## Building your toolbelt

- Using math to make meaning in the physical world.

- Dimensional analysis
- Functional dependence / scaling
- Special cases / limiting cases
- Reading the physics in the representation (graphs)
- Reading the physics in the representation (equations)
- Changing physics equations to math (and back)
- The implications game
- Parsing
- Telling the story
- The equation game


## The structure of science knowledge



## The Dimensional Analysis Tool

- Since we are mapping physical measurements into math, most of the quantities we use in physics are NOT NUMBERS. They are MEASUREMENTS.
- This means they depend on an arbitrary scale we have chosen.
- In order that the equations we write keep their validity (the equation still holds) when we change our arbitrary scale dimensions must match on both sides of the $=$.
- Dimensions are arbitrary and depend on what choices we choose to think about changing. (e.g., moles, angles)


## Equations in physics (science) are NOT the same as equations in math!

We have four different kinds of measurements that we use so far:

- A measurement with a ruler (a length)
- A measurement with a clock (a time)
- A measurement with a scale (a mass)

- A measurement of electric strength (a charge)


## When we ask a symbol: "What

 measurements are you made of and how?" we will indicate it by using double square brackets:$$
[\text { U_ }
$$

## When we combine measurements we express it by showing how

 those measurements are combined

A velocity is found by dividing a length measurement by a time measurement

An acceleration is found by dividing a velocity measurement by a time measurement

Measurements, being a number with a unit, combine like algebraic symbols when combined by multiplying or dividing.

When we have correct equations
for symbols that we know
it can tell us what measurements
were combined to create that symbol.

$$
\begin{aligned}
& F=m a \quad \text { so } \quad \llbracket F \rrbracket=\llbracket m a \rrbracket \\
& \text { so } \\
& \llbracket F \rrbracket=\llbracket m \rrbracket \llbracket a \rrbracket=\overline{(0)}(10))
\end{aligned}
$$

Since we don't want to be always drawing little scales, rulers, and clocks, we write them as " M ", " L ", and " T " but be careful not to confuse them with algebraic symbols that have values!
(Also, from laziness, we only write single instead of double brackets.)

## So read these as follows:

$$
\begin{aligned}
& {[v]=\mathrm{L} / \mathrm{T}} \\
& {[F]=\mathrm{ML} / \mathrm{T}^{2}}
\end{aligned}
$$

To get a velocity, divide a ruler measurement by a clock measurement

To get a force, multiply
a scale measurement by a ruler measurement and divide by two clock measurements

Keep separate your statement of what measurement tools
you are using (dimensional analysis)
from your actual values!

- These are not numbers!


A student measures distance $x$ to be 5 meters and area $A$ to be $25 \mathrm{ft}^{2}$.
Discuss with neighbors which of the following are true; then vote for all that are true.

$$
\begin{aligned}
& \text { 1. }\left[x^{2}\right]=[A] \\
& \text { 2. }[5 x]=A \\
& \text { 3. } x^{2}=[A] \\
& \text { 4. } x^{2}=A
\end{aligned}
$$

5. None of the above

# Which equation represents the quantity on the left? 

A. The area of a circle.

$$
\text { 1. } \quad 2 \pi R
$$

B. The volume of a sphere.

$$
\text { 2. } \quad 4 \pi R^{2}
$$

3. $\frac{4}{3} \pi R^{3}$
D. The surface area of a sphere.
C. The circumference of a circle.
4. $\pi R^{2}$

## The Special Case / Extreme Case Tool

- When we are working with symbolic equations, it often helps to think of specific cases (putting in numbers!) or considering extreme cases where we have strong intuitions as to what the result should be.


## Example

A torus is the mathematical name for the shape of a donut or bagel. Its volume can be expressed as a function of the inner and outer radii of the torus (distance from the center to the inner and outer edge). Which of the following equations could be the correct equation for the volume of the torus?

$$
\begin{aligned}
& \text { A. } V=(2 \pi R)\left(\pi r^{2}\right) \\
& \text { B. } V=\frac{\pi^{2}}{4}(R+r)(R-r)^{2} \\
& \text { C. } V=\pi R r \\
& \text { D. } V=\frac{\pi^{2}}{4}(R r)^{2}
\end{aligned}
$$



## Example

You fly east in an airplane for 100 km .
You then turn left 60 degrees and fly 200 km .
About how far are you from your starting point?
A. 170 km
B. 200 km
C. 260 km
D. 300 km
E. 370 km


## The Estimation Tool

- In the estimation game, you use whatever personal knowledge you have (and think you can trust) to build numbers in complex situations.
- This can help you
- Decide what you need to include and what to ignore when modeling
- Develop intuitions for large numbers (Use scientific notation!)
- Be careful! Memorized (one-step) numbers often get crossed up. Find things you can trust and build crosslinks when possible.


## The basic methods for estimations

1. Give enough words so we can follow your reasoning.
2. Decide what it is you need to calculate (formulas). Cross-check formulas to be sure they're correct.
3. Do you estimations from things you know (Know your basics!) and tell how you got them.
4. Keep units throughout and USE THEM to make sure you've done the correct calculation.
5. Don't keep more than 2-3 sig figs anywhere in the calculation.
6. NEVER write an untrue equation. (" $=$ as $\rightarrow$ ")
7. Be sure you answer all the questions asked!
8. Throughout, treat your numbers as MEASUREMENTS and check them for reasonableness!

## My personal scales



## Useful numbers (bio)

| Bio Scalles |  |
| :--- | :--- |
| Size of a typical animal cell | $\sim 10-20$ microns $\left(10^{-5} \mathrm{~m}\right)$ |
| Size of a bacterium, <br> chloroplast, or mitochondrion | $\sim 1$ micron $\left(10^{-6} \mathrm{~m}\right)$ |
| Size of a medium-sized virus | $\sim 0.1$ micron $\left(10^{-7} \mathrm{~m}\right)$ |
| Thickness of a cell membrane | $\sim 5-10 \mathrm{~nm}\left(10^{-8} \mathrm{~m}\right)$ |

## Every analysis we make in science is a model

- This means we are ignoring some things and paying attention to others.
- We need to choose wisely which is which
- The art in science is in picking what really matters and what can safely be ignored - for the particular issue being considered at the time.
- That requires intuition about estimations


## Estimation is fundamental to all modeling

A. A copper pot with a mass of 2 kg is sitting at room temperature $\left(20^{\circ} \mathrm{C}\right)$. If 200 g of boiling water $\left(100^{\circ} \mathrm{C}\right)$ are put in the pot, after a few minutes the water and the pot come to the same temperature. What temperature?
B. In the transformation that occurred in part A, how much thermal energy left the water? How much entered the copper?
C. If there were already 50 g of water in the pot (at room temperature) before the 200 g of hot water was added, what would the common temperature reached have been?

> In solving the above problems, you almost certainly made a number of simplifying but unrealistic assumptions that could affect the result. Name three

## The Functional Dependence / Scaling Tool

- This is one of your most important tools.
- Different dependences show you that things may change in different ways when different things change, with some effects being much more important than others.
- A critical example in biology is Fick's Law.
- The fact that how long it takes something to diffuse a given distance is proportional to the square root of the time rather than the time is responsible for lots of structures in organismal anatomy.


## Example

One of the essential elements of the animal immune system is the macrophage: a cell that ingests an destroys harmful bacteria. The bacterium might contain molecules of a chemical safe for it, but harmful to the macrophage. But the macrophage is bigger, so the density of the harmful molecules will be less. (Assume they are both spherical.) Suppose the macrophage has a diameter of $20 \mu \mathrm{~m}$ and the bacterium has a diameter of $1 \mu \mathrm{~m}$. If the density of these molecules in the bacterium is $D$, what will be their approximate density in the macrophage once the bacterium has been ingested, broken up, and distributed throughout the macrophage?
A. $D$
B. $D / 20$
C. $D / 400$
D. $D / 8000$
E. Something else

## The Parsing Tool

- Complicated expressions - whether equations or words - are often made up by combining simpler pieces.
- The complexity comes from combining many parts rather than from the complexity of the basic elements.
- Parsing is learning to focus down on the basic elements and build an understanding of the complex structure a bit at a time (instead of giving up as soon as it looks "too complicated"!).

Two pulses on an elastic string move towards each other in opposite directions. The pulses have peaks that are displaced 0.5 cm from their equilibrium positions and the pulses move along the string with a speed of $200 \mathrm{~cm} / \mathrm{s}$.


If the shape of each pulse is a Gaussian, $f(x)=A e^{-(x / a)^{2}}$ and if $d=2.0 \mathrm{~cm}$, which of the following equations might correctly represent the shape of the string at later times?
A. $f\left(x-v_{0} t\right)+f\left(x+v_{0} t\right)$
B. $f\left(x-v_{0} t\right)-f\left(x+v_{0} t\right)$
C. $f\left(x+d-v_{0} t\right)+f\left(x-d+v_{0} t\right)$
D. $f\left(x+d-v_{0} t\right)-f\left(x-d+v_{0} t\right)$
E. $f\left(x-d-v_{0} t\right)+f\left(x+d+v_{0} t\right)$
F. $f\left(x-d-v_{0} t\right)-f\left(x+d+v_{0} t\right)$
G. Something else (write it)

## Telling the story tool

- A critical element in understanding how equations (and the physics) works is understanding mechanism - How things happen.
- Components (characters)
- Interactions (relationships)
- Time sequence (plot)
- Causes and principles (message / moral)
- Learning to tell the story is a critical piece of learning to do any science.


## Example

- Consider a small rocket placed on a pad containing an electrical igniter. The rocket is attached to a small packet of chemical explosive in its tail. The igniter lights a short fuse that ignites the chemical explosive shooting the rocket upward. It rises straight up about 50 feet, then falls to the ground where it bounces and comes to a stop.
- Consider three times: $t_{0}=$ just after the explosion has completed but the rocket has not risen much, $t_{1}=$ the rocket is just at the top, and $t_{2}=$ the rocket has fallen to the ground and come to a stop.
- Identify what has happened to the various energies of the rocket (not including the explosive packet or fuse) from the beginning to the end of the time segments indicated. Put,+- , or 0 in each of the boxes to indicate the particular component of the rocket's energy had increased, decreased, or remained the same. Put $U$ (for unknown) if there is not enough information to decide.


## The Equation Game

- What physical system are we talking about?
- What variables and parameters do we use to describe the system?
- What physical principles are relevant? Do they lead to any equations? (Check dims.)
- What models are relevant?

Do they lead to any equations? (Check dims.)

- What do we know and what are we supposed to find?
- Work with symbols to solve. Don't put in numbers until the end!


