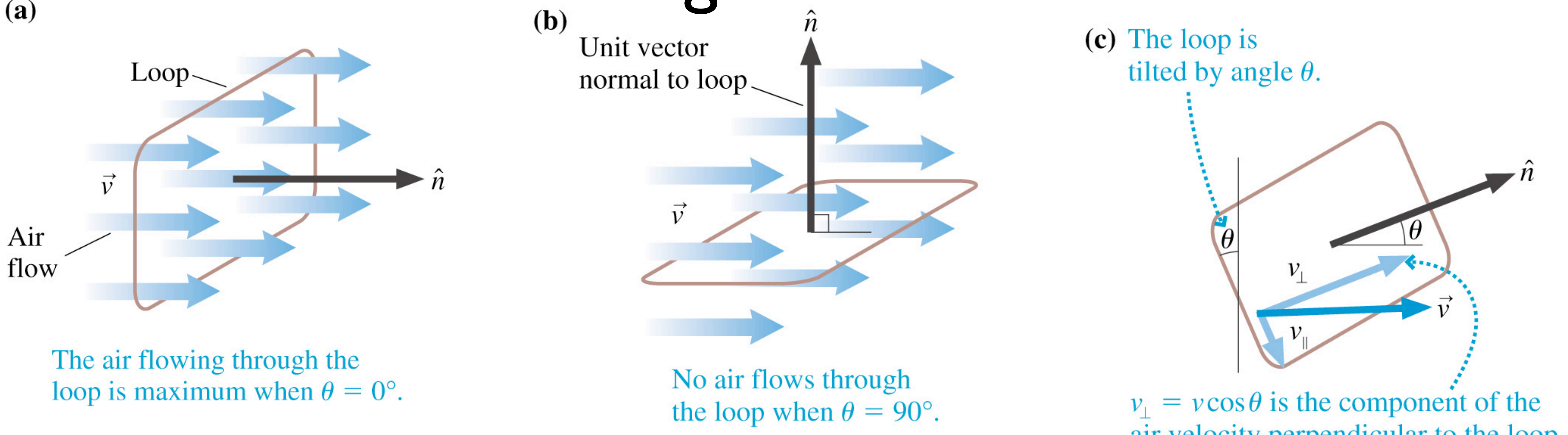
Lecture 20

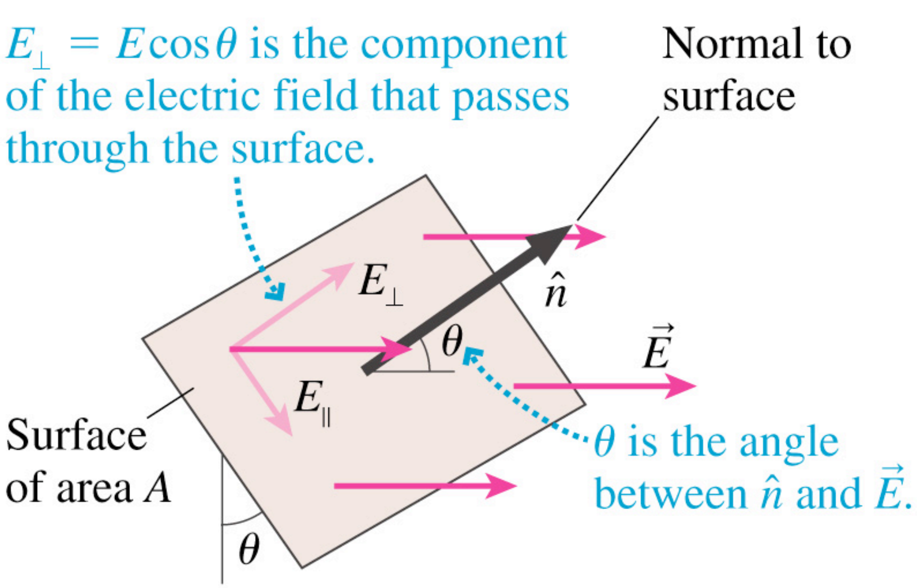
- calculating electric flux
- electric flux through closed surface required for Gauss's law: calculate $\bar{E}$ more easily; applies to moving charges
- Uses of Gauss's Law: charged sphere, wire, plane and conductor

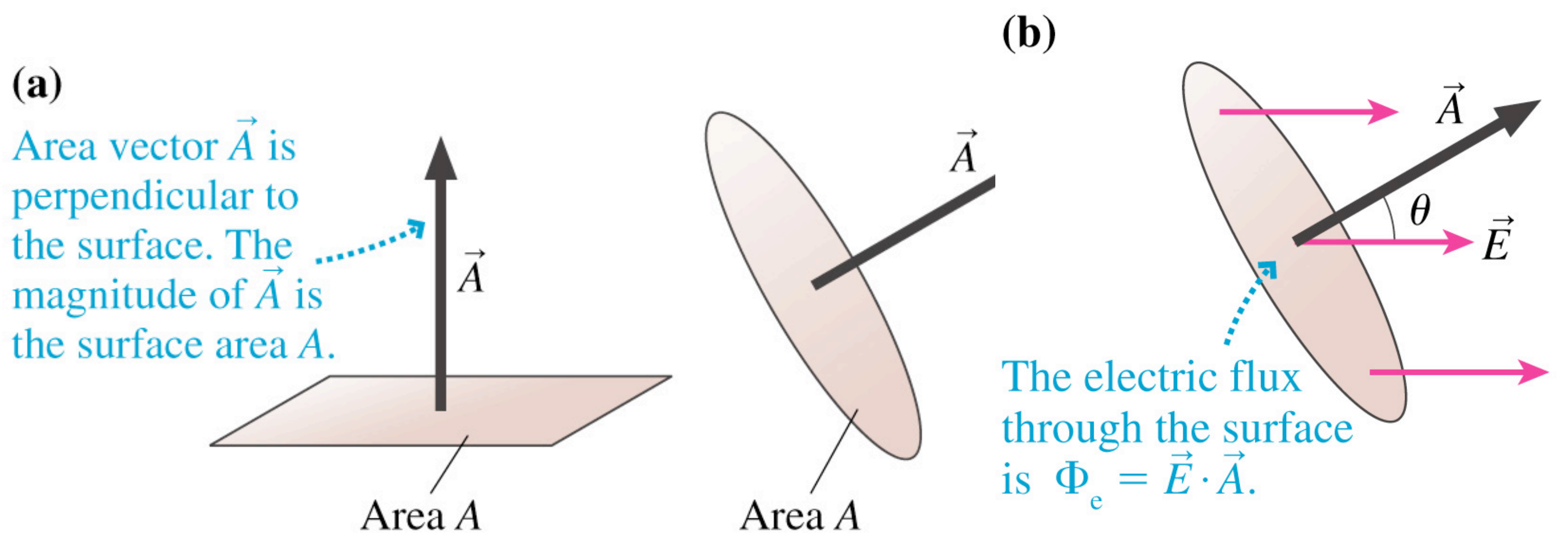
Calculating Electric Flux I



Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley
Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley

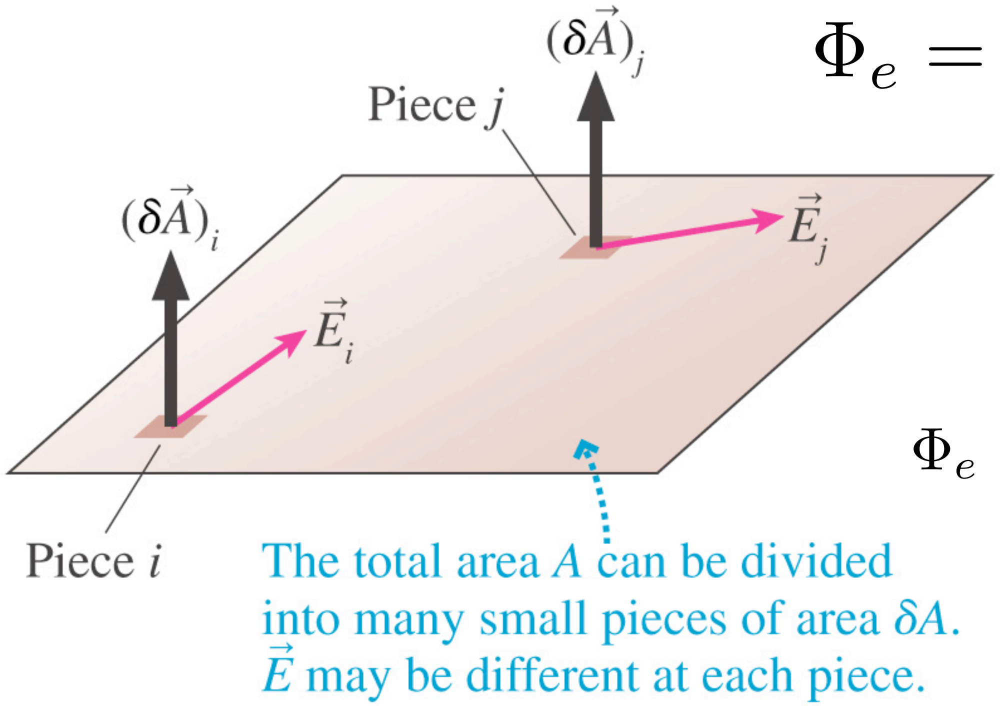
- **Analogy:** volume of air per second $(\mathrm{m}^3/\mathrm{s}) = v\ A = vA\cos\theta$
- **Electric Flux (amount of $\bar{E}$ thru' surface):** $\Phi_e = E\ A = EA\cos\theta$
- **Area vector:** $\bar{A} = A\hat{n} \rightarrow \Phi_e = \vec{E}\cdot\vec{A}$ (electric flux of a constant electric field)





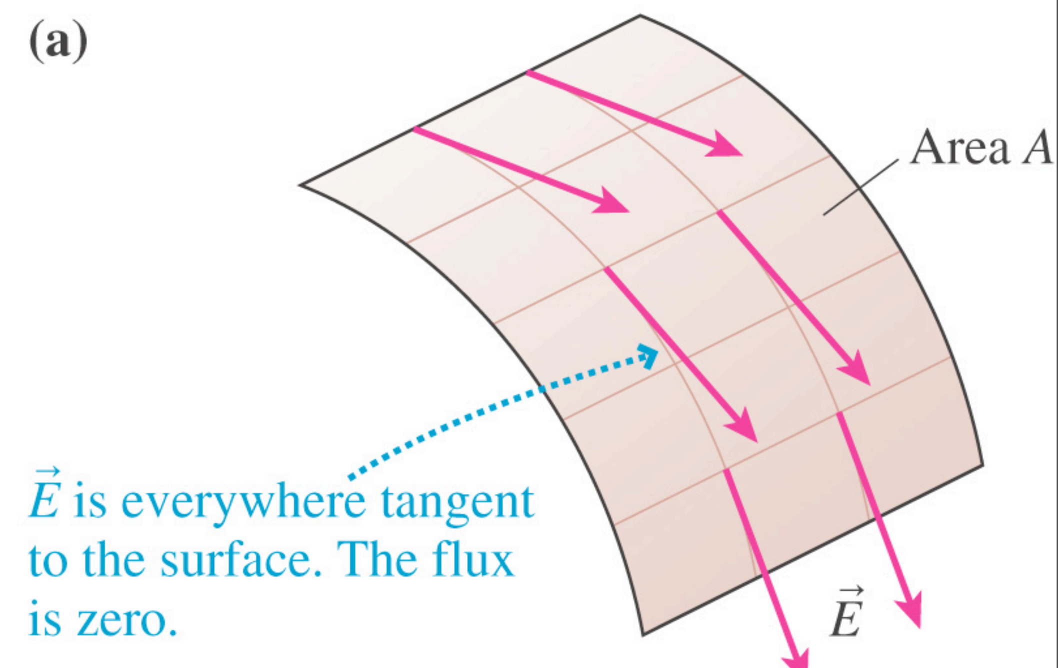
Calculating Electric Flux II

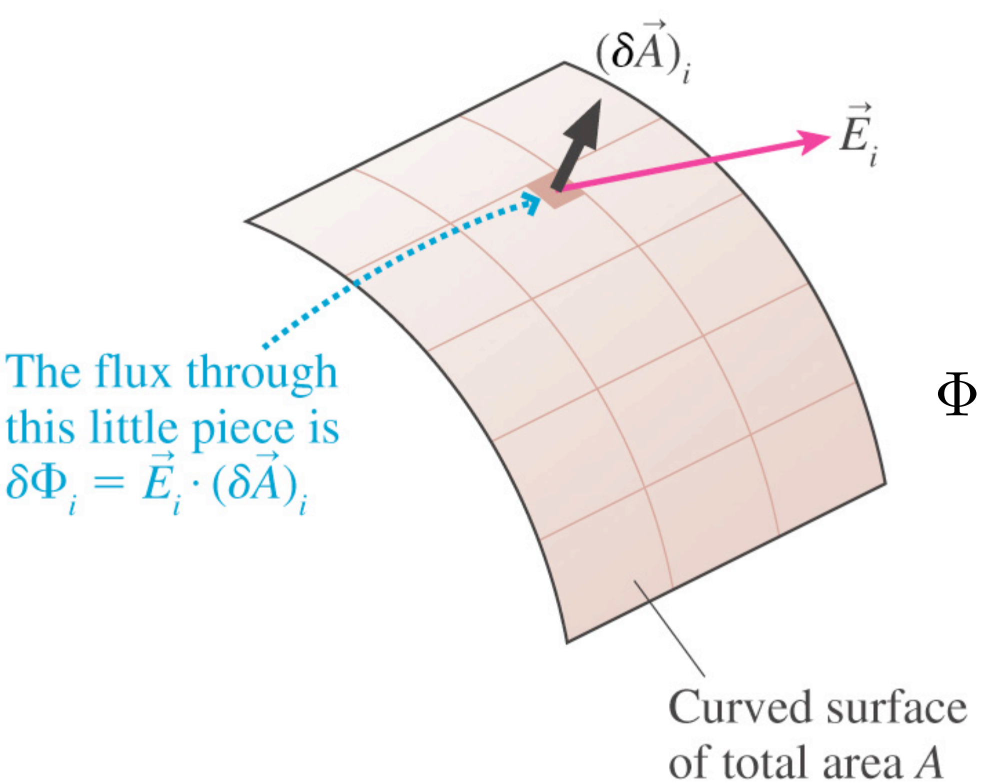
$$\Phi_e = \sum_i \delta\Phi_e = \sum_i \bar{E}_i . \left(\delta \bar{A}\right)_i \rightarrow \Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



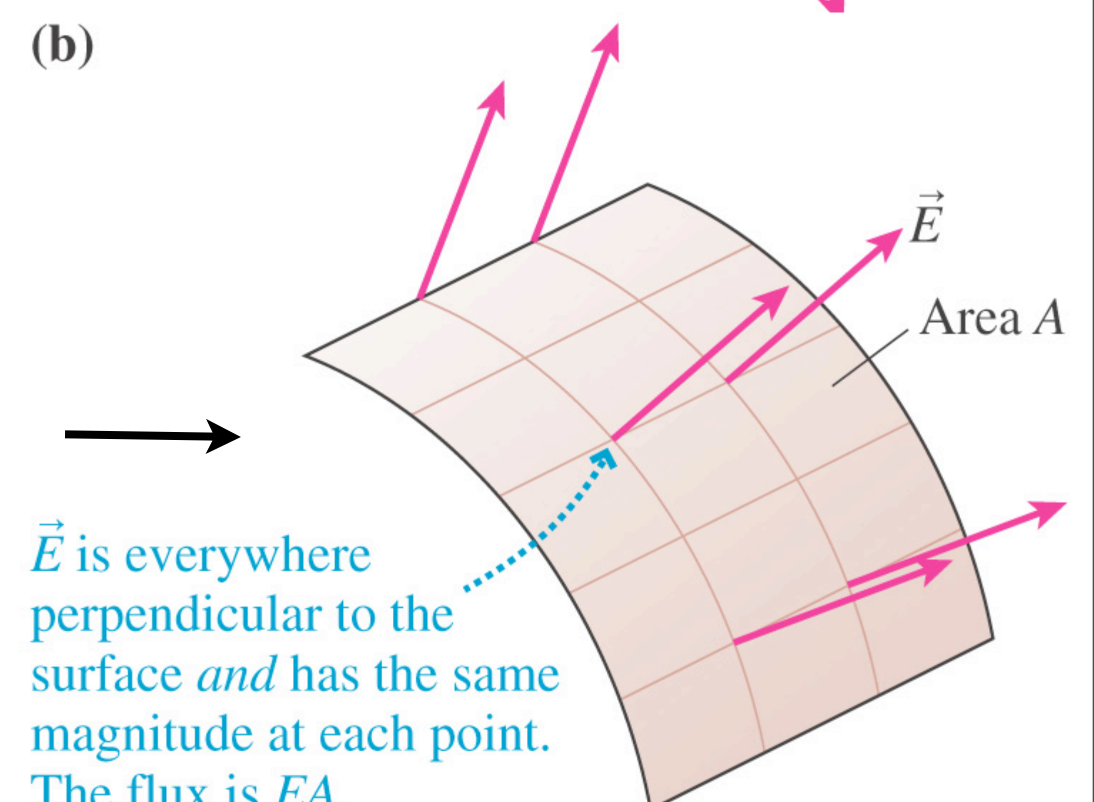
Uniform $\bar{E}$, flat surface:

$$\Phi_e = \int_{surface} \bar{E}.d\bar{A} = \int_{\dots} E\cos\theta dA = E\cos\theta \int_{\dots} dA = E\cos\theta A$$



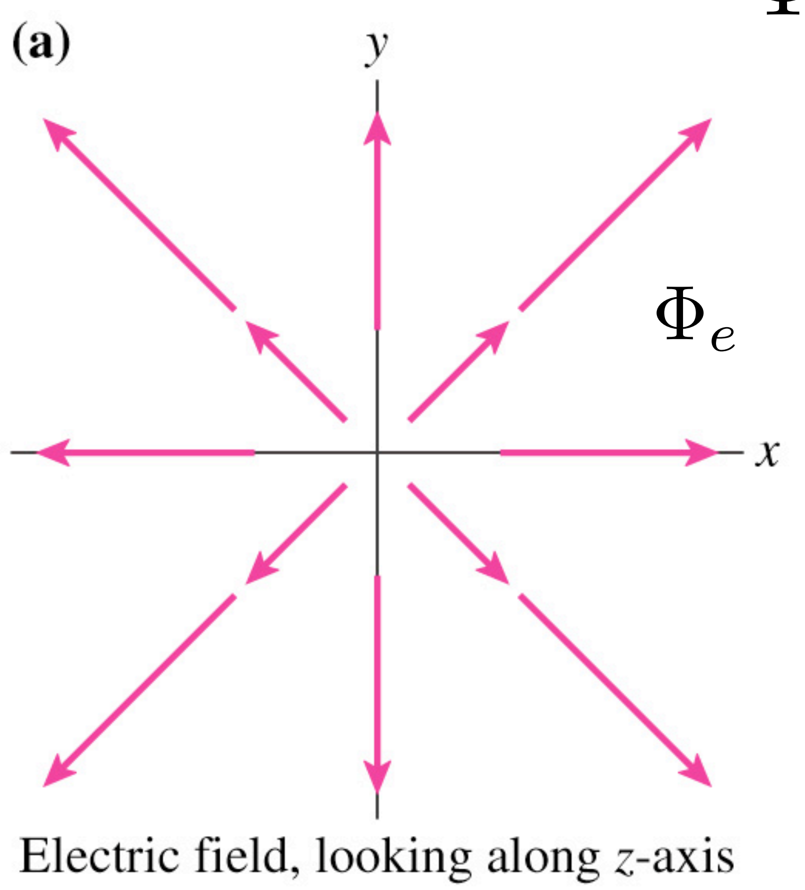


$$\Phi_e = \int_{surface} \bar{E}.d\bar{A} = \int_{\dots} EdA = E\int_{\dots} dA = EA$$



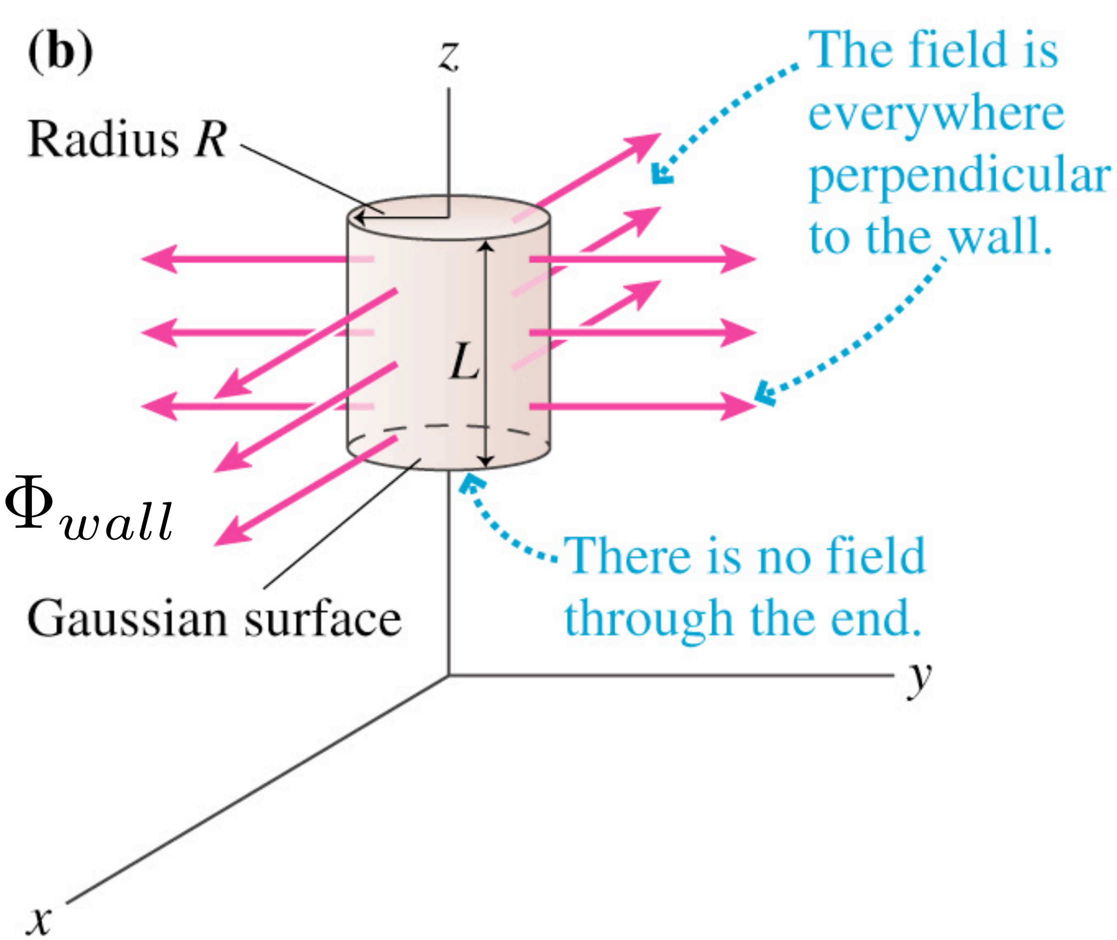
Calculating Electric Flux III

- Closed surface ($d\bar{A}$ points toward outside: ambiguous for single surface): $\Phi_e = \int \bar{E}.d\bar{A}$
- strategy: divide closed surface into either tangent or perpendicular to $\bar{E}$
- example: cylindrical charge distribution, $\bar{E} = E_0 \left(r^2/r_0^2\right)\hat{r}$ ($\hat{r}$ in xy-plane)



$$\Phi_{wall} = EA_{wall}$$

$$\begin{aligned}
\Phi_e &= \int \bar{E}.d\bar{A} \\
&= \Phi_{top} + \Phi_{bottom} + \Phi_{wall} \\
&= 0 + 0 + EA_{wall} \\
&= EA_{wall} \\
&= \left(E_0 \frac{R^2}{r_0^2}\right)(2\pi RL)
\end{aligned}$$



Flux due to point charge inside...

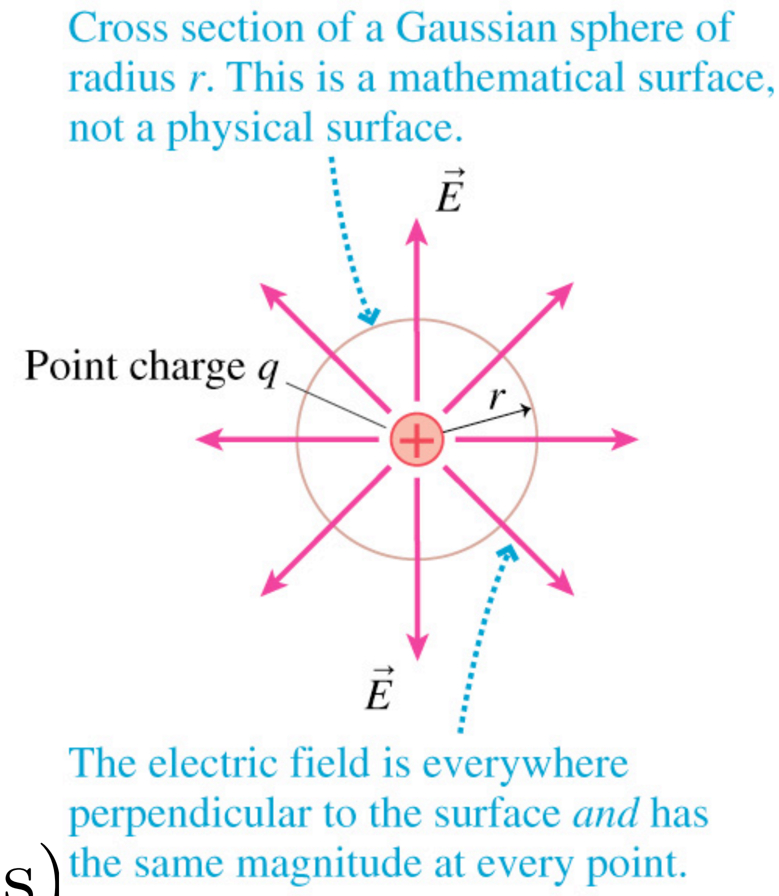
- Gaussian surface with same symmetry as of charge distribution and hence $\bar{E}$

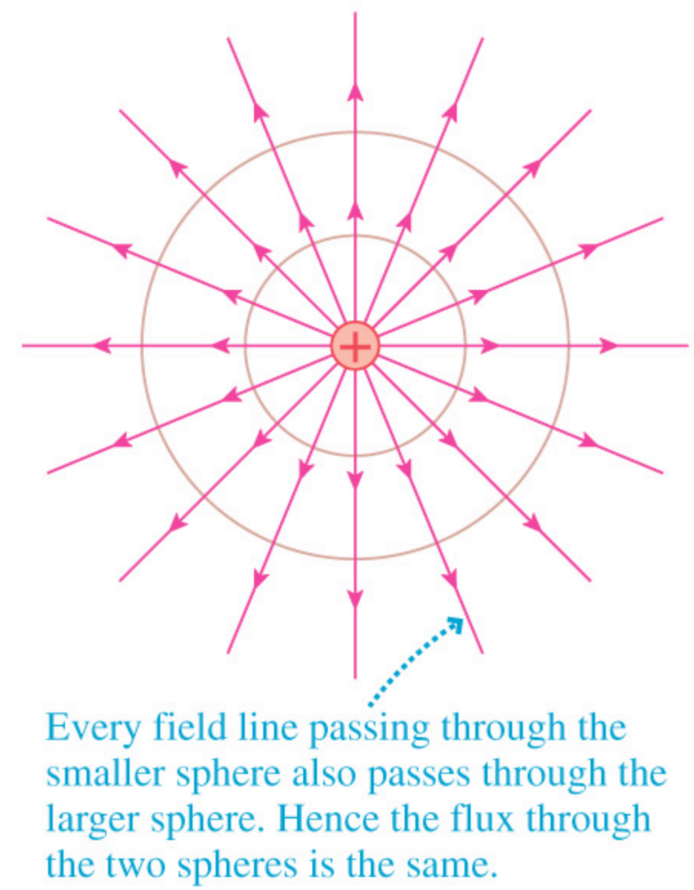
$$\Phi_e = \int \bar{E}.d\bar{A} = EA_{sphere}$$
$$(\bar{E} \perp \text{ and same at all points})$$
$$E = \frac{q}{4\pi\epsilon_0 r^2};\ A_{sphere} = 4\pi r^2$$
$$\Rightarrow \Phi_e = \frac{q}{\epsilon_0}$$

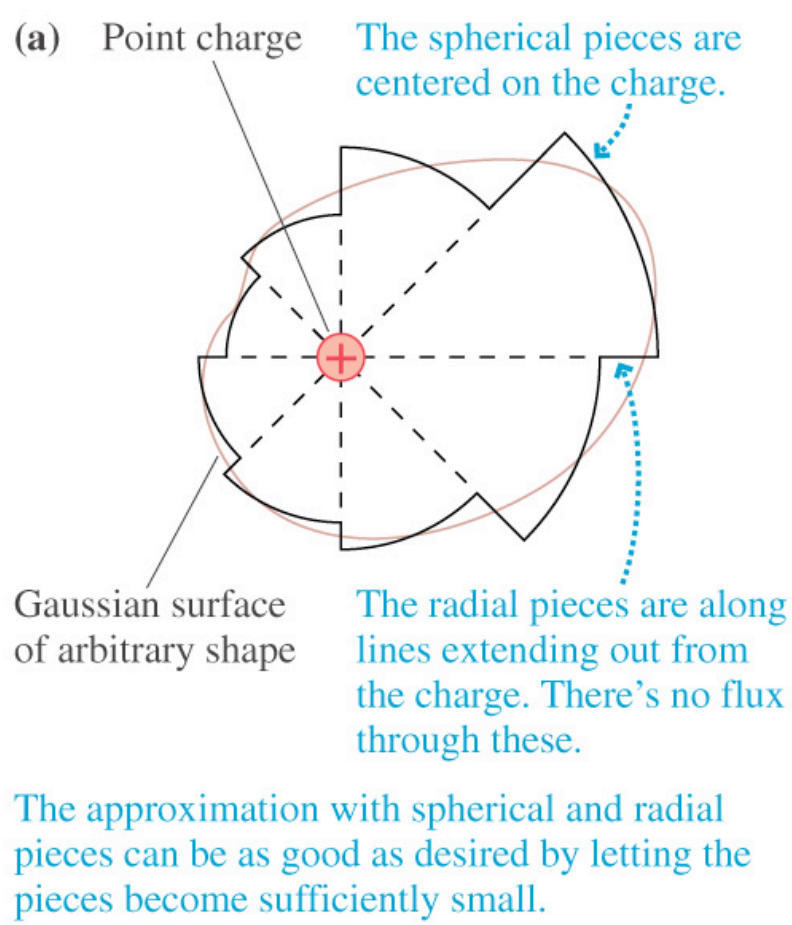
- flux independent of radius

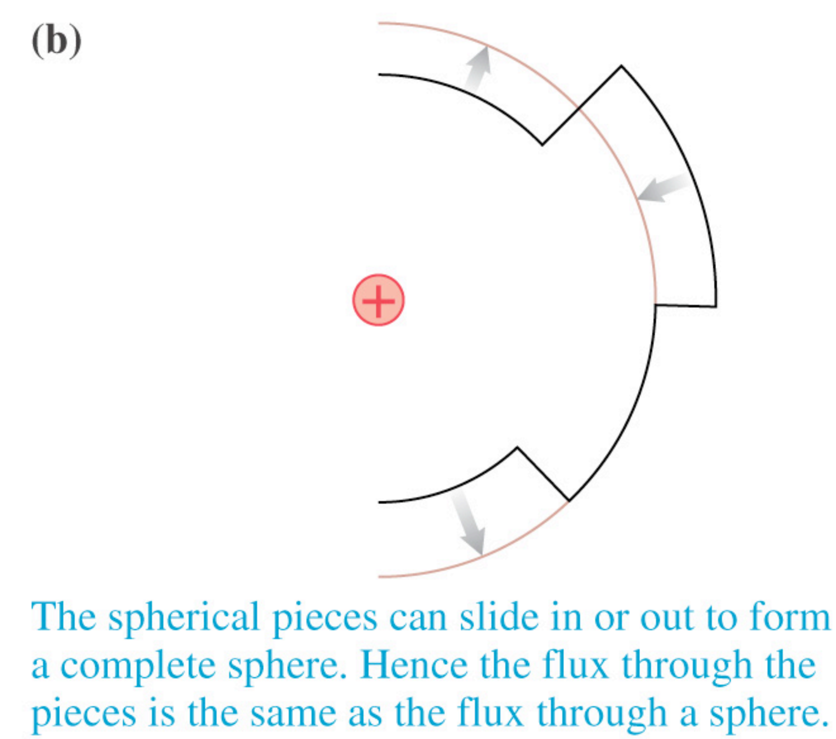
- approximate arbitrary shape...

$$\Phi_e = \int \bar{E}.d\bar{A} = \frac{q}{\epsilon_0}$$

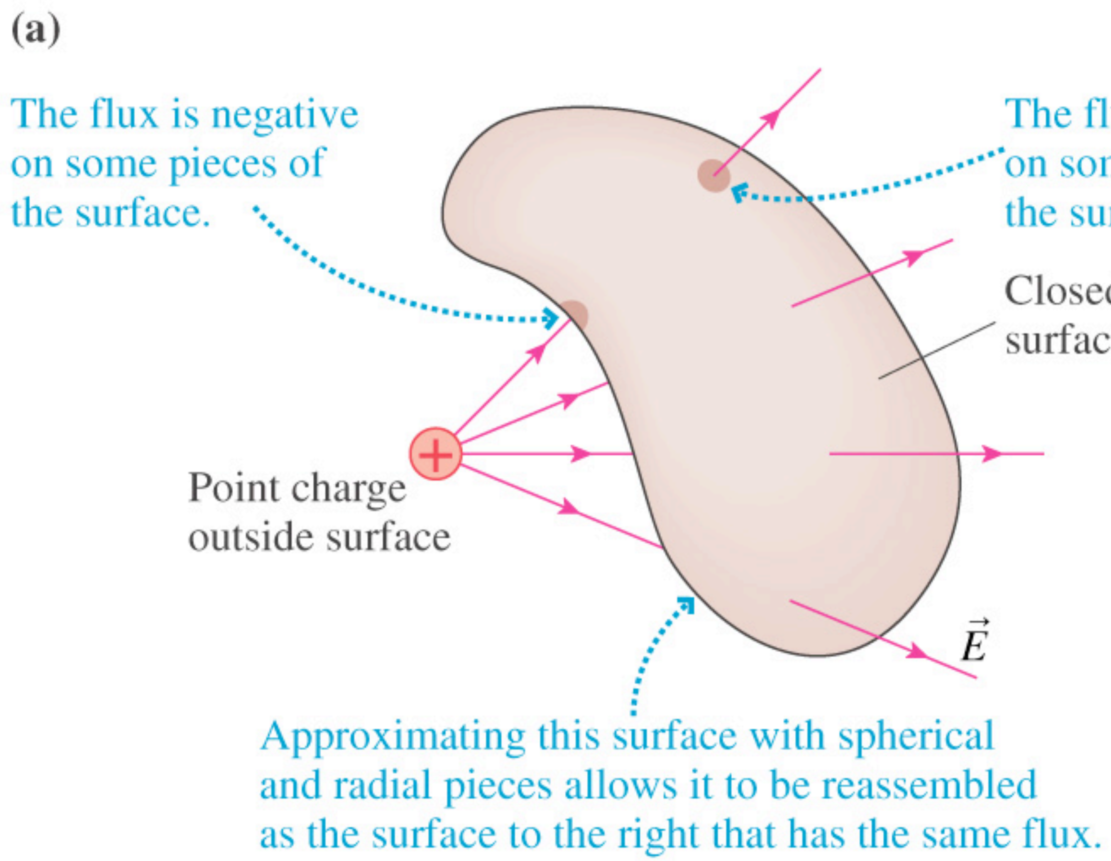


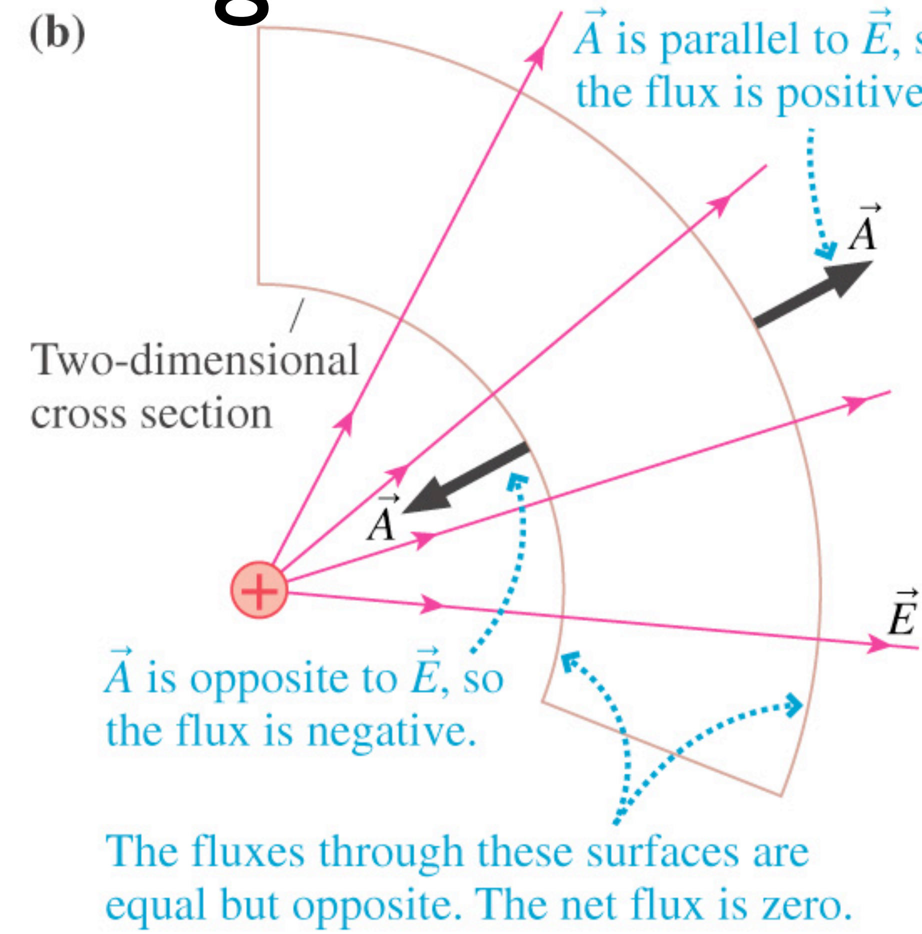


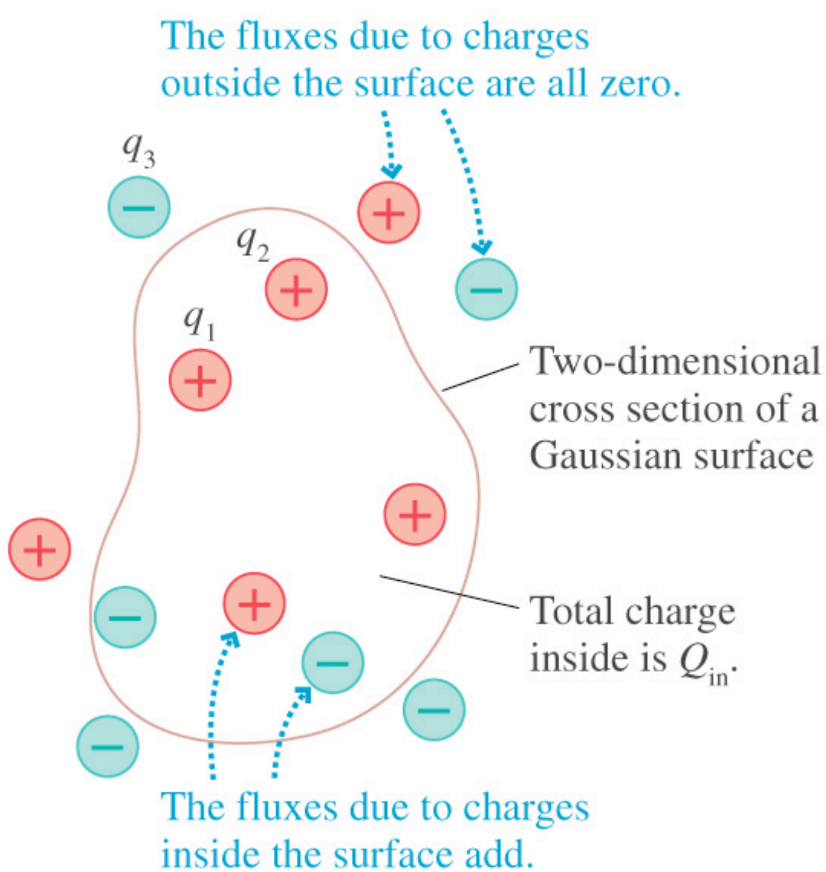




Charge outside..., multiple charges...Gauss's Law







$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \ldots \text{ (superposition)} \Rightarrow$$

$$\begin{aligned}
\Phi_e &= \int \bar{E}_1.d\bar{A} + \int \bar{E}_2.d\bar{A} + \ldots \\
&= \Phi_1 + \Phi_2 + \ldots \\
&= \left(\frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} \ldots \text{for all charges inside}\right) \\
&\quad +(0 + 0 + \ldots \text{for all charges outside})
\end{aligned}$$

$$Q_{in} = q_1 + q_2 + \ldots \text{ for all charges inside} \Rightarrow$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Using Gauss's Law

- Gauss's law derived from Coulomb's law, but states general property of $\bar{E}$: charges create $\bar{E}$; net flux "flow") thru' any surface surrounding is same
- quantitative: connect net flux to amount of charge

Strategy

- model charge distribution as one with symmetry (draw picture) symmetry of $\bar{E}$
- Gaussian surface (imaginary) of same symmetry (does not have to enclose all charge)
- $\bar{E}$ either tangent $(\Phi_e = 0)$ or perpendicular to $(\Phi_e = EA)$ surface

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Charged Sphere: $\bar{E}$ outside and inside

- charge distribution inside has spherical symmetry (need not be uniform)

  $\Phi_e = \int \bar{E}.d\bar{A} = \frac{Q_{in}}{\epsilon_0}$

  $EA_{sphere} = E 4\pi r^2$

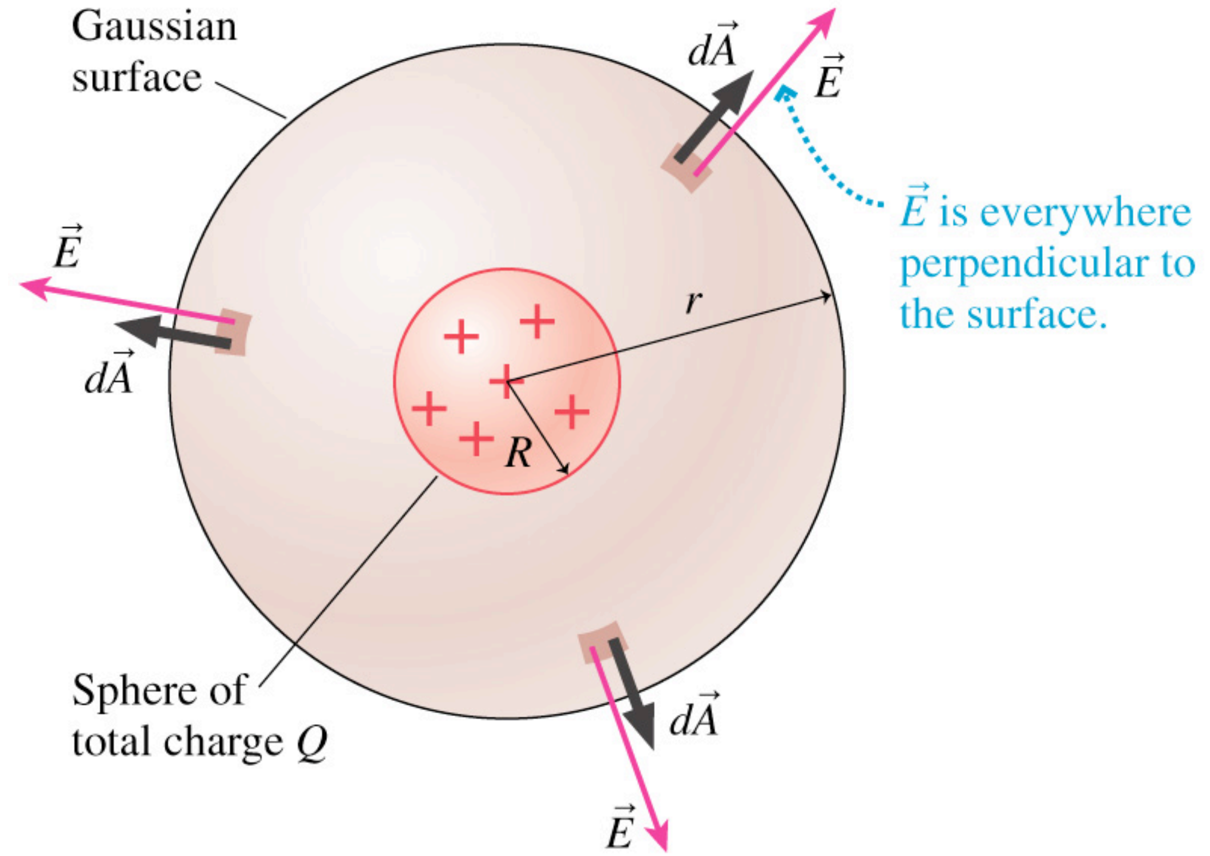
  (don't know E, but same at all points on surface)

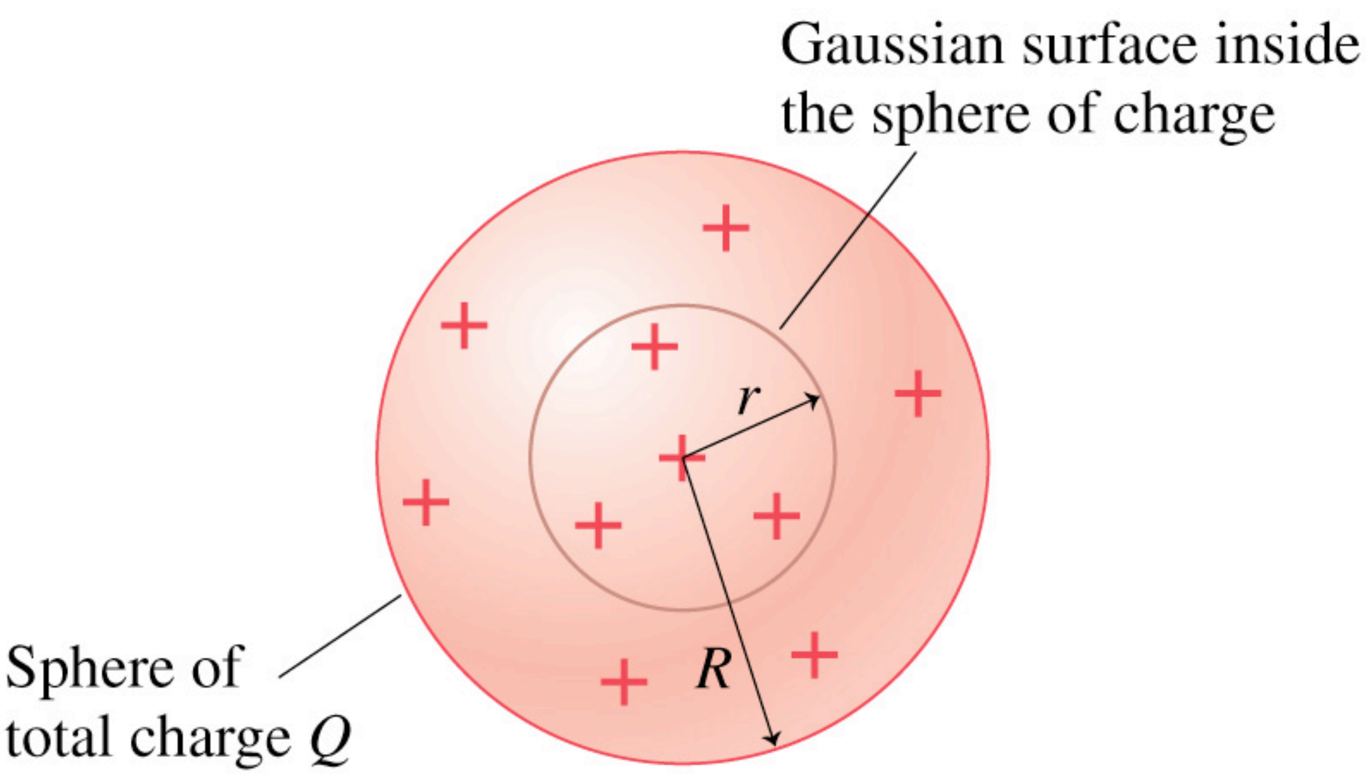
  $+ \; Q_{in} = Q$

  $\Rightarrow E = \frac{Q}{4\pi\epsilon_0}$

  (same as point charge)

- flux integral not easy for other surface
- using superposition requires 3D integral!
- spherical surface inside sphere $\Rightarrow Q_{in} \neq Q$





Charged wire and plane

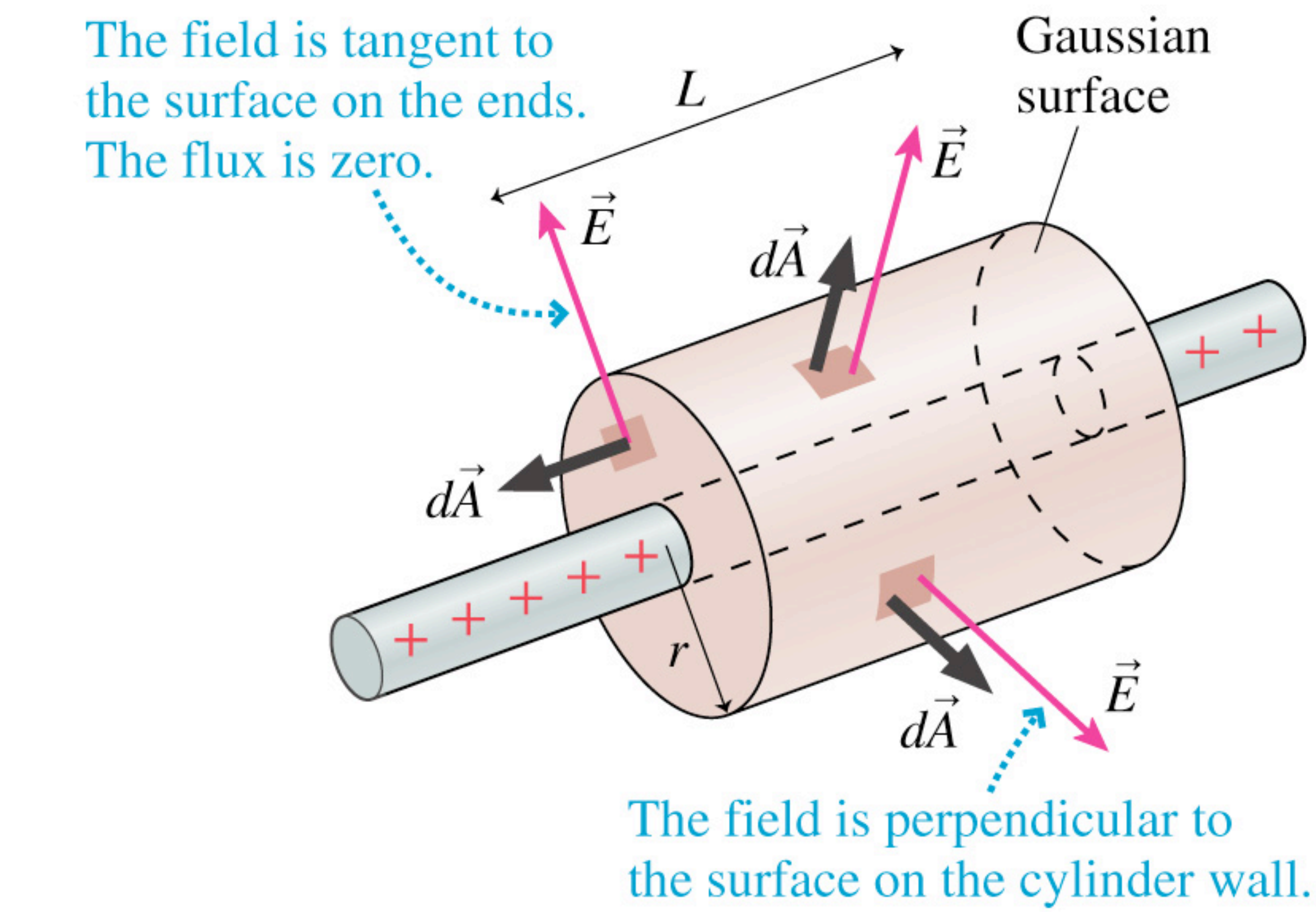
- model as long line...

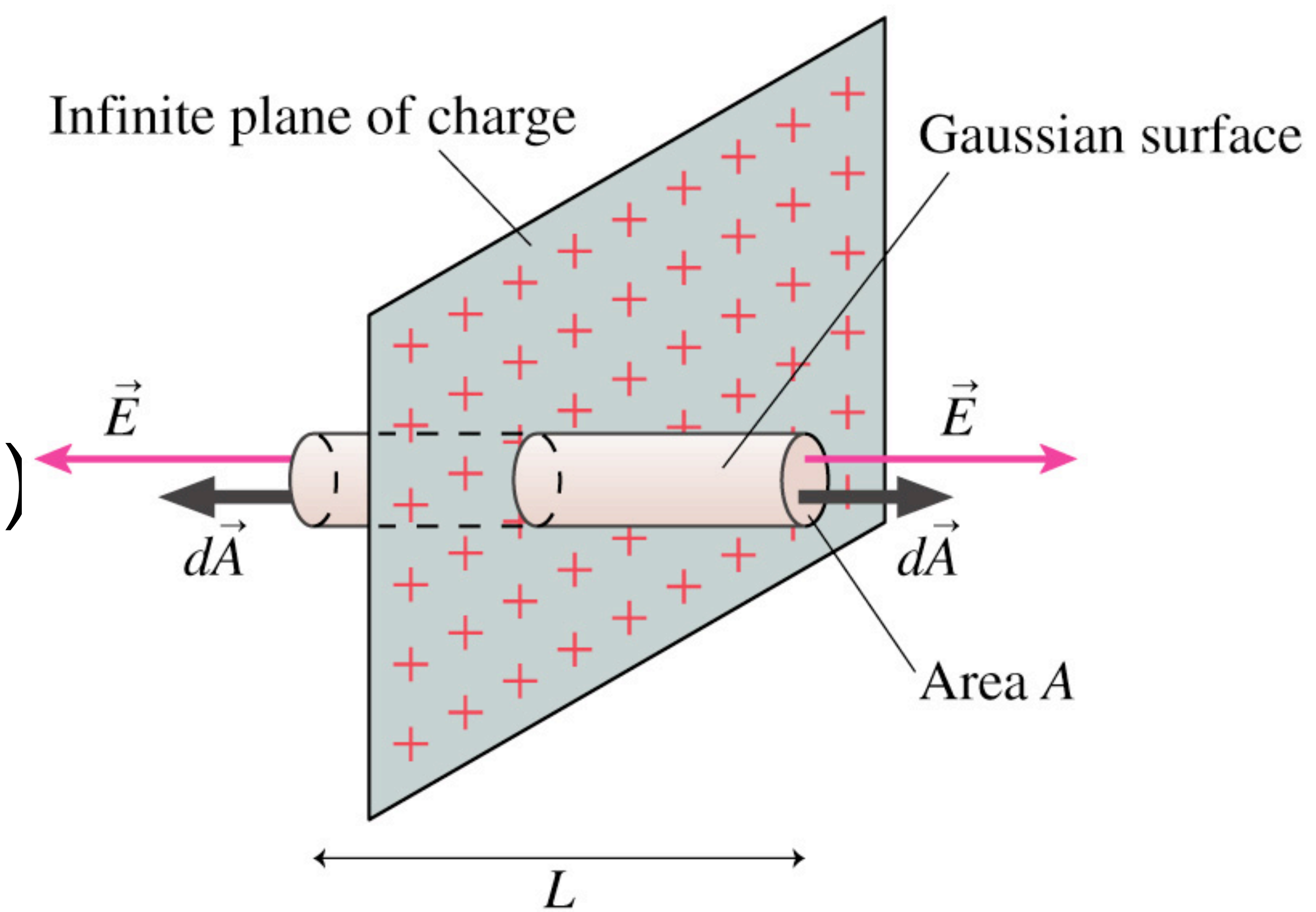
$$\begin{aligned}\Phi_e &= \Phi_{top} + \Phi_{bottom} + \Phi_{wall} \\ &= 0 + 0 + EA_{cyl.} \\ &= E2\pi r L\end{aligned}$$

$$\Phi_e = \frac{Q_{in}}{\epsilon_0};\ Q_{in} = \lambda L$$

$$\Rightarrow E_{wire} = \frac{\lambda}{2\pi\epsilon_0 r}$$

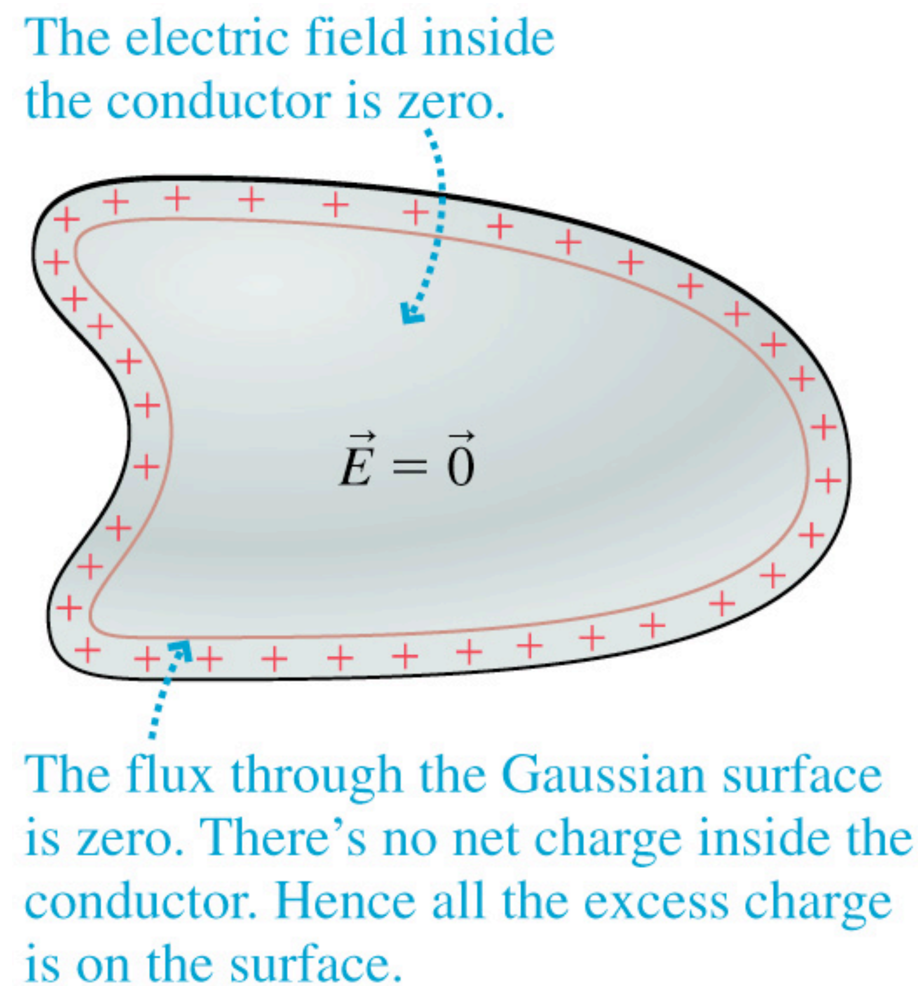
- independent of L of imaginary...
- cylinder encloses only part of wire's charge: outside does not contribute to flux, but essential to cylindrical symmetry (easy flux integral); cannot use for finite length ($\bar{E}$ not same on wall)
- Gauss's law effective for highly symmetrical: superposition always works...





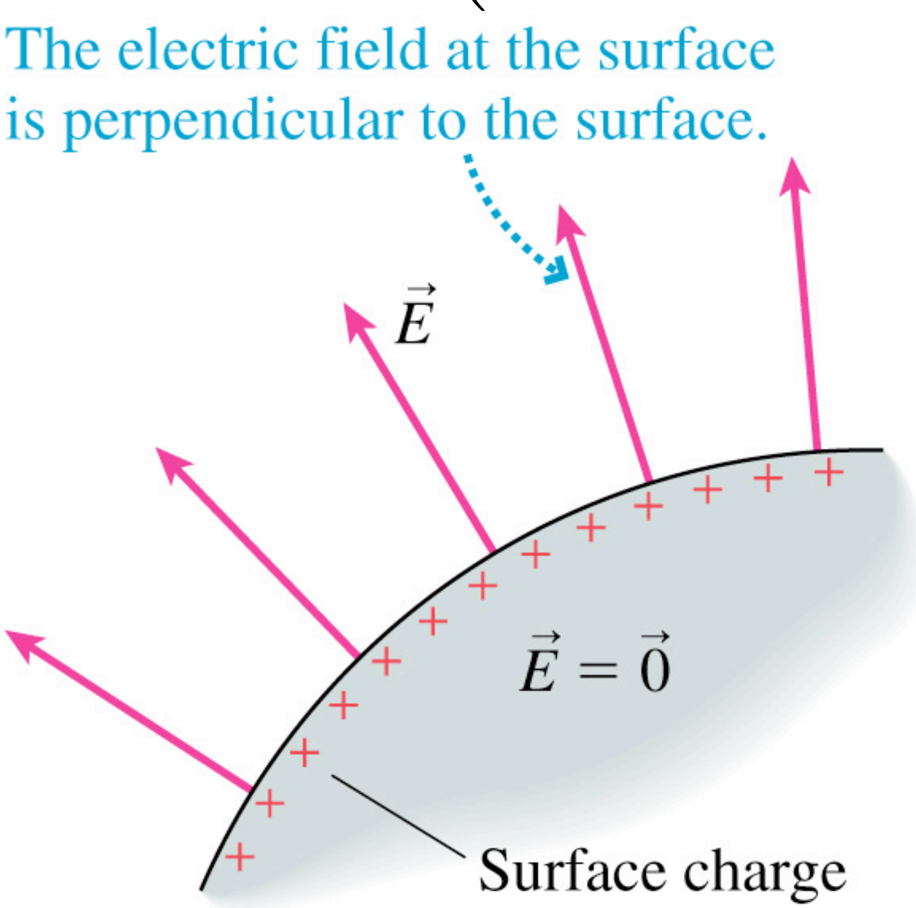
Conductors in Electrostatic Equilibrium: $\bar{E}$ at surface

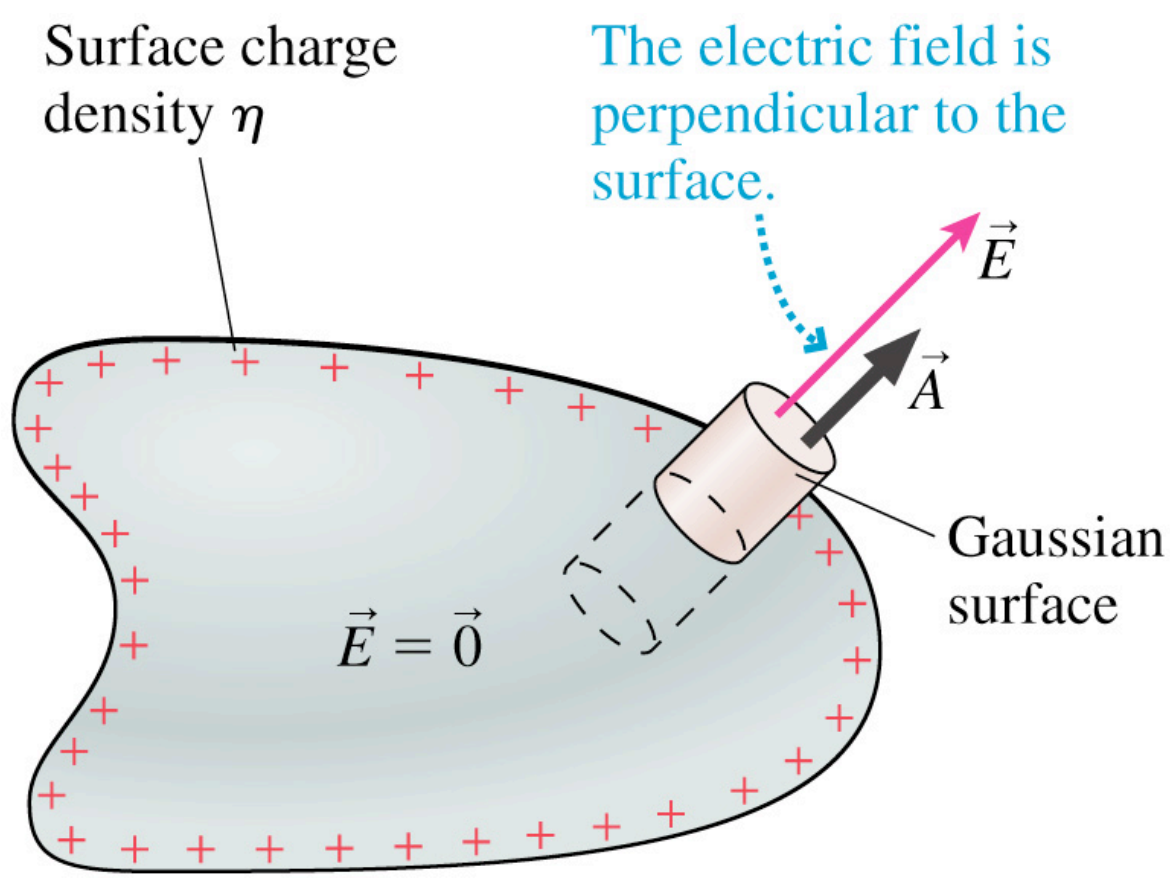
- $\bar{E}_{in} = 0$ if not, charges (free to move) would...



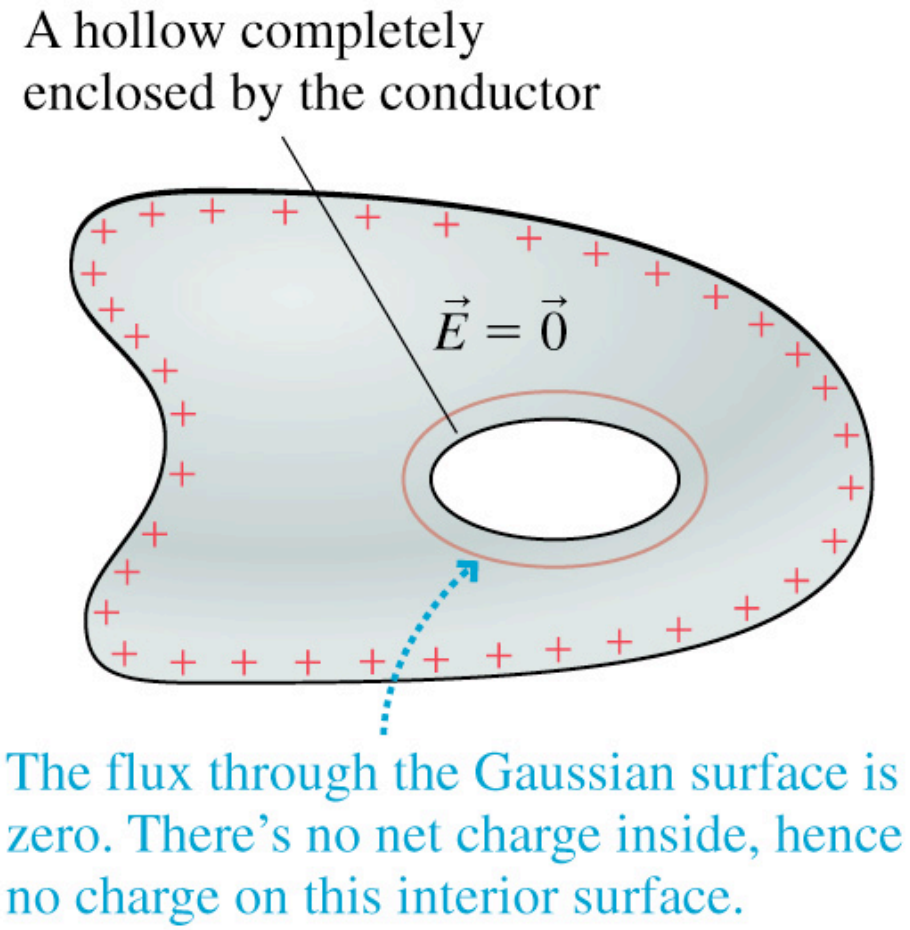
- net charge $\rightarrow \bar{E} \neq 0$ outside
- If $\bar{E}$ tangent to surface, charges move...

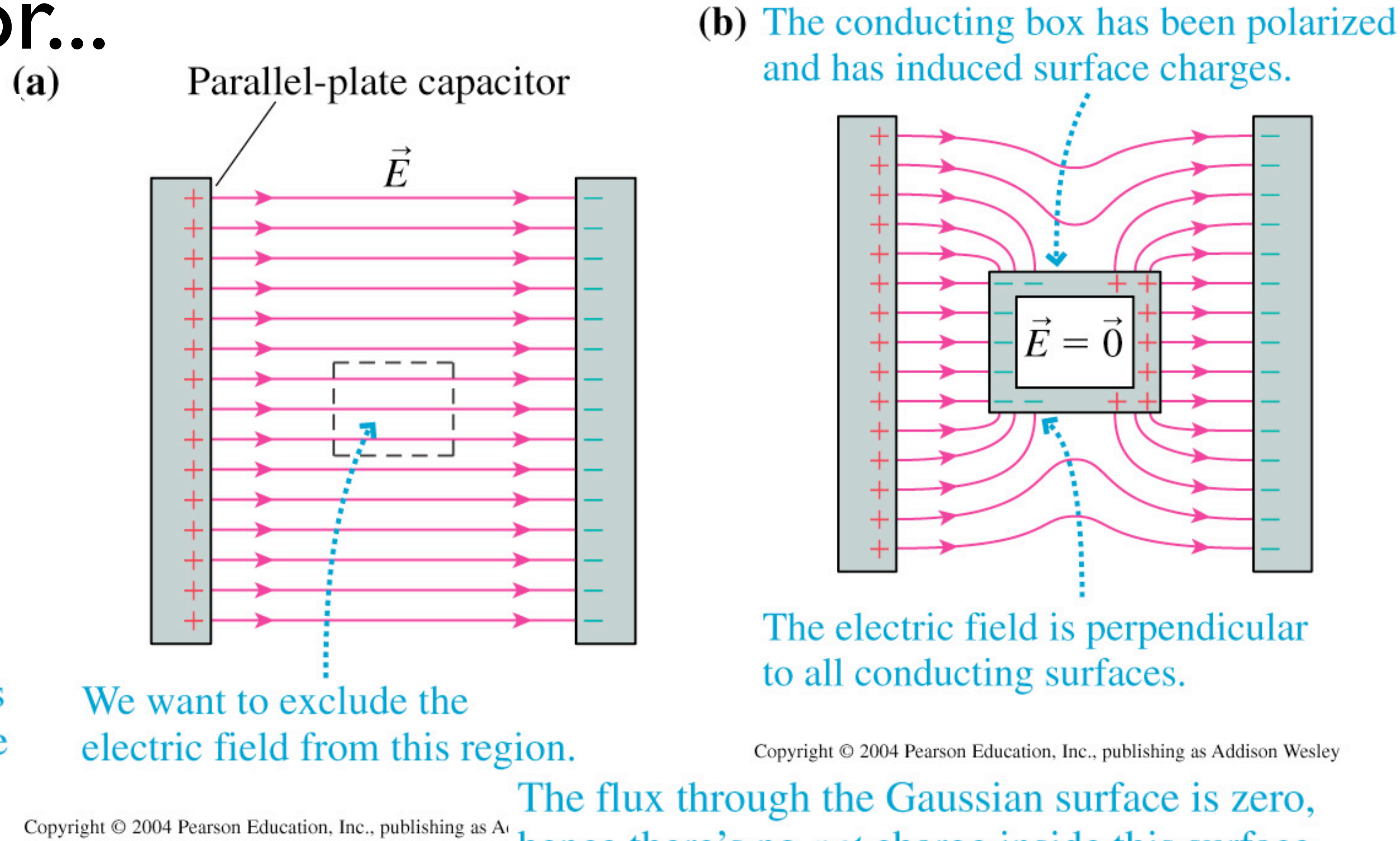
$\Phi_e = AE_{surface}$ for outside face
$+0$ for inside face $(\bar{E}_{in} = 0)$
$+0$ for wall $(\bar{E} \perp \text{surface})$
$\Phi_e = \frac{Q_{in}}{\epsilon_0}$; $Q_{in} = \eta A \Rightarrow$
$\bar{E}_{surface} = \left(\frac{\eta}{\epsilon_0}, \perp \text{ to surface}\right)$

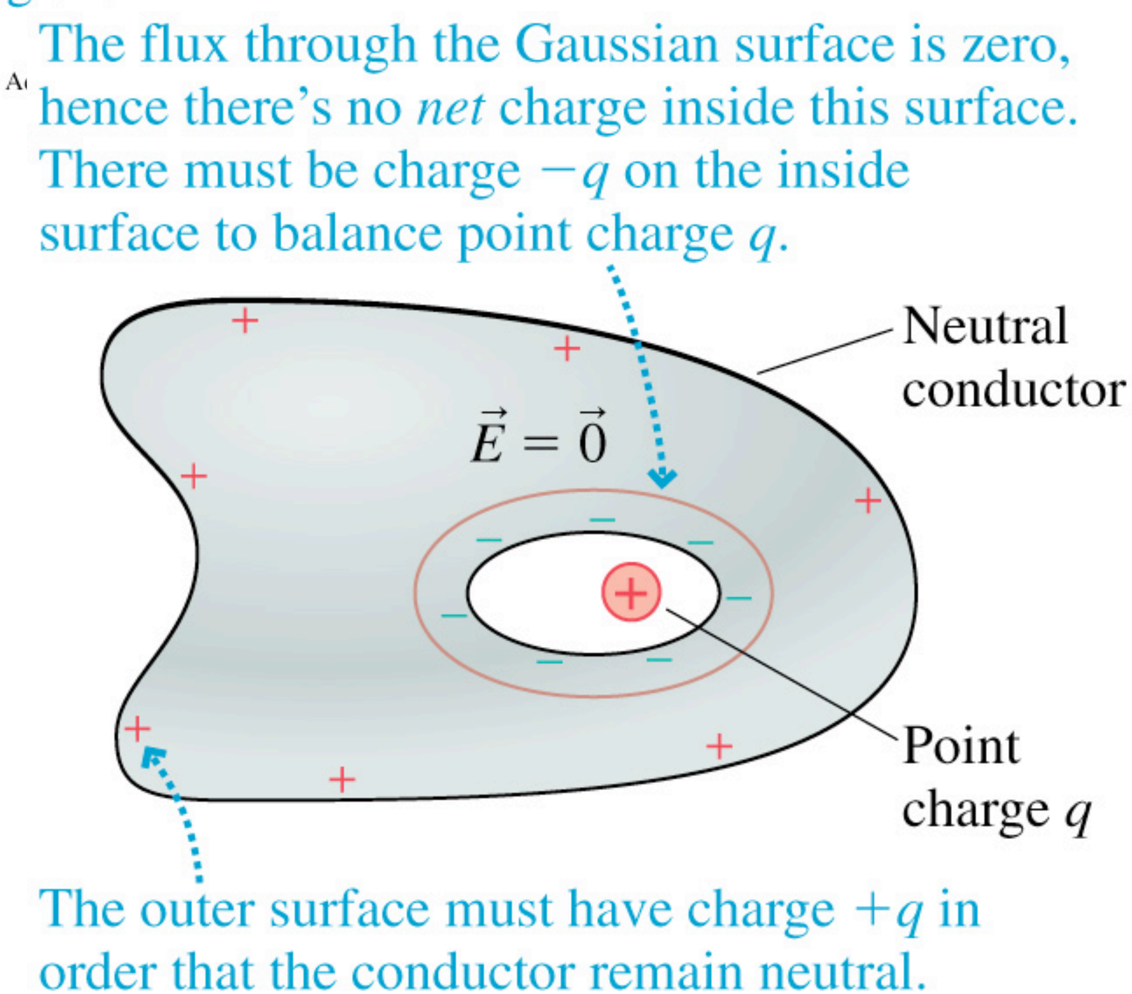




Within conductor...







- excess charge on exterior surface
- $\bar{E} = 0$ inside hole ($\bar{E} = 0$ inside conductor and no charge in hole): screening
- charge inside hole of neutral conductor polarizes...