PHYS 260 Supplimentary Notes

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CONTENTS

I. OSCILLATORS

A. Characteristic Frequencies

$$x''(t) + 2\gamma_0\, x'(t) + \omega_0^2 x(t) = 0 \tag{I.1}$$
$$\lambda^2 + 2\gamma_0\, \lambda + \omega_0^2 = 0 \tag{I.2}$$

For $\gamma_0 < \omega_0$:

$$\lambda = -\gamma_0 \pm \imath\sqrt{\omega_0^2 - \gamma_0^2} \tag{I.3}$$
$$\gamma = \gamma_0 \tag{I.4}$$
$$\omega = \sqrt{\omega_0^2 - \gamma_0^2} \tag{I.5}$$

For $\gamma_0 > \omega_0$:

$$\lambda = -\gamma_0 \pm \sqrt{\gamma_0^2 - \omega_0^2} \tag{I.6}$$
$$\gamma_{\mathrm{fast}} = \gamma_0 + \sqrt{\gamma_0^2 - \omega_0^2} \tag{I.7}$$
$$\gamma_{\mathrm{slow}} = \gamma_0 - \sqrt{\gamma_0^2 - \omega_0^2} \tag{I.8}$$
$$\omega = 0 \tag{I.9}$$

B. Driving Forces

$$x''(t) + 2\gamma_0\, x'(t) + \omega_0^2 x(t) = F_0 \cos(\omega_{\mathrm{ext}} t) \tag{I.10}$$

Form of under-damped solution (transient + response):

$$x(t) = x_{\mathrm{tran}}(t) + x_{\mathrm{res}}(t) \tag{I.11}$$
$$x_{\mathrm{tran}}(t) = A_{\mathrm{tran}} \cos(\omega t + \phi_{\mathrm{tran}})\, e^{-\gamma_0 t} \tag{I.12}$$
$$x_{\mathrm{res}}(t) = A_{\mathrm{res}} \cos(\omega_{\mathrm{ext}} t + \phi_{\mathrm{res}}) \tag{I.13}$$

where ω is given by (I.5).

Response amplitude:

$$A_{\mathrm{res}} = \frac{F_0/m}{\sqrt{\left(\omega_{\mathrm{ext}}^2 - \omega_0^2\right)^2 + \left(2\gamma_0\omega_{\mathrm{ext}}\right)^2}} \tag{I.14}$$

II. FLUIDS

A. Continuity

Continuity equation for incompressible fluid:

$$\sum_{i\in\mathrm{in}} \mathbf{v}_i \cdot \mathbf{A}_i = \sum_{i\in\mathrm{out}} \mathbf{v}_i \cdot \mathbf{A}_i \tag{II.1}$$

assuming no sources or sinks within the closed surface.

B. Drag

Stokes drag (high viscosity, laminar flow):

$$F_D = -6\pi\eta R v \tag{II.2}$$

Rayleigh drag (low viscosity, incompressible):

$$F_D = -\frac{1}{2} C\rho A v^2 \tag{II.3}$$

III. WAVES

The linear wave equation:

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2} D(x,t) = \frac{\partial^2}{\partial x^2} D(x,t) \tag{III.1}$$

Traveling wave solutions

$$D(x,t) = g(x \mp vt) \tag{III.2}$$

IV. THERMODYNAMICS

A. Perfect Blackbody Radiation

Plank's Law:

$$\frac{dI}{df} = \frac{2\pi h}{c^2} \frac{f^3}{e^{\frac{hf}{kT}} - 1} \tag{IV.1}$$

How this relates to the Steffan-Boltzmann law:

$$I = \int_0^\infty \frac{dI}{df} df = \underbrace{\frac{2\pi^5 k^4}{15 c^2 h^3}}_{\sigma} T^4 \tag{IV.2}$$

B. Boltzmann's Distribution

Defined in terms of zeroed ground-state energy and ground-state probability (as presented in Serway):

$$P(E) = P_0\, e^{-\frac{E}{kT}} \tag{IV.3}$$

$$P_0 = P(0) \tag{IV.4}$$

$$E_0 = 0 \tag{IV.5}$$

Defined in terms of ground-state probability and energy more generally:

$$P(E) = P_0\, e^{-\frac{E-E_0}{kT}} \tag{IV.6}$$

$$P_0 = P(0) \tag{IV.7}$$

V. ELECTROSTATICS

Electrostatic field of a point charge

$$\mathbf{E}(\mathbf{r}_E) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \mathbf{r} \tag{V.1}$$

$$\mathbf{r} = \mathbf{r}_E - \mathbf{r}_q \tag{V.2}$$

Electrostatic field of a microscopic dipole

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \left[3 \underbrace{(\mathbf{p} \cdot \hat{\mathbf{r}})\, \hat{\mathbf{r}}}_{\mathbf{p} \| \mathbf{r}} - \mathbf{p} \right] \tag{V.3}$$