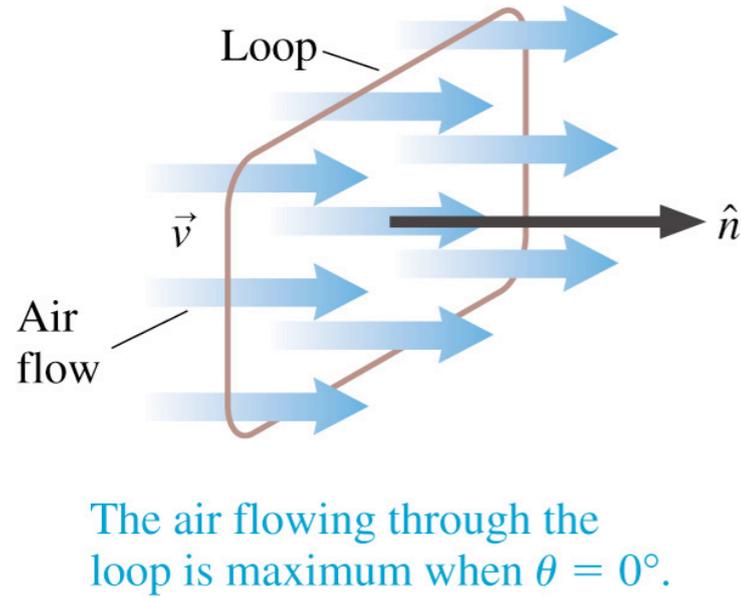


Lecture 20

- calculating electric flux
- electric flux through closed surface
required for Gauss's law: calculate \bar{E} more easily; applies to moving charges
- Uses of Gauss's Law: charged sphere, wire, plane and conductor

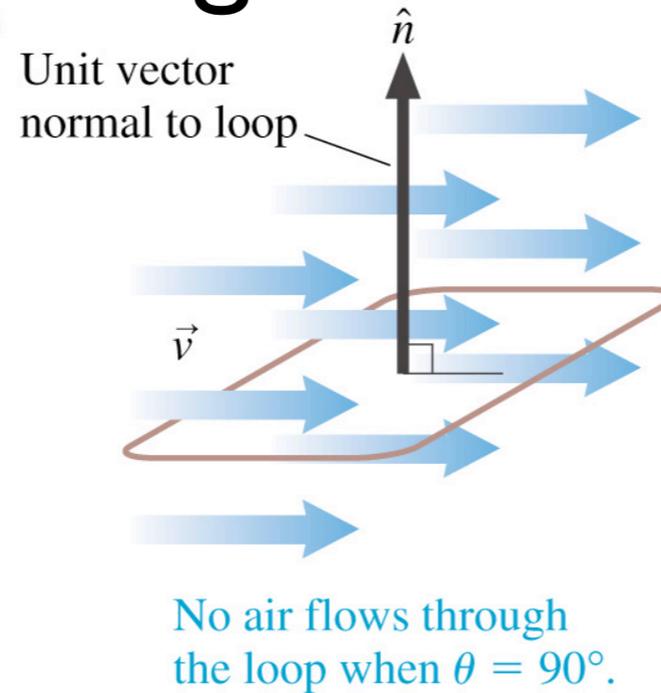
Calculating Electric Flux I

(a)



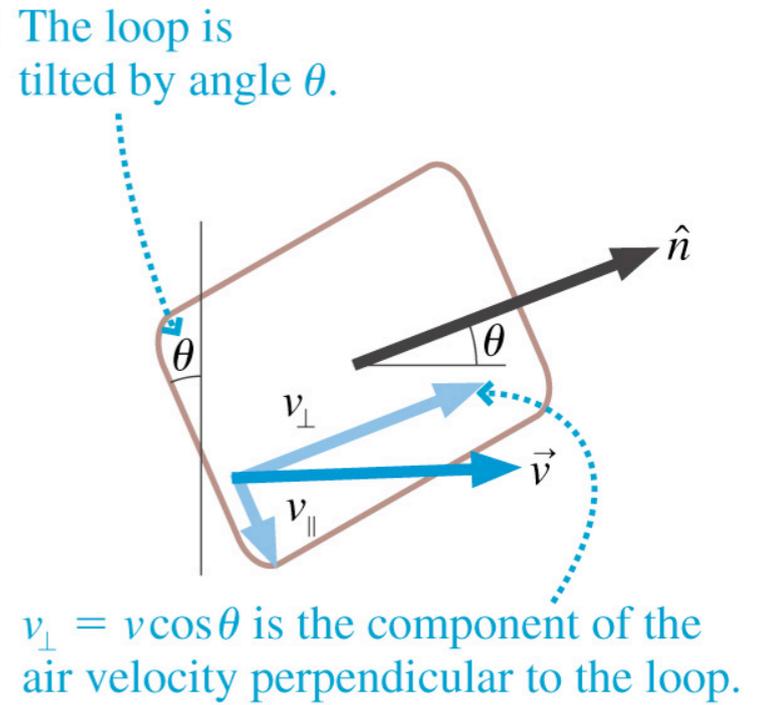
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(b)



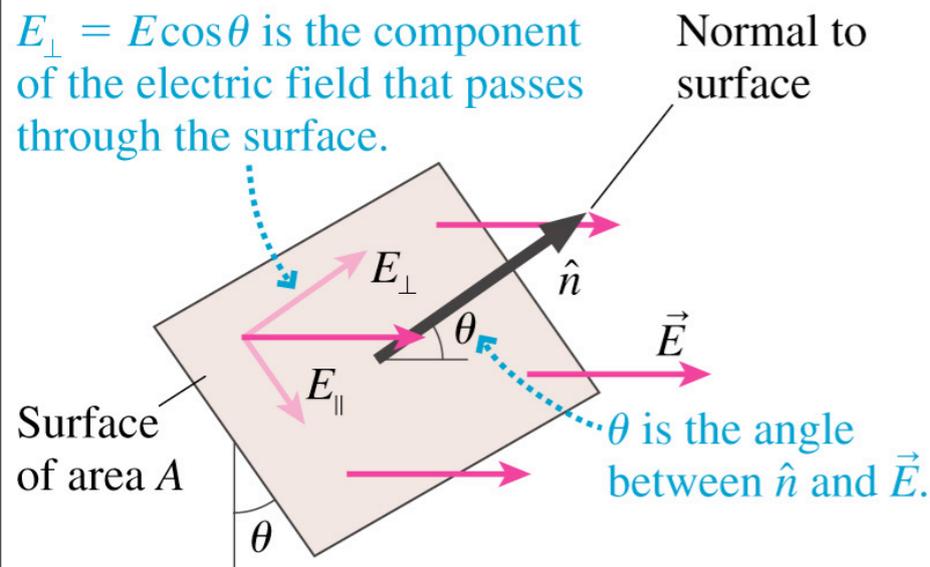
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(c)



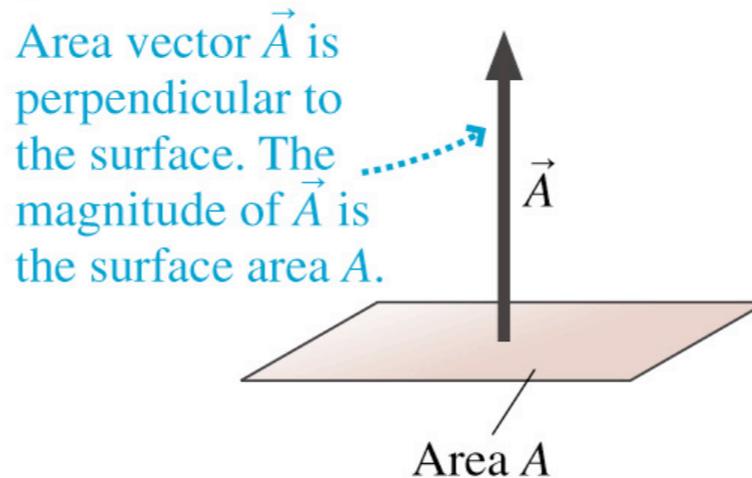
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- **Analogy:** volume of air per second (m^3/s) = $v A = v A \cos \theta$
- **Electric Flux (amount of \vec{E} thru' surface):** $\Phi_e = E A = E A \cos \theta$
- **Area vector:** $\vec{A} = A \hat{n} \rightarrow \Phi_e = \vec{E} \cdot \vec{A}$ (electric flux of a constant electric field)

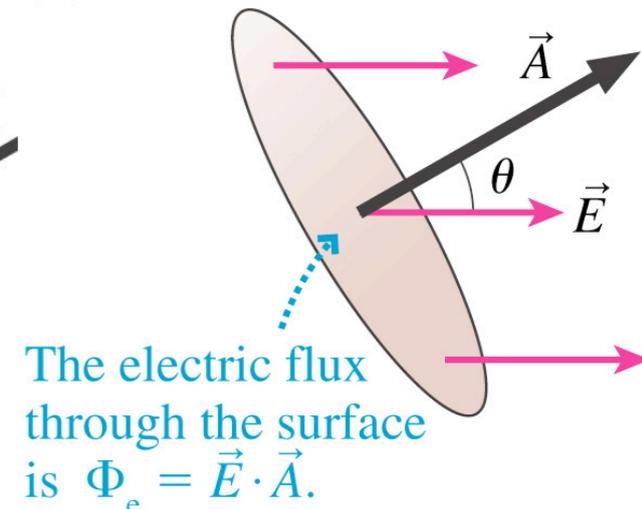


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(a)

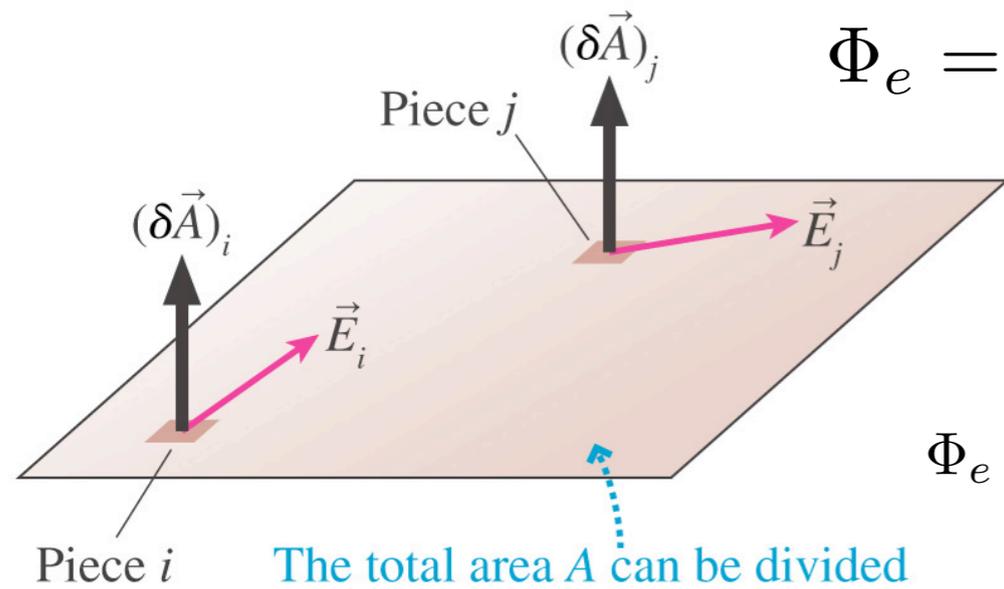


(b)



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Calculating Electric Flux II



The total area A can be divided into many small pieces of area δA . \vec{E} may be different at each piece.

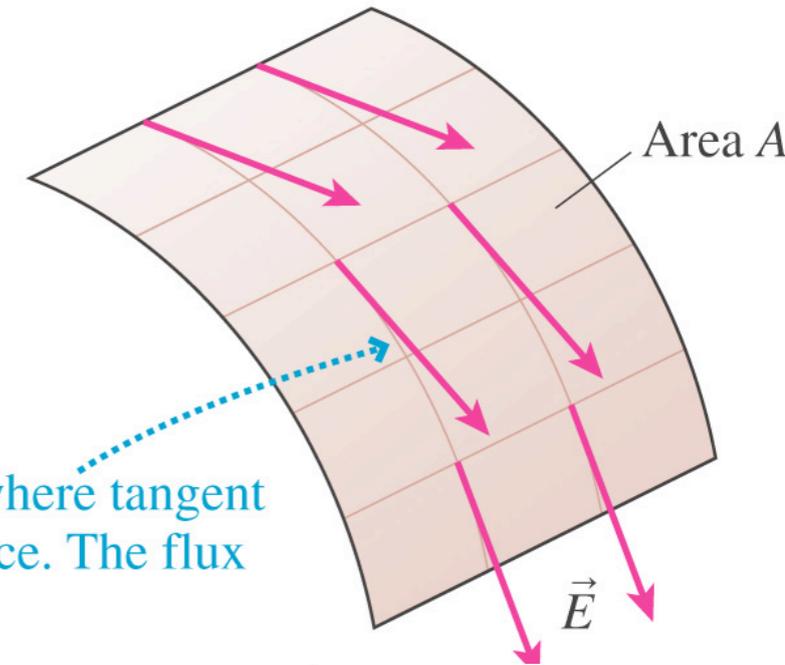
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$$\Phi_e = \sum_i \delta\Phi_e = \sum_i \vec{E}_i \cdot (\delta\vec{A})_i \rightarrow \Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

Uniform \vec{E} , flat surface:

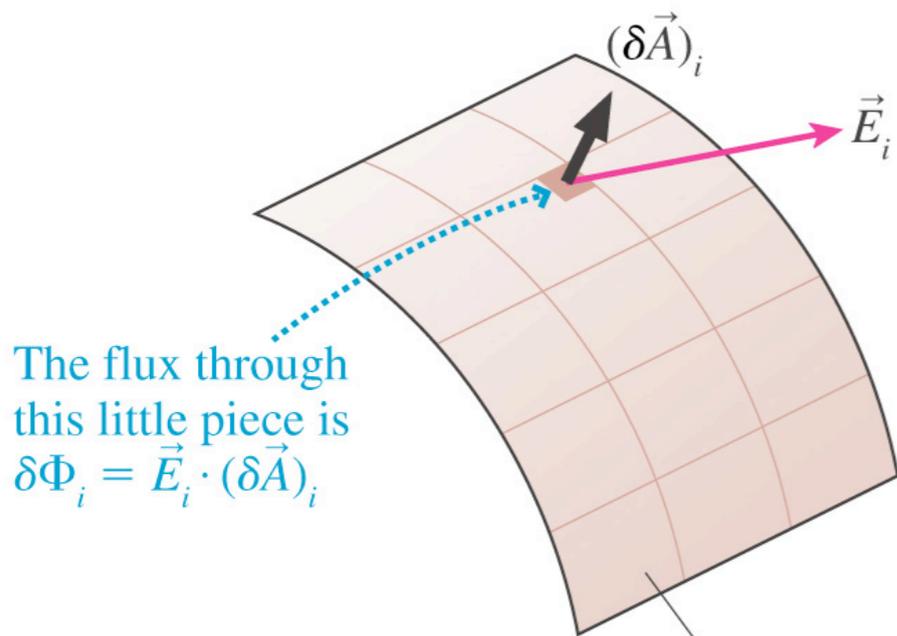
$$\begin{aligned} \Phi_e &= \int_{\text{surface}} \vec{E} \cdot d\vec{A} \\ &= \int \dots E \cos \theta dA \\ &= E \cos \theta \int \dots dA \\ &= E \cos \theta A \end{aligned}$$

(a)



\vec{E} is everywhere tangent to the surface. The flux is zero.

(b)



The flux through this little piece is $\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$

Curved surface of total area A

$$\begin{aligned} \Phi_e &= \int_{\text{surface}} \vec{E} \cdot d\vec{A} \\ &= \int \dots E dA \\ &= E \int \dots dA = EA \end{aligned}$$

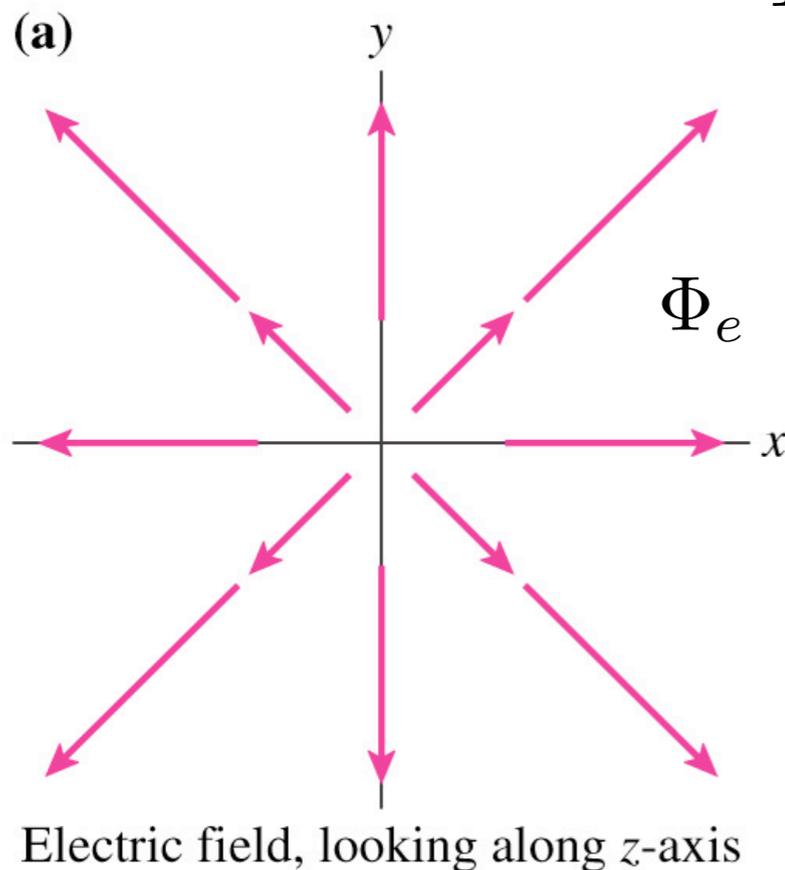
\vec{E} is everywhere perpendicular to the surface and has the same magnitude at each point. The flux is EA .

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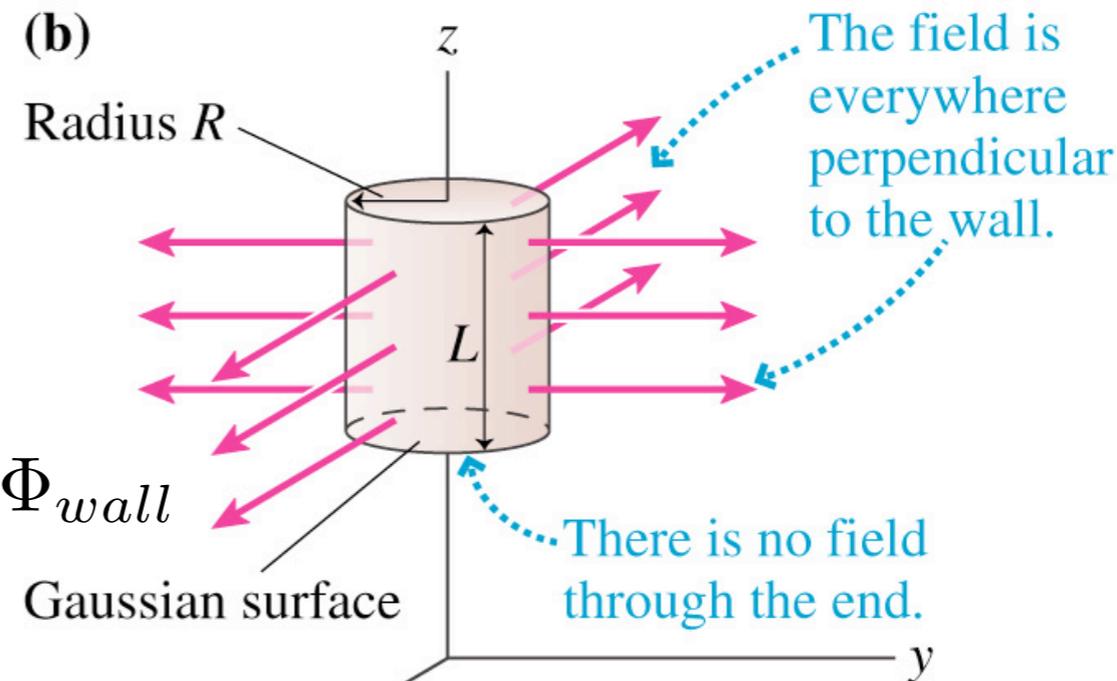
Calculating Electric Flux III

- Closed surface ($d\bar{A}$ points toward outside: ambiguous for single surface): $\Phi_e = \int \bar{E} \cdot d\bar{A}$
- strategy: divide closed surface into either tangent or perpendicular to \bar{E}
- example: cylindrical charge distribution, $\bar{E} = E_0 (r^2/r_0^2) \hat{r}$ (\hat{r} in xy -plane)

$$\Phi_{wall} = EA_{wall}$$



$$\begin{aligned} \Phi_e &= \int \bar{E} \cdot d\bar{A} \\ &= \Phi_{top} + \Phi_{bottom} + \Phi_{wall} \\ &= 0 + 0 + EA_{wall} \\ &= EA_{wall} \\ &= \left(E_0 \frac{R^2}{r_0^2} \right) (2\pi RL) \end{aligned}$$



Flux due to point charge inside...

- Gaussian surface with same symmetry as of charge distribution and hence \vec{E}

$$\Phi_e = \int \vec{E} \cdot d\vec{A} = EA_{sphere}$$

($\vec{E} \perp$ and same at all points)

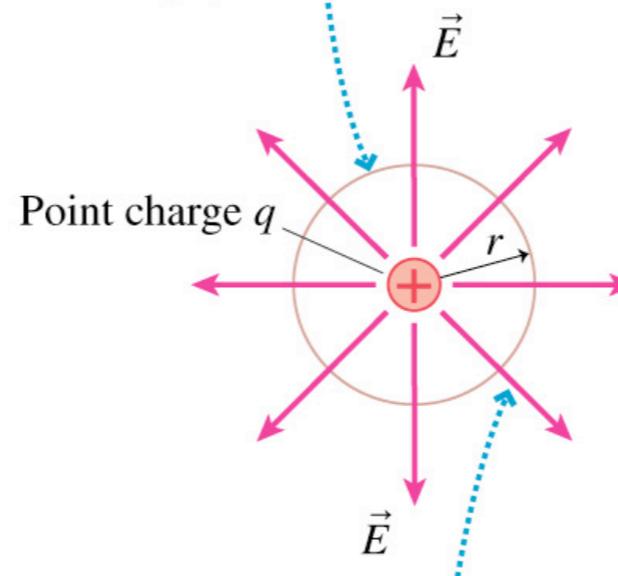
$$E = \frac{q}{4\pi\epsilon_0 r^2}; A_{sphere} = 4\pi r^2$$

$$\Rightarrow \Phi_e = \frac{q}{\epsilon_0}$$

- flux independent of radius
- approximate arbitrary shape...

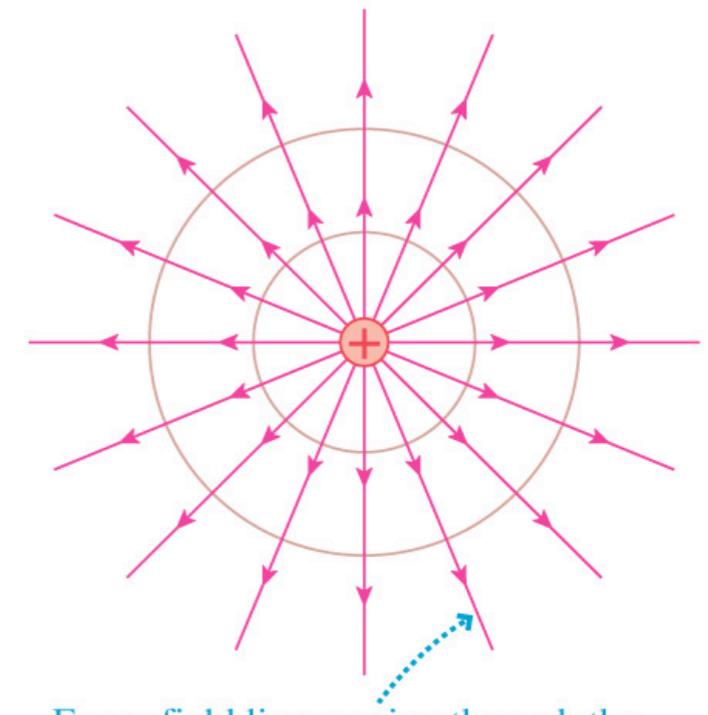
$$\Phi_e = \int \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

Cross section of a Gaussian sphere of radius r . This is a mathematical surface, not a physical surface.



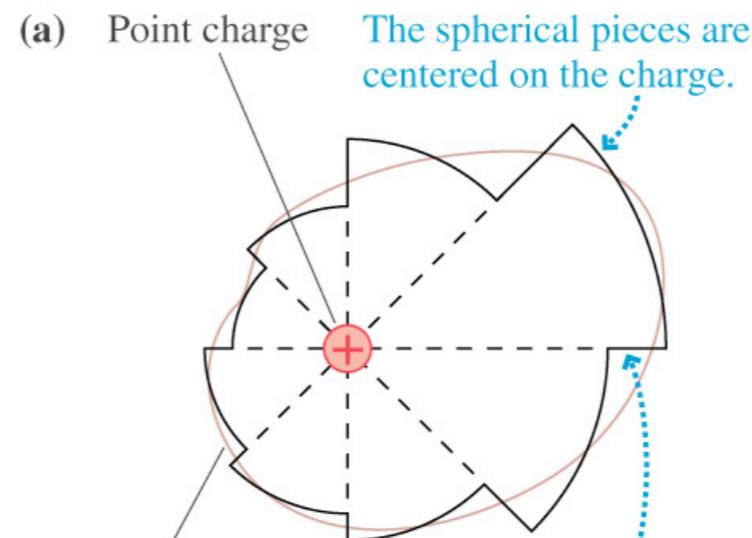
The electric field is everywhere perpendicular to the surface and has the same magnitude at every point.

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Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.

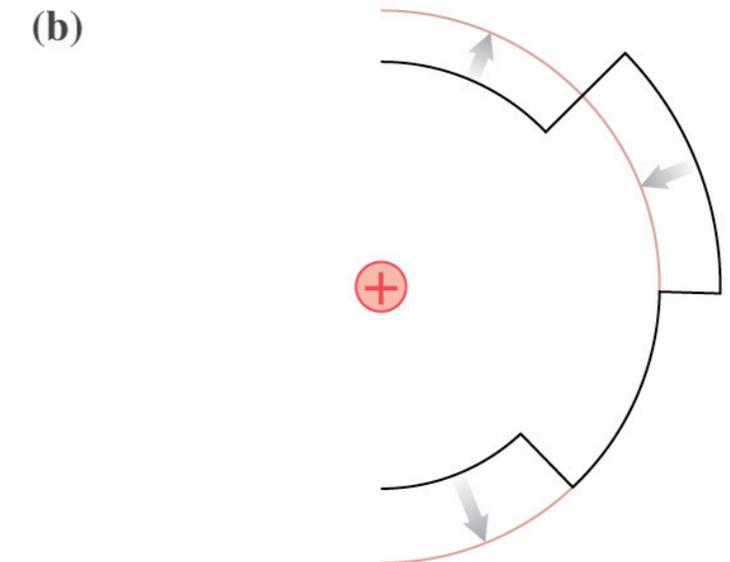
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Gaussian surface of arbitrary shape

The radial pieces are along lines extending out from the charge. There's no flux through these.

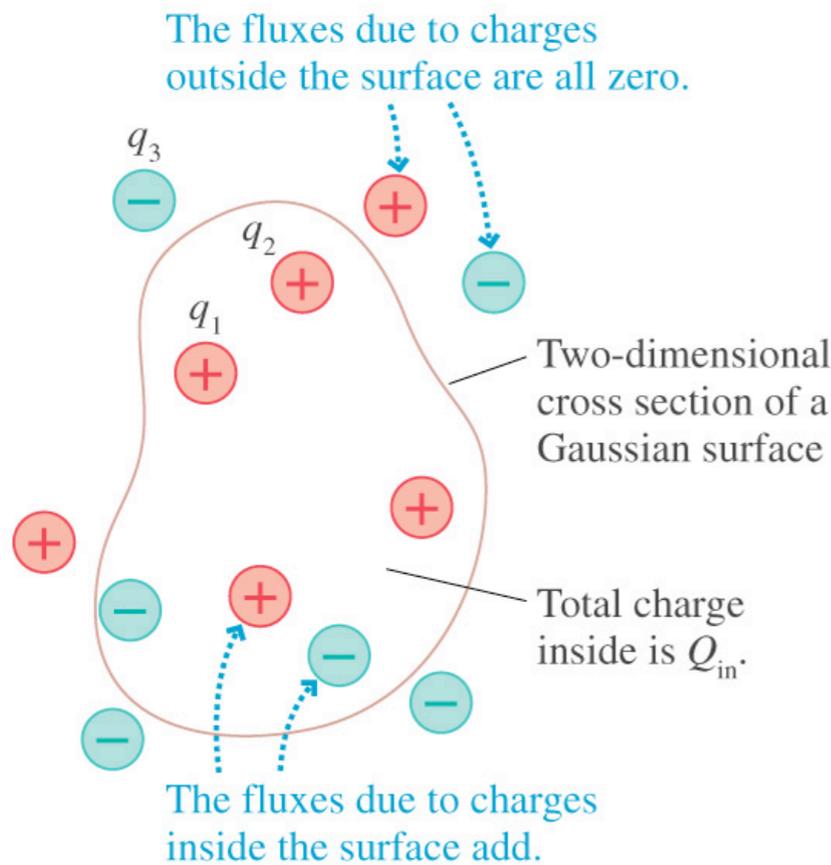
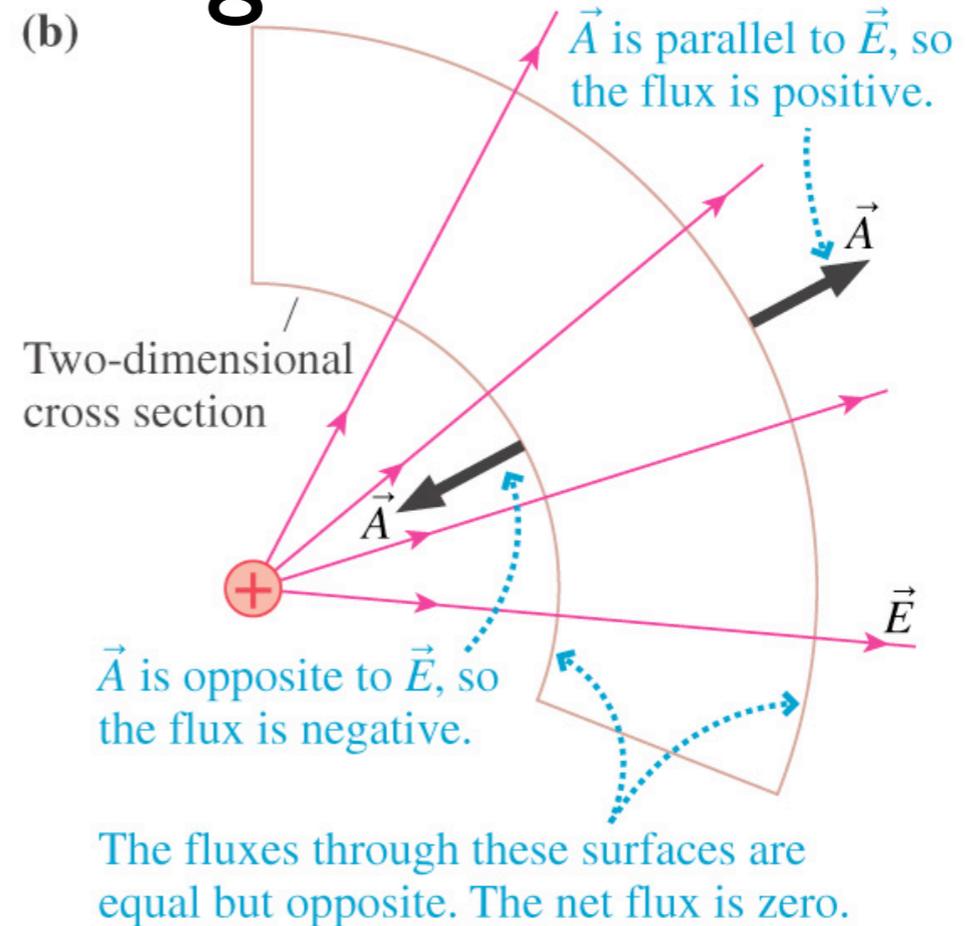
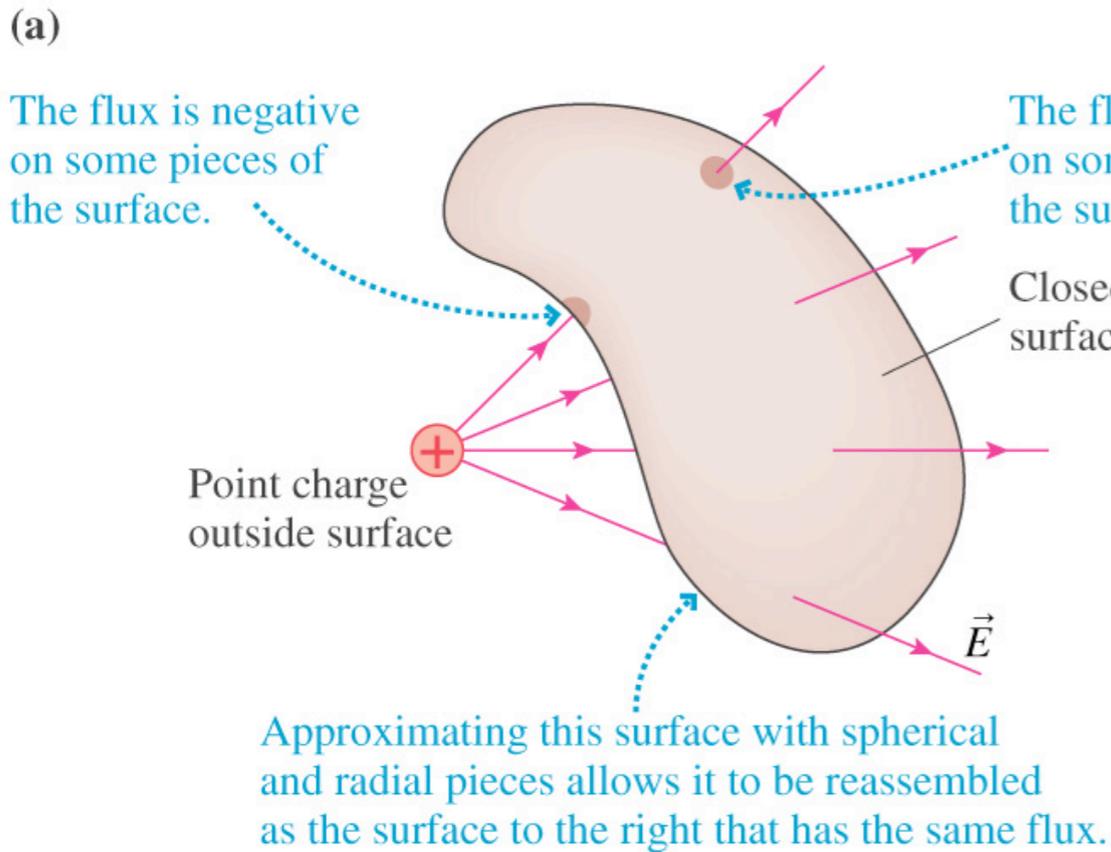
The approximation with spherical and radial pieces can be as good as desired by letting the pieces become sufficiently small.



The spherical pieces can slide in or out to form a complete sphere. Hence the flux through the pieces is the same as the flux through a sphere.

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Charge outside..., multiple charges... Gauss's Law



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$$\begin{aligned} \vec{E} &= \vec{E}_1 + \vec{E}_2 + \dots \text{ (superposition) } \Rightarrow \\ \Phi_e &= \int \vec{E}_1 \cdot d\vec{A} + \int \vec{E}_2 \cdot d\vec{A} + \dots \\ &= \Phi_1 + \Phi_2 + \dots \\ &= \left(\frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0} \dots \text{for all charges inside} \right) \\ &\quad + (0 + 0 + \dots \text{for all charges outside}) \\ Q_{in} &= q_1 + q_2 + \dots \text{ for all charges inside } \Rightarrow \end{aligned}$$

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Using Gauss's Law

- Gauss's law derived from Coulomb's law, but states general property of \vec{E} : charges create \vec{E} ; net flux "flow" thru' any surface surrounding is same
- quantitative: connect net flux to amount of charge

Strategy

- model charge distribution as one with symmetry (draw picture) symmetry of \vec{E}
- Gaussian surface (imaginary) of same symmetry (does not have to enclose all charge)
- \vec{E} either tangent ($\Phi_e = 0$) or perpendicular to ($\Phi_e = EA$) surface

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

Charged Sphere: \vec{E} outside and inside

- charge distribution inside has spherical symmetry (need not be uniform)

$$\Phi_e = \int \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

$$EA_{sphere} = E4\pi r^2$$

(don't know E, but same at all points on surface)

$$+ Q_{in} = Q$$

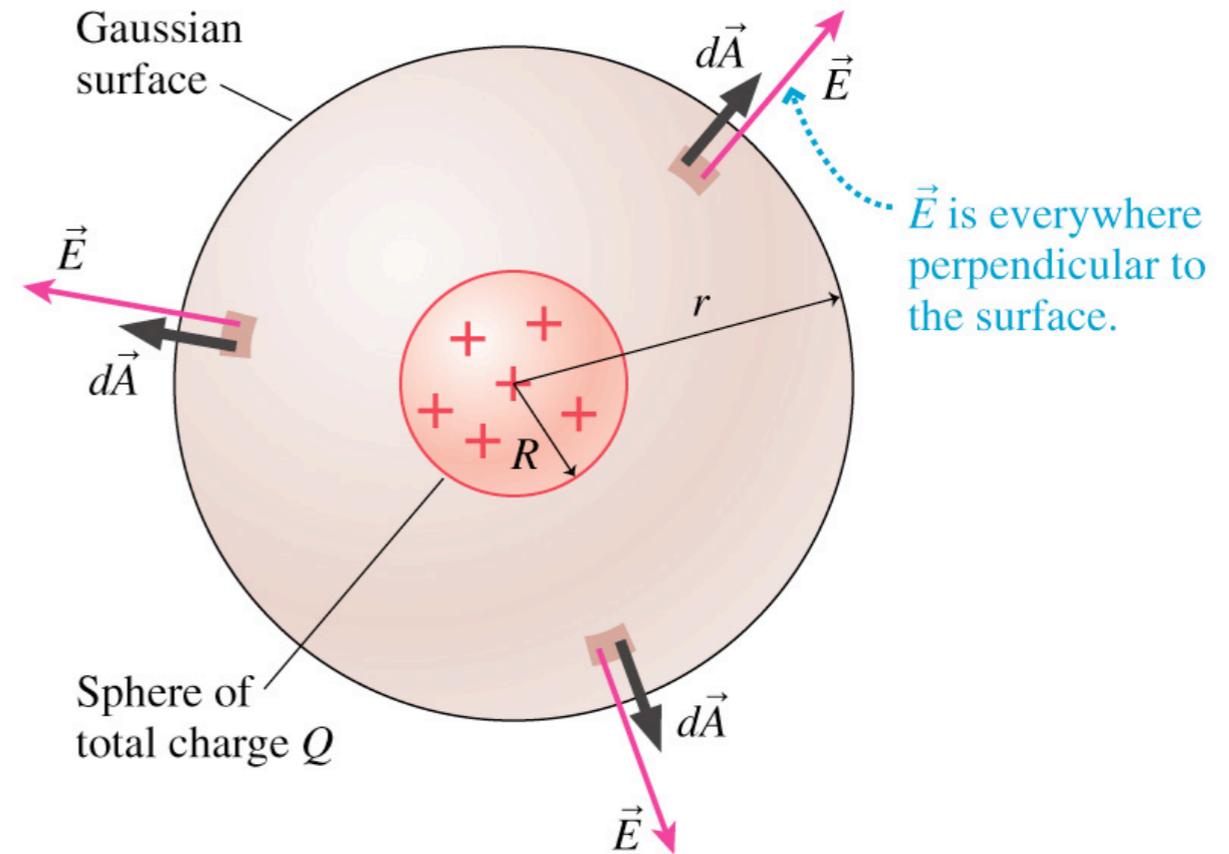
$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2}$$

(same as point charge)

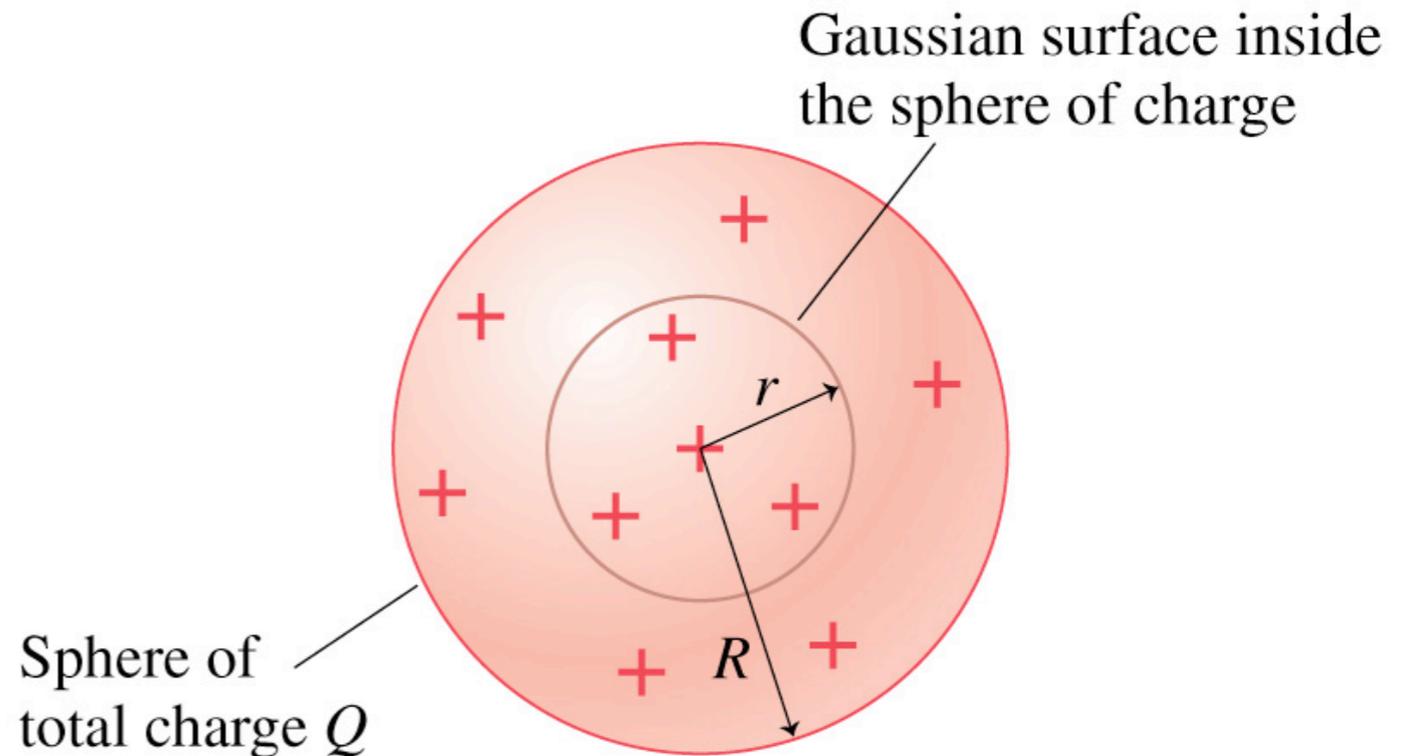
- flux integral not easy for other surface

- using superposition requires 3D integral!

- spherical surface inside sphere $\Rightarrow Q_{in} \neq Q$



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Charged wire and plane

- model as long line...

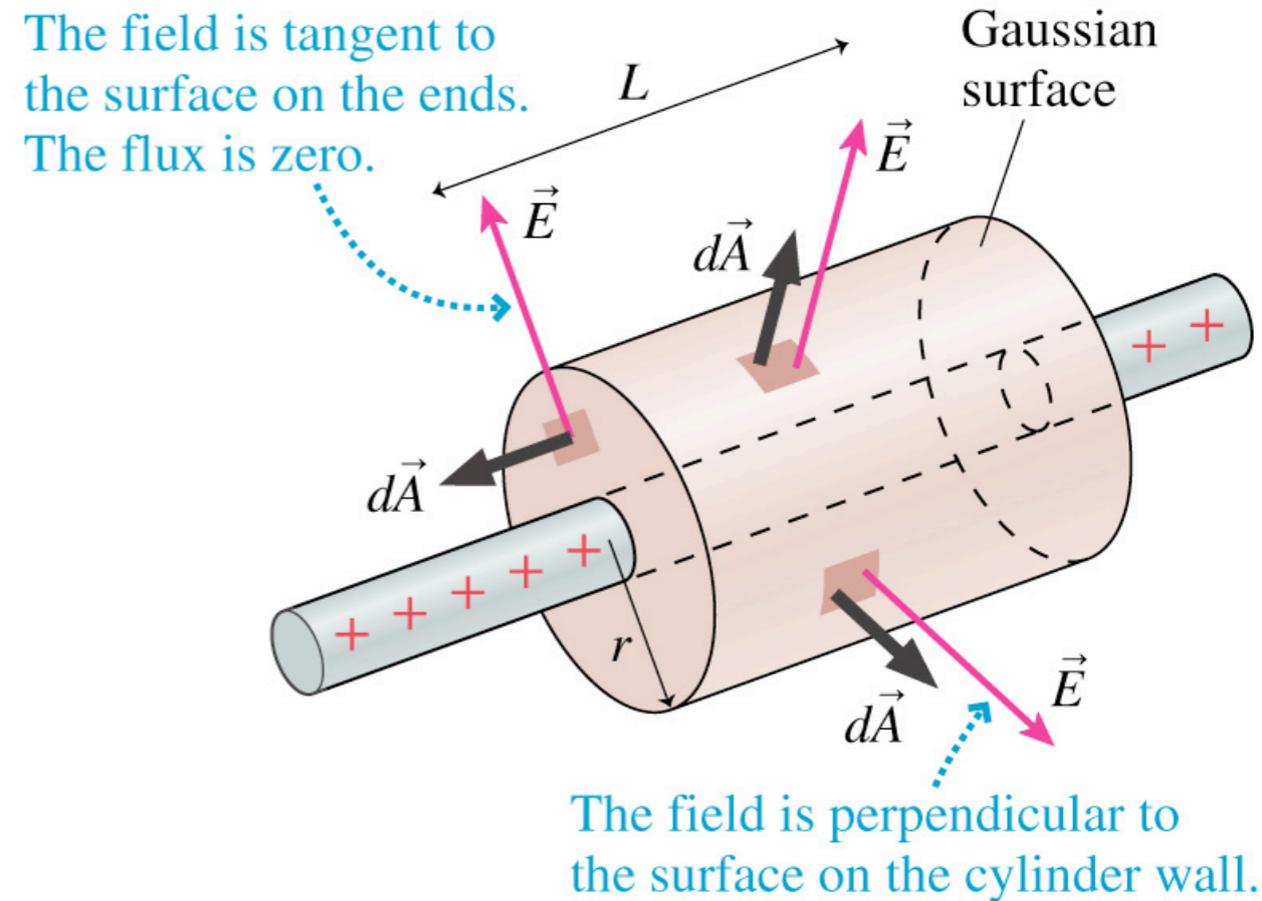
$$\begin{aligned}\Phi_e &= \Phi_{top} + \Phi_{bottom} + \Phi_{wall} \\ &= 0 + 0 + EA_{cyl}.\end{aligned}$$

$$= E2\pi rL$$

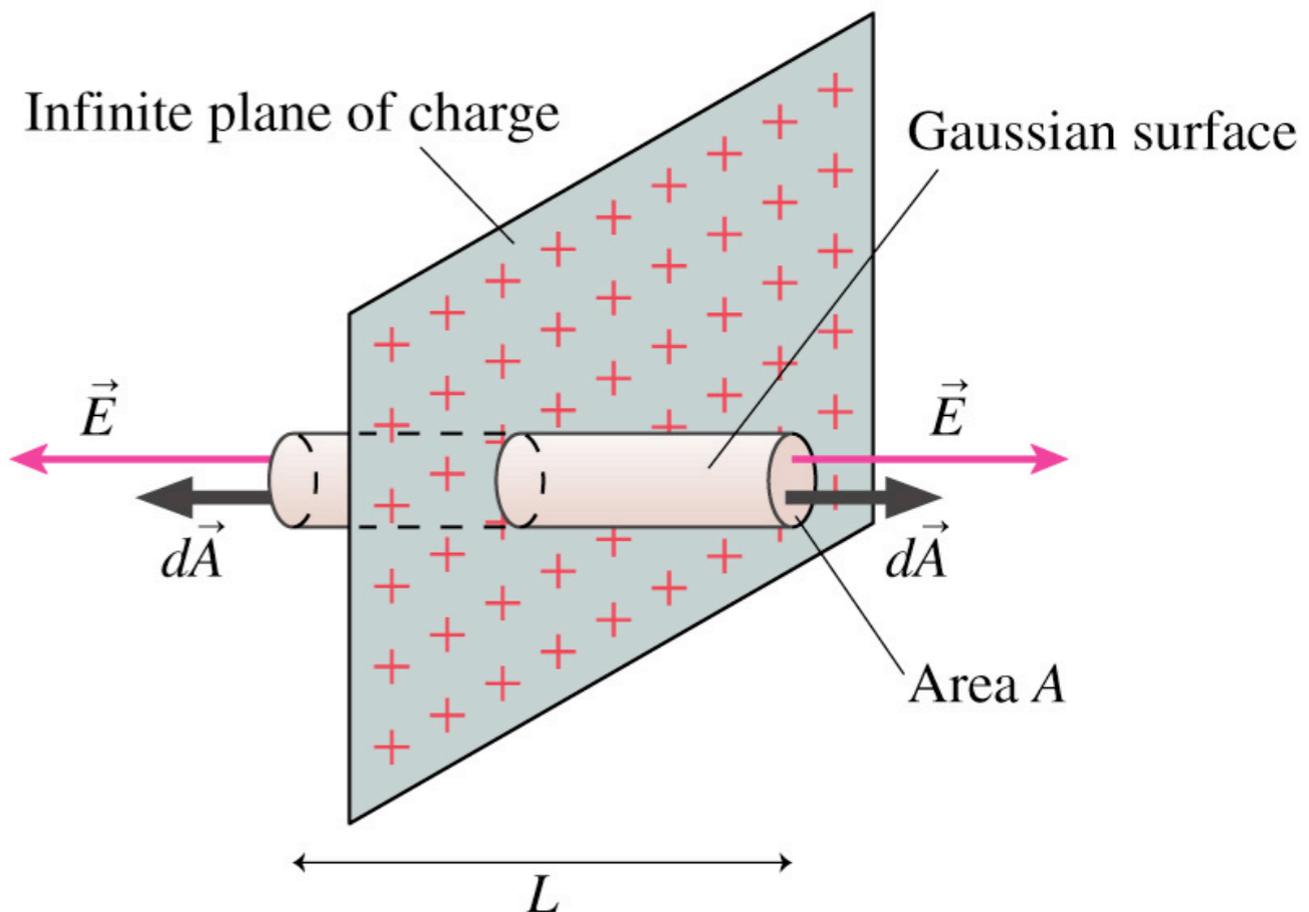
$$\Phi_e = \frac{Q_{in}}{\epsilon_0}; Q_{in} = \lambda L$$

$$\Rightarrow E_{wire} = \frac{\lambda}{2\pi\epsilon_0 r}$$

- independent of L of imaginary...
- cylinder encloses only part of wire's charge: outside does not contribute to flux, but essential to cylindrical symmetry (easy flux integral); cannot use for finite length (\vec{E} not same on wall)
- Gauss's law effective for highly symmetrical: superposition always works...



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Conductors in Electrostatic Equilibrium: \vec{E} at surface

• $\vec{E}_{in} = 0$ if not, charges (free to move) would...

The electric field inside the conductor is zero.

• net charge $\rightarrow \vec{E} \neq 0$ outside

• If \vec{E} tangent to surface, charges move...

$\Phi_e = AE_{surface}$ for outside face

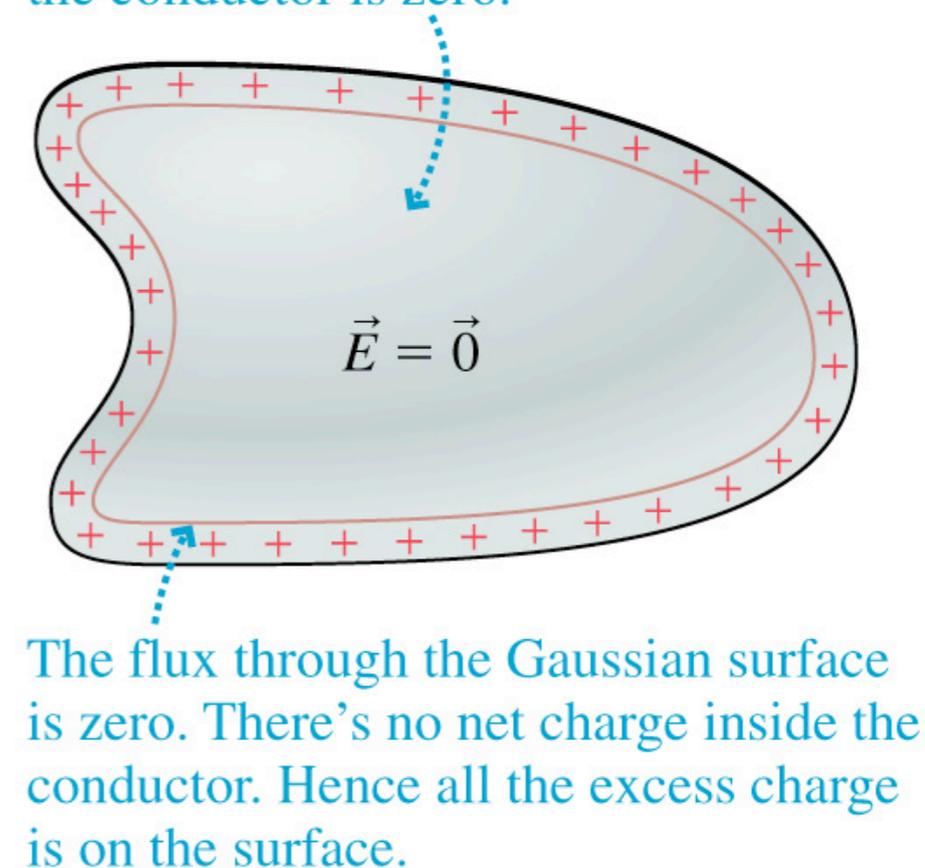
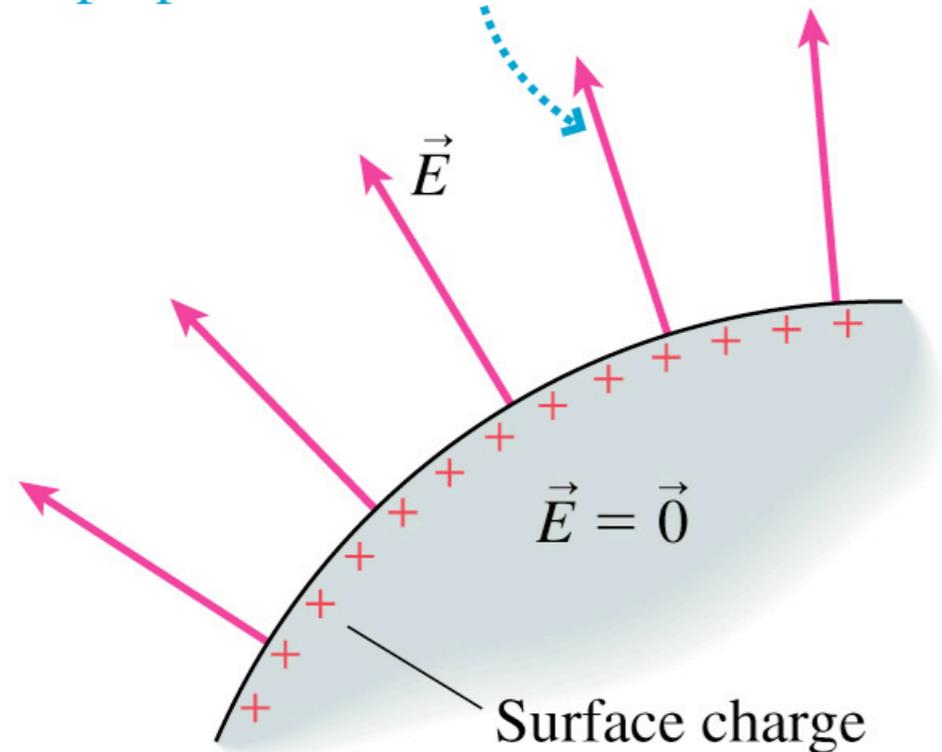
+0 for inside face ($\vec{E}_{in} = 0$)

+0 for wall ($\vec{E} \perp$ surface)

$\Phi_e = \frac{Q_{in}}{\epsilon_0}$; $Q_{in} = \eta A \Rightarrow$

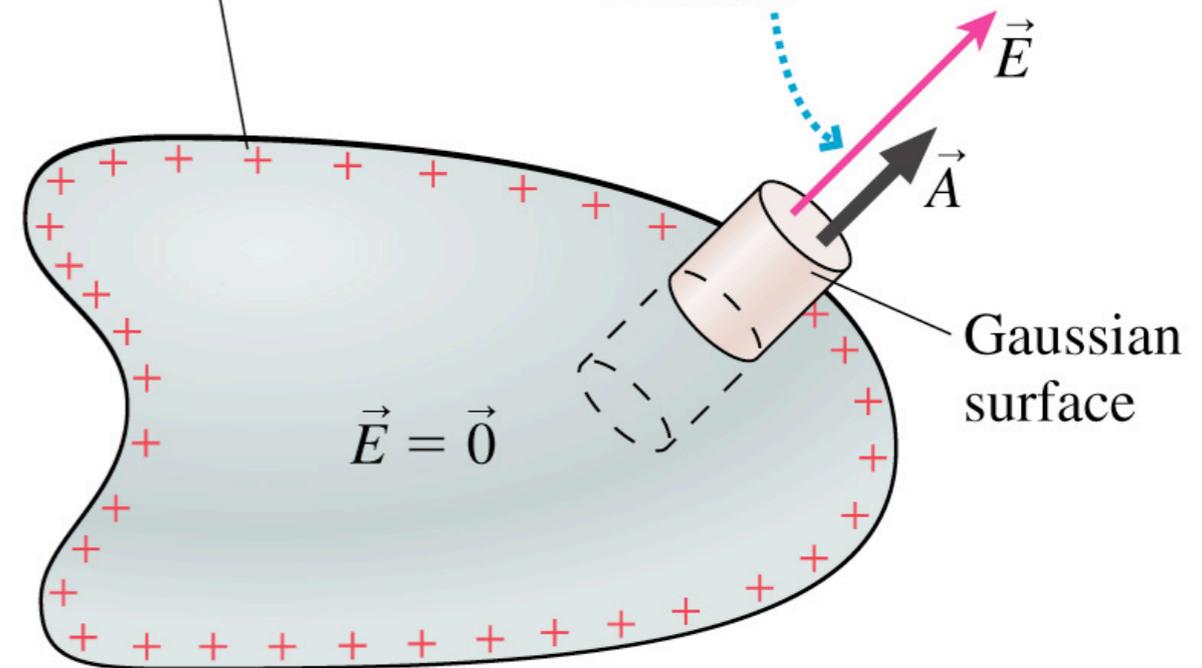
$\vec{E}_{surface} = \left(\frac{\eta}{\epsilon_0}, \perp \text{ to surface} \right)$

The electric field at the surface is perpendicular to the surface.



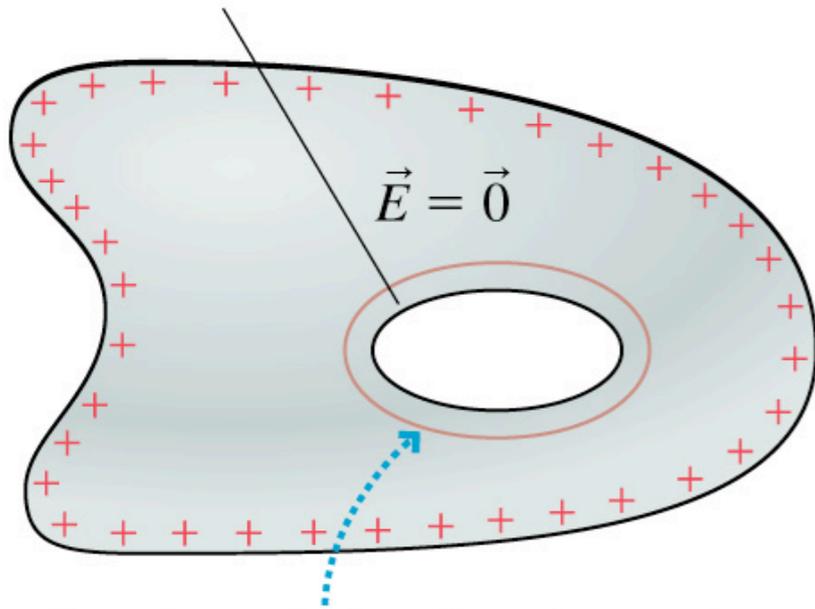
Surface charge density η

The electric field is perpendicular to the surface.



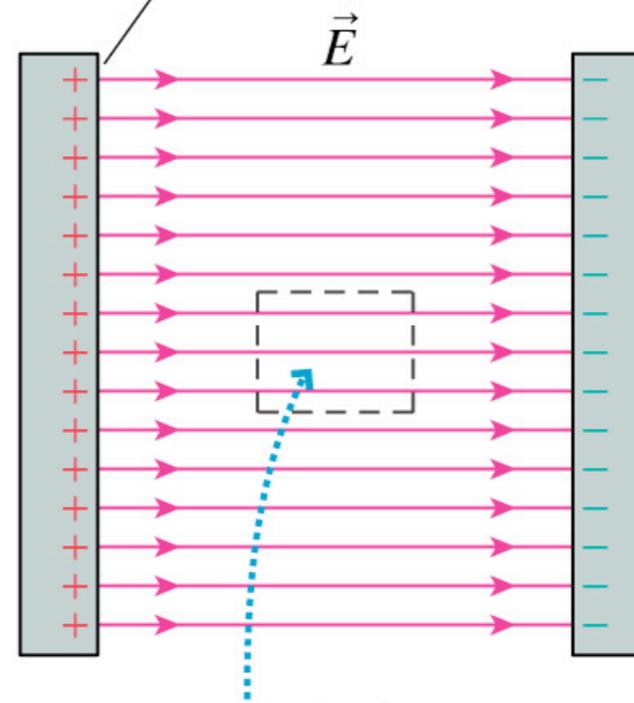
Within conductor...

A hollow completely enclosed by the conductor



(a)

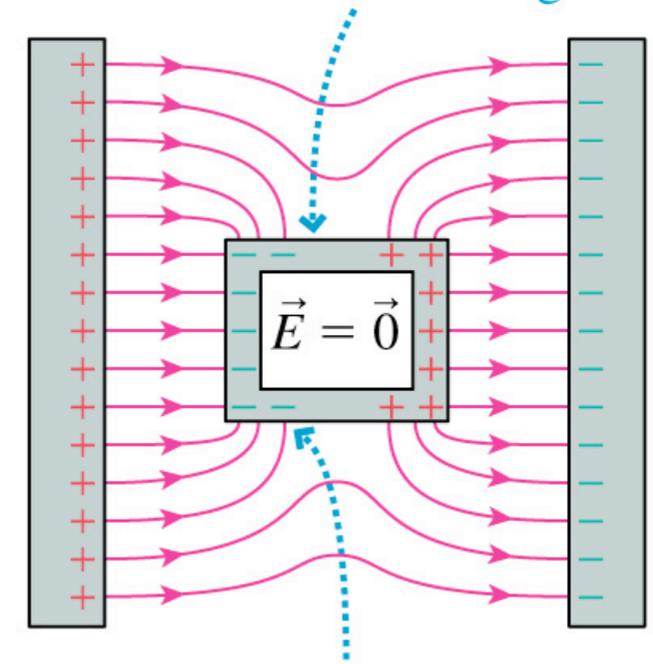
Parallel-plate capacitor



We want to exclude the electric field from this region.

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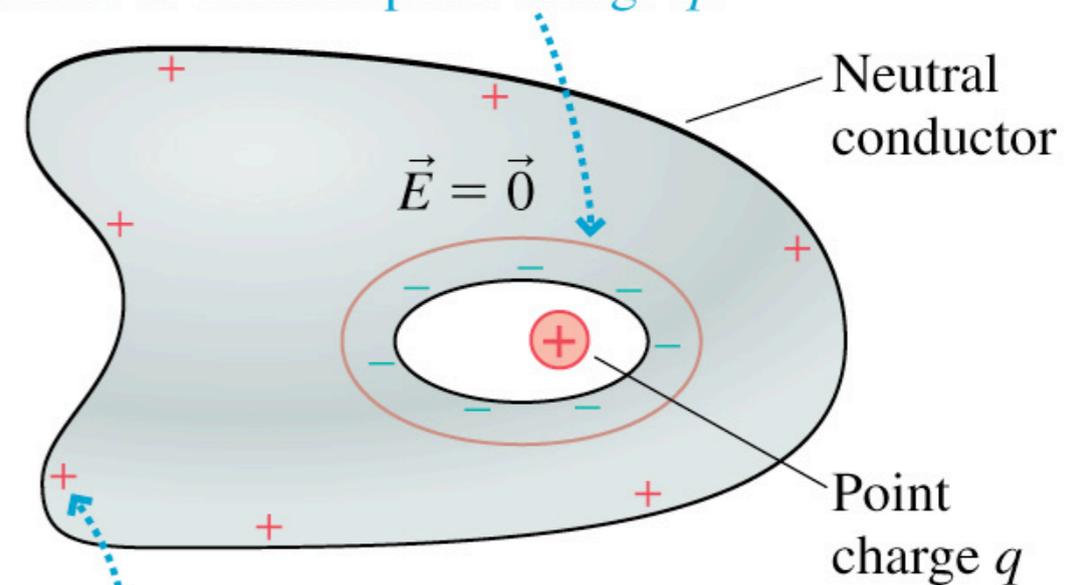
(b) The conducting box has been polarized and has induced surface charges.



The electric field is perpendicular to all conducting surfaces.

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The flux through the Gaussian surface is zero, hence there's no net charge inside this surface. There must be charge $-q$ on the inside surface to balance point charge q .



The outer surface must have charge $+q$ in order that the conductor remain neutral.

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- excess charge on exterior surface
- $\vec{E} = 0$ inside hole ($\vec{E} = 0$ inside conductor and no charge in hole): screening
- charge inside hole of neutral conductor polarizes...