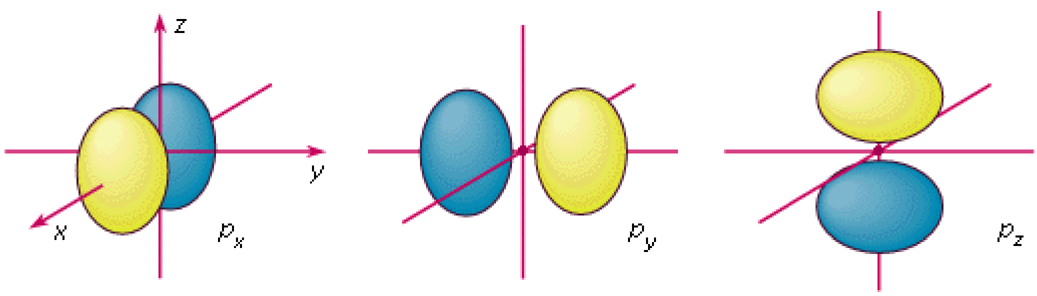
There are still some unresolved issues regarding Bohm's ontology. For example, as discussed elsewhere, the standard planetary model of the atom where the electrons orbit the nucleus just as the planets orbit the Sun is impossible, because according to classical electromagnetism such an electron is in a state of non-uniform accelerated motion and must radiate away its energy, causing it to spiral into the nucleus. However, when we think about the electron in its wave aspect, then when the waves are in a standing wave pattern, this corresponds to the allowed orbits of the Bohr model and the electrons do not radiate.

When the idea of treating the electron as a wave is fully developed by Quantum Mechanics, the orbits are more complicated then indicated in the document referenced in the previous paragraph.

To the right we show the "wave function" for the electron in its ground state orbital. It can be seen that it is spherically symmetric. In an earlier discussion we called this the orbit for which the quantum number *n* is equal to 1.

In Bohm's ontology, the electron is a particle. But for this orbit the electron is stationary, with the electric force trying to pull it into the proton being just balanced by the quantum potential. Thus, this electron will certainly not radiate away energy.





For the state with principle quantum number 2, there is a spherically symmetric wave function that looks just like the one shown before for $n = 1$. But there are also three other orbitals, which look as shown above.

For the first two of these "p" orbitals, the electrons are moving and accelerating and would be expected to radiate away energy. The last p_z orbital turns out to represent an electron that is stationary.

This is clearly in conflict with the fact that the electrons in the atom do not radiate energy except when they change from one allowed orbit to another allowed orbit.

In fact, this difficulty manifests in another form in the double slit experiment for electrons. If the electron is a particle that changes its trajectory when it goes through the slits, it too should radiate away energy. One of Bohm's colleagues, Vigier, recently said that the wavelength of this radiation is very large and so the energy loss is negligible; some people believe that Vigier is wrong. Work on this problem is currently being pursued; one of the people working on it is Professor John Sipe of this Department. I became aware of this controversy in attempting to find the answer to a question asked by former JPU200Y student Sharmilla Reid.

CELLULAR AUTOMATA

A *cellular automaton* provides another approach to the study of the emergence of structures based on rules.

One of the best known automata is the *Game of Life*, devised by John Conway in 1970. This example is played on a large checkerboard-like grid. One starts with a configuration of cells on the board that are populated, and then calculates the population in succeeding generations using three simple rules:

1. **Birth**: an unoccupied cell with exactly 3 occupied neighbors will be populated in the next generation.
2. **Survival**: an occupied cell with 2 or 3 occupied neighbors will be populated in the next generation.
3. **Death**: in all other cases a cell is unoccupied in the next generation.

Despite the simplicity of the rules, truly amazing patterns of movement, self-organising complexity, and more arise in this game.

To the right is a Flash animation of the simplest possible configuration that changes from generation to generation but never grows or dies out.

- Click on the **Step** button to step from generation to generation. In this mode the number of occupied neighbors of each cell is shown.
- Click on **Play** to resume playing the animation.

There are many resources available on the web to explore this fascinating "game" in more detail.

If you are reading the *pdf* version of this document, Flash animations are not available from pdf.

It has been proposed that these sorts of automata may form a useful model for how the universe really works. Contributors to this idea include Konrad Zuse in 1967, Edward Fredkin in the early 1980's, and more recently Stephen Wolfram in 2002. Wolfram's work in particular is the outcome of nearly a decade of work, which is described in a mammoth 1200 page self-published book modestly titled **A New Kind of Science**.

There are two key features of cellular automata that are relevant for this discussion:

1. The rules are always strictly deterministic.
2. The evolution of a cell depends only on its nearest neighbors.

This seems to put a cellular automaton model of Physics in conflict with Bell's Theorem, which asserts that a logical local deterministic model of the universe can not be correct.

Advocates of the cellular automaton model attempt to argue that there is no essential conflict, just an apparent one. Arguments include:

- That the apparent randomness of quantum phenomena is only *pseudo-random*. To me, they seem to be re-introducing the idea of *hidden variables* via the back door. Plamen Petrov in one of the proponents of this argument.
- That there is some sort of higher-dimensional thread outside of the normal four dimensions of space and time. This "thread" will somehow allow for super-luminal connections. Wolfram and others have proposed this idea.
- Other Wolfram supporters have argued that the speed of light is or can be much greater than the "usual" value that we are used to. Whether or not it needs to be infinite is not clear.

In the previous **Bohm's Ontology of Quantum Mechanics** sub-section, we saw that Bohm's attempt to keep causality ended up with a totally non-local mechanism encapsulated in a *Quantum Potential*. Even there, we saw at the end that there are serious problems with the model.

It may be that there are even more serious problems with the Cellular Automaton model for the way the universe works. The controversy continues to be very active as of this writing (Spring, 2003). A semi-random list of further readings is:

- http://arxiv.org/PS_cache/quant-ph/pdf/0206/0206089.pdf
- http://www.math.usf.edu/~eclark/ANKOS_reviews.html
- http://digitalphysics.org/Publications/Petrov/Pet02m/Pet02m.htm

FINALLY ...

Einstein died many years ago, and so is not here to defend himself against claims of what he would or would not do today. Nonetheless, I tend to think that if he were alive today, Bell's theorem would force him to accept Quantum Mechanics.

AUTHOR

Two Analogies to Bell's Theorem

This document introduces two simple analogies to the *entangled quantum pairs* that are the subject of Bell's Theorem.

The first analogy is intended to illustrate some of the key features of these pairs in a more everyday context. The analogy is in some ways an over-simplification of the actual situations that are the subject of Bell's Theorem.

This analogy gets us only part way towards understanding what Einstein called the "spooky action at a distance" that seems to be inherent in Quantum Mechanics in general and Bell's Theorem in particular. Below are a list of references to these matters which take the discussion further but are accessible the the non-physicist.

The second analogy is much more realistic.

Analogy 1

Background

There are a couple of facts which we will need for our discussion. One is from human biology, and the other from physics.

Issues in the Development of People

The context of our analogy is the *nature* versus *nurture* debate about the development of people. Adherents of the nurture position believe that at birth humans are essentially a blank slate, and that their environment as they grow and develop is the only factor that determines characteristics of the individual. Thus matters of choice of profession, mate, musical preferences, morality, etc. are determined by society. Believers in the nature position, on the other hand, say the genetics is crucial in development, and that the characteristics of an individual are determined at birth.

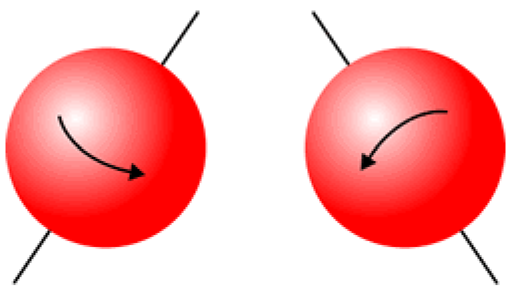
The data are fairly clear that both genetics and environment are approximately equally important in the development of an individual.

One of the types of studies that lead to this conclusion involve identical twins who were separated at birth. Such twins have almost completely identical DNA, and sometimes were raised in very different social environments. Nonetheless, there are often strong correlations between the later behavior of such twins: if one is, say, a firefighter than often the other is also a firefighter. Other characteristics that twins tend to share, even if raised in very different environments, include physical characteristics of their choice of mate, preferences in music, and more.

Later it will be important to note that the correlations are not 100%. Just because one twin is a firefighter does not guarantee that the other is too. Similarly, if one twin really hates the music of Twisted Sister does not guarantee that the other twin will also despise that type of "music." Nonetheless, the correlations are sufficiently strong that it is almost certain that they did not arise by pure chance.

Pairs of Spinning Particles in Physics

Most elementary particles, such as electrons, photons, etc., have an intrinsic angular momentum which is usually called *spin*. For our purposes, we can imagine the particle as a small ball that is spinning about some axis.

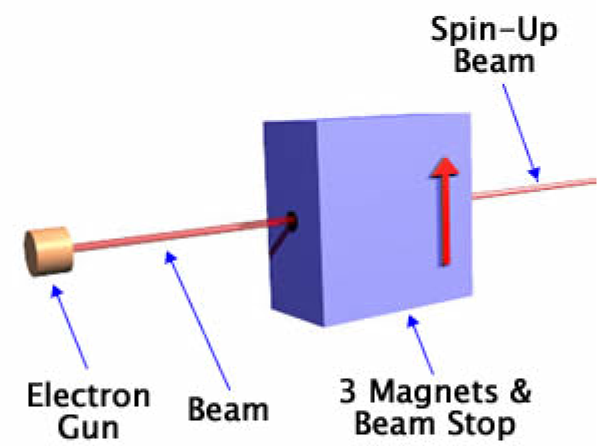


It turns out that the spin of an electron has only two states, which we call *up* and *down*. The origins of this terminology are not important for our purposes here. In the figure, the electron on the left is spin-up and the electron on the right is spin-down.

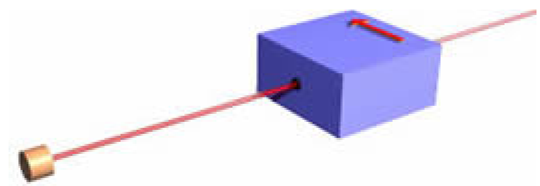
It is possible to construct a "filter" that selects only spin-up electrons. Again the details of what is in the box are not important for our purposes.

What is important here is that one-half of the electrons from the electron gun will emerge from the filter with the same speed in the same direction as before they entered the box, and one-half of the electrons will not emerge. Which is the case for an individual electron is random.

You will also want to notice that we have painted an arrow on the side of the box to indicate what direction is *up*.



The apparatus actually defines the direction of up. If we rotate it by some angle, again one-half of the incident electrons emerge from the filter, and which is the case for an individual electron is random.



There are some radioactive materials for which when an individual atom decays it simultaneously emits two electrons in opposite directions. These pairs of electrons have a total spin of zero: if one electron is spin-up, its companion electron is spin-down and vice versa. Such pairs of particles are called *entangled quantum pairs*.

The Analogy

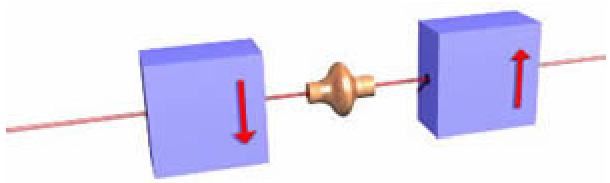
We shall begin by assuming that the *nurture* position on human development is correct. This assumption means that we would expect that for identical twins separated at birth, any later correlations in their choices of profession, mate, etc. are due to similarities in the environment in which the twins were raised. Each twin's environment is *local* to the separate individual, and we are assuming that this local environment causes the later behavior of the individual. In Physics-speak, we call this assumption **local causality**.

There are two kinds of correlation experiments that we can do.

1. Correlations in the choice of profession, or the taste in music, or some other characteristic.
2. Correlations of the choice of profession of one twin with the musical taste of the other twin. This is a more sophisticated experiment, and the analysis will require more statistical information.

The studies of the correlations of twin behavior, then, show that the assumption of local causality is incorrect.

We can also do correlation experiments for the entangled quantum pairs of electrons that we discussed above. The source of the total spin-zero entangled electron pairs is in the center of the figure, and the two electrons from each decay go in opposite directions.



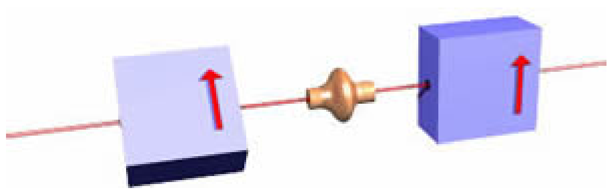
Note that the two filters have opposite orientations.

It turns out that one-half of the electrons traveling to the right pass through its filter and one-half do not, and which is the case for an individual electron is random. Similarly one-half of the electrons traveling to the left pass through its filter and one-half do not, and which is the case for an individual electron is random. However, when we examine the correlations, if the right-hand electron passes through its filter its companion left-hand electron also passes through its filter. If the right-hand electron does not pass through its filter its companion left-hand electron does not pass through its filter. This is a consequence of the fact that the two electrons have a total spin of zero, so if one is spin-up the other is spin-down *provided the direction of "up" is the same for both measurements*. Here the direction of "up" is opposite for the two filters.

For the identical twins, correlations in the same characteristic such as profession showed that local causality is not true in the nature vs. nurture debate. However, this correlation in electron spins does not violate local causality. This circumstance is more analogous to the following:

We carefully saw a coin in half along its plane, so one piece has the "head" and the other has the "tails." We put each piece in an envelope and walk the two envelopes away from each other. If we open one envelope and see a heads, we are guaranteed that the other piece contains a tails.

However, when we set the electron filters at orientations other than opposite each other we see strange correlations. To the right we have the right-hand filter oriented at zero degrees, and the left-hand filter tipped by 45 degrees.



In fact, the conflict with local causality for entangled electron spin correlations only shows up when we set the right and left hand filters at different angles. This is analogous the the twin correlation measurements where we try to correlate the profession of one with the musical taste of the other. The actual tests involve orienting the filters at zero degrees, 45 degrees, and 90 degrees.

GJ Comment: Certainly, the analysis can proceed with the three angles, 0, 45, and 90 degrees, However, the Mermin 1985 article analyzes the case of SG oriented at 0,120, -120 degrees .

The electron that goes through SG0 will result in a green light, and an electron that passes in SG1 will result in a green light.

Let the angle between the two SG's d=0 degrees. The probability of measuring GG = 1/2 Cos[d/2]^2=1/2, and the probability of measuring RR=1/2 Cos[d/2]^2=1/2. The probability of measuring the same color then is 1 when the switches are set to either 11 or 22 or 33.

If the switches are completely randomized, there are 9 possible orientations of the switches. So, P11+P22+P33 = 3/9 for same color and when switches happen to be the same.

If SG0 is set to 0 degrees (position 1) and SG1 is set to +120 degrees (position 2), the probability of obtaining GG = 1/2 Cos[120/2]^2=1/8, RR = 1/2 Cos[120/2]^2 =1/8. Same outcomes (since d is the same) for positions 12, 21, 23, 32, 13, and 31. Probability of measuring the same color when switches are set to a different position is 1/4. So P12+P21+P13+P31+P23+P32 = (3/2)/9 for same color when switches happen to be different.

Therefore, the total probability for obtaining the same color = 4.5/9 for any random switch configuration. The total probability for obtaining different colors for any random switch configuration is 4.5/9. Since the probability is zero for obtaining different colors in orientation 11, 22, and 33, the probability of obtaining different colors in orientation 12, 21, 13, and 31 is necessarily 1.5/9.

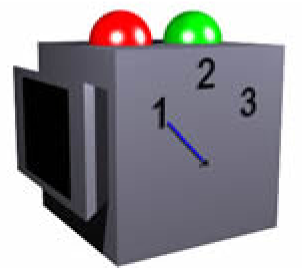
This is consistent with Mermin 1985 Physics today scenario:

> ▶ If one examines only those runs in which the switches have the same setting (figure 4), then one finds that the lights always flash the same colors.
> ▶ If one examines all runs, without any regard to how the switches are set (figure 5), then one finds that the pattern of flashing is completely random. In particular, half the time the lights flash the same colors, and half the time different colors.

We explore in more detail how this conflict arises in the next section.

The Second Analogy: More Boxes

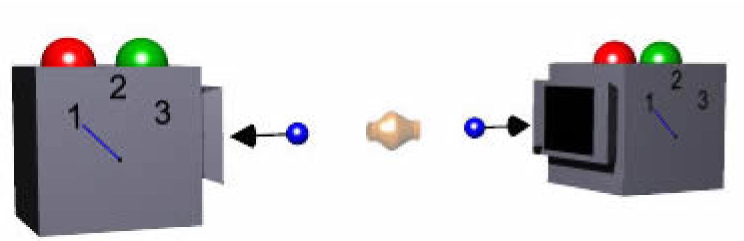
A Flash animation that duplicates much of the discussion of this section is available. It requires Flash 6, and has a file size of 78k. You may access it by clicking here. It will appear in a separate window.



We imagine a box, such as is shown to the right. Although nobody has every made such a box, there is no reason why it could not be constructed. It has a red and green light on top, and a switch that can be set to three positions: 1, 2, and 3. The apparatus is self contained, and has batteries inside to drive the lights and whatever mechanism is inside.

The box is a detector, and one of the lights will light up when a particle enters it from the left.

We have two of these detectors, and place them on either side of some device which emits pairs of particles in opposite directions.



We have bazillions of pairs of these particles go through the detectors, and set the switch positions randomly for each pair. If the boxes are measuring electron spin, then the switches could correspond to orienting the spin filters at zero, 45, and 90 degrees, and the pairs of particles could be entangled electron pairs. Soon we shall attempt to build another more classical model of what is being measured, and will run into trouble with it.

We record which lights flash for each pair and what are the switch positions. There are two cases:

1. If both switches on the boxes are set to the the same positions, either 1 or 2 or 3, the same light flashes on both boxes. Either both red lights flash or both green lights flash. Half of the time both red lights flash, the other half of the time both green lights flash.
2. If the switches are set to different positions both detectors flash the same color one-quarter of the time, either both red or both green. One half of the time when both colors flash they are both red, the other half of the time they are both green. Three-quarters of the time the detectors flash different colors, either red on the left and green on the right or green on the left and red on the right; in this case each of the two possibilities occur half of the time.

Explaining Case 1

Imagine that when the switch is in position 1 it measures the speed of the object, when it is position 2 it measures the size, and in position 3 it measure the shape of the object.

Switch Position	Measures	Green Light	Red Light
1	Speed	Flashes when particle is going fast	Flashes when particle is going slow
2	Size	Flashes when particle is big	Flashes when particle is small
3	Shape	Flashes when the particle is a sphere	Flashes when the particle is not a sphere

Then the experimental results are easy to explain:

- The pairs of particles always have the same speeds, the same size, and the same shape.
- Half of the time both the particles are moving fast, half of the time both are moving slow.
- Half of the time both the particles are big, half of the time they are both small.
- Half of the time both particles are spheres, half of the time they are not.
- There are eight different states the pairs of particles can be in, each occurring with equal frequency:
    1. Fast big spheres.
    2. Slow big spheres.
    3. Fast little spheres.
    4. Slow little spheres.
    5. Fast big non-spheres
    6. Slow big non-spheres
    7. Fast little non-spheres.
    8. Slow little non-spheres.

What About Case 2?

There are six settings of the switches which are different.

For the case of fast big spheres here are the possible switch settings and the results:

Left Switch	Left Light	Right Switch	Right Light
1	Green	2	Green

2	Green	1	Green
2	Green	3	Green
3	Green	2	Green
1	Green	3	Green
3	Green	1	Green

So for this case all the switch settings end up with the both green lights flashing. For slow small non-spheres, similarly, both red lights will flash for all six switch positions. We expect one-quarter of the bazillion pairs of particles to be either fast big spheres or slow small non-spheres. So far so good: the experimental result is that the lights flash the same color one-quarter of the time.

But imagine the case of pairs of fast big non-spheres. Here are the possible switch settings and the results:

Left Switch	**Left Light**	**Right Switch**	**Right Light**
1	Green	2	Green
2	Green	1	Green
2	Green	3	Red
3	Red	2	Green
1	Green	3	Red
3	Red	1	Green

Only two of the six possible settings have both lights flash the same color, green in this case. But the switch settings are made at random, so we expect each of the six possible results in the above table to occur with equal frequency. So both lights flash the same color one-third of the time.

The same argument can be made for the other five pairs that are not big fast spheres or small slow non-spheres: both lights will flash the same color one-third of the time.

Imagine we take data for 24 bazillion pairs of particles. We expect each of the eight possible states of speed, size, and shape to occur with equal frequency, so our sample will have 3 bazillion pairs of each type. We then expect the following results when the switches are set to different positions:

Switches are in different positions				
Type	**Number**	**Number of Pairs For Which the 2 Lights Flash the Same Color**	**Color**	**Fraction**
fast big spheres	3 bazillion	3 bazillion	both green	1
slow big spheres	3 bazillion	1 bazillion	both green	1/3
fast little spheres	3 bazillion	1 bazillion	both green	1/3
slow little spheres	3 bazillion	1 bazillion	both red	1/3
fast big non-spheres	3 bazillion	1 bazillion	both green	1/3
slow big non-spheres	3 bazillion	1 bazillion	both red	1/3
fast little non-spheres	3 bazillion	1 bazillion	both red	1/3
slow little non-spheres	3 bazillion	3 bazillion	both red	1
Total	**24 bazillion**	**12 bazillion**	**half both red, half both green**	**1/2**

So when we summarise the data for all the pairs of particles that we measured, we would not expect to have different colors flashing on the two detectors one-quarter of the time, but instead one-half of the time. But the experimental result is one-quarter, not one-half.

This example has been thinking about classical objects, which is tantamount to assuming local causality. Thus we see that these correlation measurements violate local causality, in exactly the same way the the electron spin measurements of entangled electron pairs violate local causality.

This entire section is a slight simplification of Mermin's analysis, which is listed in the references. I close with the conclusion of that lovely paper:

I shall not describe how contemporary physical theory accounts for the behavior of the device except to note that although, in its own way, the explanation is very simple, it is far from obvious, and, some might argue, hardly an explanation at all. Instead, I only emphasize again that we live in a world where such a device can be built; nature is stranger and more wonderful than we had once thought or could possible [sic] have imagined. Ponder the device a little more if that seems too extreme a conclusion.

Further Study

Here are some documents on particle spin, Bell's Theorem, and the Nature vs. Nurture debate which are accessible to the layperson.

Spin

- A non-mathematical treatment, by the author of this document, is available on the web at: http://www.upscale.utoronto.ca/PVB/Harrison/SternGerlach/SternGerlach.html (html) http://www.upscale.utoronto.ca/PVB/Harrison/SternGerlach/SternGerlach.pdf (pdf)
- A wonderful discussion which does do a bit of the mathematics is Richard P. Feynman, Robert B. Leighton and Matthew Sands, **The Feynman Lectures on Physics**, Vol. III, Chapters 5 - 6, ISBN: 0201500647.

Bell's Theorem

- A brilliant example, which is non-mathematical but subtle, is N.D. Mermin, American Journal of Physics **49**, 940 (1981). Many institutions, including the University of Toronto, have subscriptions to this journal so they may be accessed from any computer whose IP number corresponds to the subscribing institution. The American Journal of Physics is available on-line at: http://scitation.aip.org/ajp/
- A clever proof, using simple Venn diagrams, is B. d'Espagnat, Scientific American **241**, 158 (November 1979).
- A "chaotic ball" analogy is C.H. Thompson and H. Holstein, Foundations of Physics Letters **9**, 357 (1996), http://www.arxiv.org/format/quant-ph/9611037 .
- A mostly non-mathematical treatment, by the author of this document, is available on the web at:
http://www.upscale.utoronto.ca/PVB/Harrison/BellsTheorem/BellsTheorem.html (html)
http://www.upscale.utoronto.ca/PVB/Harrison/BellsTheorem/BellsTheorem.pdf (pdf)

Nature versus Nurture

- Judith Rich Harris, **The Nurture Assumption** (Free Press,1999), ISBN**:** 0684857073
- Steven Pinker, **The Blank Slate** (Viking, 2002), ISBN: 0670031518

Author