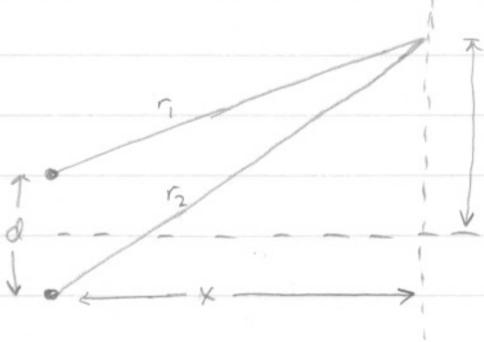
Solutions to hw 8

$$\Delta\phi = \frac{2\pi}{\lambda}(r_2 - r_1)$$

$$r_1^{\,2} = x^2 + \left(y - \frac{d}{2}\right)^2$$

$$r_2^{\,2} = x^2 + \left(y + \frac{d}{2}\right)^2$$

my numbers : $x = 1000\,m$, $d = 272\,m$



a) $y = 400\,m \Rightarrow r_2 - r_1 = 100.3\,m$

$\Delta\phi = 4\pi \Rightarrow \lambda = 50.2\,m$

b) The next minimum occurs when $\Delta\phi = 5\pi \Rightarrow r_2 - r_1 = \frac{5}{2}\lambda$. The algebra needed to obtain an exact solution for y is tedious, but we can obtain a good approximation by writing $y = y_0 + \Delta y$ and expanding with respect to Δy, such that

$$r_\pm = \left[x^2 + \left(y_0 + \Delta y \pm \frac{d}{2}\right)^2\right]^{1/2} \approx r_\pm^{(0)} + \frac{\partial r_\pm^{(0)}}{\partial y}\Delta y = r_\pm^{(0)} + \frac{y_\pm \Delta y}{r_\pm^{(0)}}$$

$$\Delta(r_+ - r_-) \approx \left(\frac{y_0 + \frac{d}{2}}{r_+^{(0)}} - \frac{y_0 - \frac{d}{2}}{r_-^{(0)}}\right)\Delta y$$

where (0) indicates the preceding results of part a). Using

$$y_0 = 400\,m \Rightarrow r_+^{(0)} = 1134.6\,m, \quad r_-^{(0)} = 1034.3\,m$$

$$\Delta(r_+ - r_-) = \frac{\lambda}{2} = \left(\frac{536}{1134.6} - \frac{264}{1034.3}\right)\Delta y \Rightarrow \Delta y = 115.6\,m$$

we obtain a first approximation, but Δy may not be sufficiently accurate because $\Delta y / y$ is not really small. To improve upon this approximation, we compute $\Delta\phi$ for several nearby values and interpolate.

Δy	$\Delta\phi/\lambda$
116 m	2.47
120	2.486
124	2.5006

Therefore, the car must travel just a little less than 124 m further to reach the next minimum.

2) Here we do use the small-angle approximation.

a) $\delta = d \sin\theta = \frac{yd}{L}$; $y = 1.25\,cm$, $L = 150\,cm$, $d = 0.1\,mm \Rightarrow \delta = 8.33\times10^{-7}\,m$

b) $\lambda = 500\,nm \Rightarrow \delta/\lambda = 1.67$

c) intermediate

3) $I = I_{max}\cos^2\frac{\Delta\phi}{2} \Rightarrow \Delta\phi = 2\cos^{-1}\left(\sqrt{\frac{I}{I_{max}}}\right)$

a) $I_{max}/I = 0.57 \Rightarrow \Delta\phi = 1.43\,rad$ is minimum phase difference

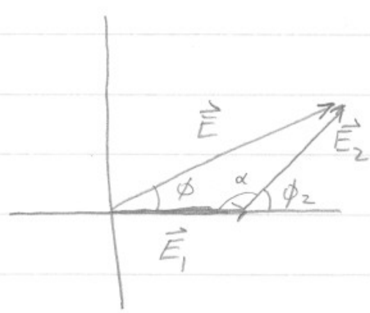
b) $\Delta\phi = \frac{2\pi}{\lambda}\Delta r \Rightarrow \Delta r = \lambda\Delta\phi/2\pi$; $\lambda = 487.3\,nm \Rightarrow \Delta r = 111\,nm$

4) $E^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\alpha$, $\alpha = \pi - \phi_2$

$\sin\phi = \frac{E_2}{E}\sin\phi_2$

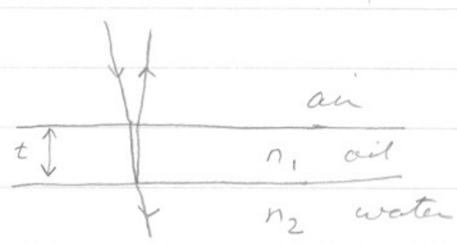
my numbers: $E_1 = 12$, $E_2 = 16.5$, $\phi_2 = 60^\circ$

$\therefore E = 24.8$, $\phi = 35.2^\circ$



5) $n_1 = 1.47$, $n_2 = 1.33$, $t = 286\,nm$

a) External reflection at the air-oil changes the phase by π, but internal reflection at the oil-water interface does not change the phase. The total phase difference between two reflected beams at normal incidence is then



$$\Delta\phi = 2\pi \frac{2n_1 t}{\lambda} - \pi$$

Constructive interference is obtained when $\Delta\phi = 2\pi m$ for integer m, such that

$$(m + \tfrac{1}{2})\lambda = 2n_1 t$$

The only solution for λ that lies in the visible spectrum is

$$m = 1 \Rightarrow \lambda = 561\ nm$$

) Light reflected from the oil-water and then oil-air interface and then transmitted into water experiences two internal reflections that do not affect the phase. Thus,

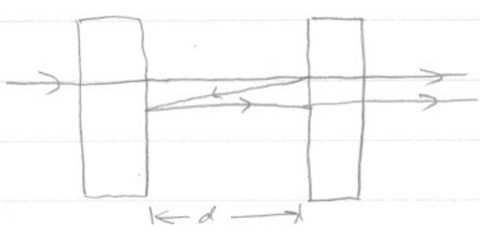
$$\Delta\phi = 2\pi \frac{2n_1 t}{\lambda} = 2\pi m \Rightarrow m\lambda = 2n_1 t$$

The only solution in the visible range is

$$m = 2 \Rightarrow \lambda = 420\ nm.$$

$$\Delta\phi = 2\pi \frac{2d}{\lambda} = 2\pi m$$

$$\therefore \quad d = m\frac{\lambda}{2}$$



$$\lambda = 580 nm, \quad m = 1 \Rightarrow d = 290 nm$$

) Interference between light reflected either from the top or the bottom of the air gap produces dark fringes when

$$\Delta\phi = \frac{2\pi}{\lambda}2t + \pi = 2\pi\left(m + \frac{1}{2}\right)$$

where m is an integer. Note that one reflection is internal and the other external, so that there is a net phase difference of π due to reflection. Using

$$\lambda = 550.1 \text{ nm}, \quad t \leq 4 \times 10^{-3} \text{ cm} \Rightarrow 0 \leq m \leq 145.43.$$

Thus, including the contact fringe ($m=0$) there are a total of 146 dark bands.

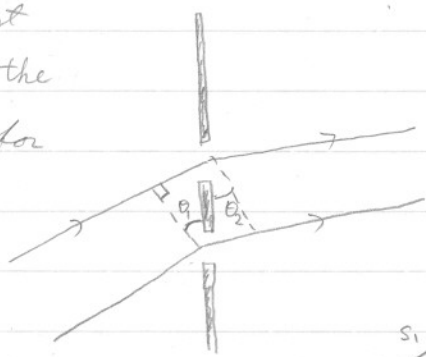
$$2\sqrt{h^2 + \left(\frac{d}{2}\right)^2} - d = \frac{\lambda}{2}$$

$$h^2 = \frac{1}{4}\left(d + \frac{\lambda}{2}\right)^2 - \left(\frac{d}{2}\right)^2$$
$$= \frac{1}{16}\left(4\lambda d + \lambda^2\right)$$

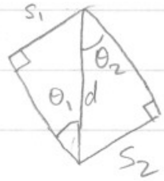
$$\therefore \; h = \frac{\lambda}{4}\sqrt{1 + 4\frac{d}{\lambda}} \approx \frac{1}{2}\sqrt{\lambda d}$$

my numbers: $d = 33$ km, $\lambda = 345$ m $\Rightarrow$ $h = 1.69$ km

P37.12) If the incident light is at an angle θ_1 wrt normal, the total path length difference for parallel rays has two contributions



$$\Delta s = s_2 - s_1$$
$$= d(\sin\theta_2 - \sin\theta_1)$$



Constructive interference requires

$$\Delta s = m\lambda = d(\sin\theta_2 - \sin\theta_1)$$

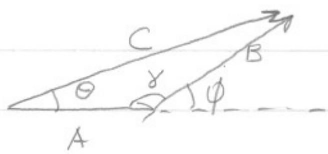
where m is an integer.

P37.22) We can add two phasors of different amplitude using the accompanying diagram. According to the law of cosines

$$C^2 = A^2 + B^2 - 2AB\cos\gamma$$

but

$$\gamma = \pi - \phi \Rightarrow \cos\gamma = -\sin\phi$$



such that

$$C^2 = A^2 + B^2 + 2AB\sin\phi$$
$$\sin\theta = \frac{B\sin\phi}{C}$$

$$\therefore\ E_{01}\sin\omega t + E_{02}\sin(\omega t + \phi) = E_0\sin(\omega t + \theta) \Rightarrow E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\sin\phi$$
$$\sin\theta = \frac{E_{02}}{E_0}\sin\phi$$

P37.44) Traversing the cell twice, the total phase difference between empty and full cells is

$$\Delta\phi = 2\pi\frac{n-1}{\lambda}2L = 2\pi N \Rightarrow N = (n-1)\frac{2L}{\lambda} \Rightarrow n-1 = \frac{N\lambda}{2L}$$