Solutions to hw 4

1) my numbers: $N = 100$, $d = 10\,mm$, $l = 9\,cm$, $\mu_m = 650\,\mu_0$

a) $\mathcal{L} = \mu_0 N^2 \frac{A}{l} = 11\,\mu H$

b) $\mu_m = 650\,\mu_0 \Rightarrow \mathcal{L} = 7.13\,mH$

2) my numbers: $\mathcal{E} = 6V$, $L = 9.0\,mH$, $R = 4.0\,\Omega$

a) $\tau = \frac{L}{R} = 2.25\,ms$

b) $I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$, $t = 250\,\mu s \Rightarrow I = 0.1577\,A$

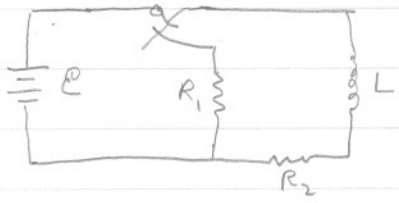
c) $t \to \infty \Rightarrow I \to \frac{\mathcal{E}}{R} = 1.5\,A$

d) $f = 1 - e^{-t/\tau} \Rightarrow t = -\tau \ln(1-f)$; $f = 0.8 \Rightarrow t = 3.62\,ms$

3) my numbers: $\mathcal{E} = 12.5V$, $R_1 = 1200\,\Omega$, $R_2 = 12.0\,\Omega$, $L = 2.0\,H$

a) $I \to \frac{\mathcal{E}}{R_2} = 1.04\,A$

b) $IR_2 = 12.5\,V$, $IR_1 = 1.25\,kV$ $\Big\}$ $\Delta V_L = 1262.5\,V$



c) $\tau = \frac{L}{R_1 + R_2} = 1.65\,ms$

$e^{-t/\tau} = \frac{12.5}{1262.5} \Rightarrow t = 7.26\,ms$

4a) $U = 4\pi \int_R^\infty dr\, r^2 \frac{B^2}{2\mu_0}$, $B = B_0\left(\frac{R}{r}\right)^2 \Rightarrow U = \frac{2\pi}{\mu_0} B_0^2 R^3$

b) $B_0 = 5.2 \times 10^{-5}\,T$, $R = 6.4 \times 10^6\,m \Rightarrow U = 3.54 \times 10^{18}\,J$

5) my numbers: $R = 110\,\Omega$, $L = 250\,mH$, $C = 2.0\,\mu F$, $\Delta V_{max} = 210\,V$, $f = 50\,Hz$

a) $X_L = \omega L = 78.5\,\Omega$ $\qquad \omega = 2\pi f$

b) $X_C = \frac{1}{\omega C} = 1.59\,k\Omega$

c) $Z = \left(R^2 + (X_L - X_C)^2\right)^{1/2} = 1.52\,k\Omega$

d) $I_{max} = \frac{V_{max}}{Z} = 138\,mA$

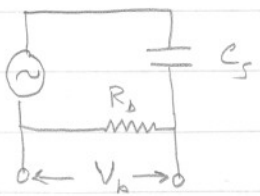
e) $\phi = \tan^{-1}\frac{X_L - X_C}{R} = -85.8^0$

my numbers: $V_{rms} = 5000\ V$, $C_s = 23\ pF$, $R_b = 40\ k\Omega$, $f = 60\ Hz$

$Z = (R^2 + X_C^2)^{1/2}$, $X_C = \frac{1}{\omega C}$ $\Rightarrow$ $Z = 1.15 \times 10^8\ \Omega$

$I_{rms} = \frac{V_{rms}}{Z} = 4.335 \times 10^{-5}\ A$

$V_b = I_{rms} R_b = 1.73\ V$



my numbers: $\mathcal{E}_{rms} = 120\ V$, $f = 60\ Hz$, $L = 28\ mH$, $R = 20\ \Omega$

a) $Z = (R^2 + X_L^2)^2$, $X_L = \omega L \Rightarrow Z = 22.6\ \Omega$ $\quad I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = 5.31\ A$

b) $\cos\phi = \frac{R}{Z} = 0.884$

c) need $X_L = X_C \Rightarrow \frac{1}{\omega C} = \omega L \Rightarrow C = \frac{1}{\omega^2 L} = 251.2\ \mu F$

d) $\mathcal{E}^0 \rightarrow \mathcal{E}^0 \cos\phi = 106\ V$

my numbers: $L = 20\ mH$, $C = 90\ nF$, $R = 10\ \Omega$, $\Delta V_{max} = 100\ V$

a) $\omega_0 = \sqrt{\frac{1}{LC}} = 2.357 \times 10^4\ rad/s = 2\pi f \Rightarrow f = 3.75\ kHz$

b) $\omega = \omega_0 \Rightarrow Z = R \Rightarrow I_{max} = \frac{V_{max}}{R} = 10\ A$

c) $Q = \frac{\omega}{\Delta\omega} = \omega_0 \frac{L}{R} = 47.1$

d) $Z_L = \omega L \Rightarrow \Delta V_L = \omega L I_{max} = V_{max} \frac{\omega L}{Z}$

$\omega = \omega_0 \Rightarrow Z = R \Rightarrow \Delta V_L = V_{max} \frac{\omega_0 L}{R} = 4.71\ kV$