April 21, 2017 Physics 132 Prof. E. F. Redish

- **<u>Theme Music:</u> Duke Ellington**
  Take the A Train
- **<u>Cartoon:</u> Cantu and Castellanos**
  Baldo

Foothold ideas: Electric potential energy and potential

- The potential energy between two charges is

$$U_{12}^{elec} = \frac{k_C Q_1 Q_2}{r_{12}}$$

- The potential energy of many charges is

$$U_{12\ldots N}^{elec} = \sum_{i<j=1}^{N} \frac{k_C Q_i Q_j}{r_{ij}}$$

- The potential energy added by adding a test charge *q* is

$$\Delta U_q^{elec} = \sum_{i=1}^{N} \frac{k_C q Q_i}{r_{iq}} = qV$$

Some basic electrical ideas

- ***Conductor*** – a material that permits some of its charges to move freely within it.
  - ***Implication***: If the charges in a conductor are not moving, the whole conductor is as the same V.

Why?

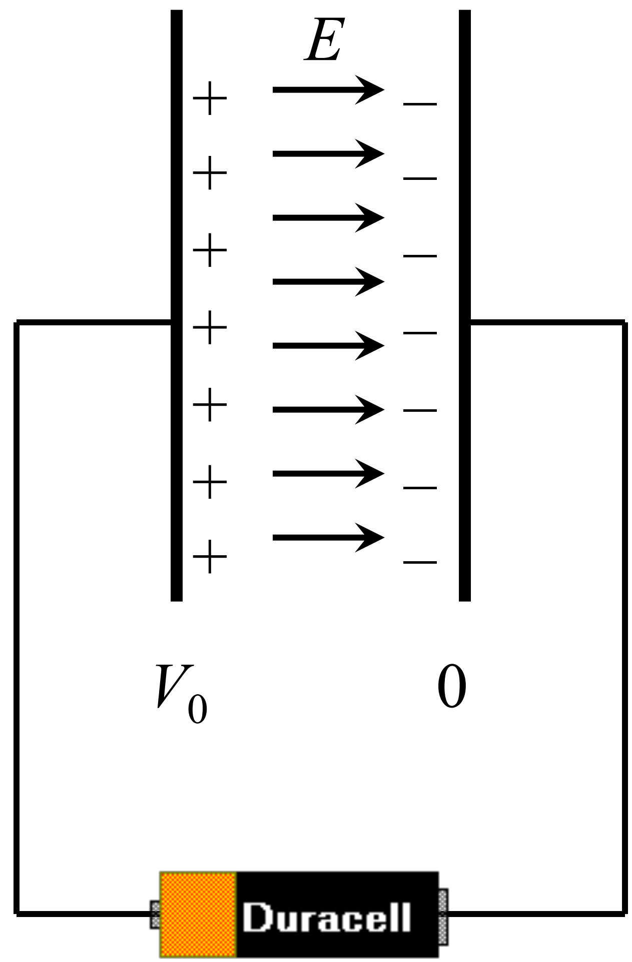
- ***Insulator*** – a material that permits some of its charges to move a little, but not freely.
- ***Battery*** – a device that creates and maintains a constant potential difference across its terminals.

$\Delta V = V_0$ **volts**

High end Low end

Charging a capacitor

- What is
  the potential difference
  between the plates?
- What is the field
  around the plates?
- How much charge
  is on each plate?



Capacitor Equations

$$\Delta V = E\Delta x = Ed$$

$$E = 4\pi k_C \sigma = 4\pi k_C \frac{Q}{A} \quad \Rightarrow \quad Q = \left(\frac{A}{4\pi k_C}\right)E$$

$$Q = \left(\frac{A}{4\pi k_C d}\right)\Delta V$$

$$Q = C\Delta V$$

What does this "Q" stand for?

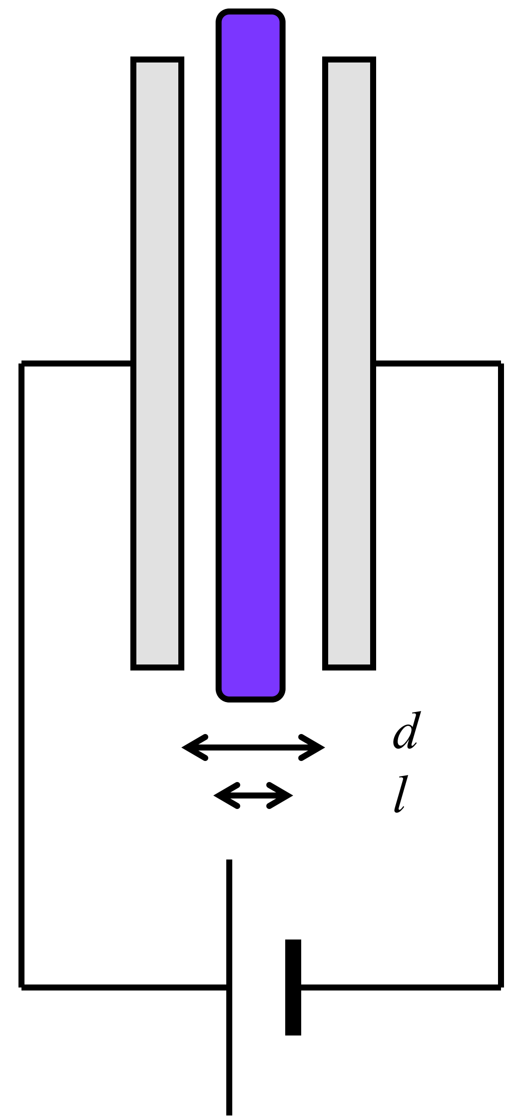
Energy stored in a capacitor

$$E = \tfrac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

Conductors

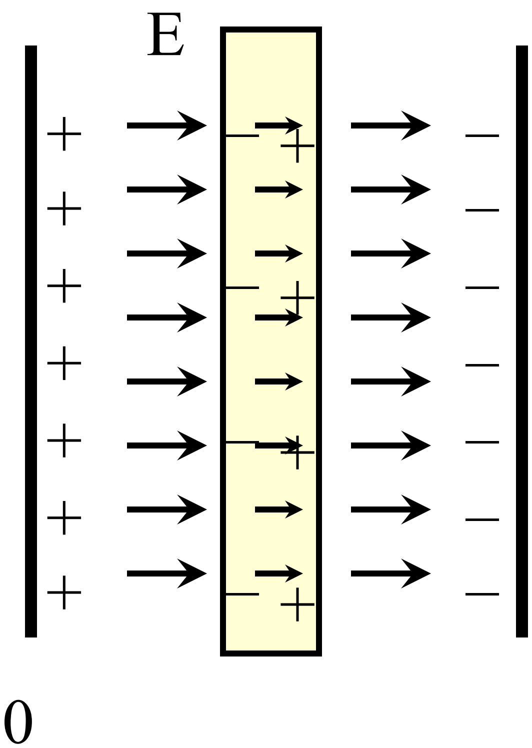
- Putting a conductor inside a capacitor eliminates the electric field inside the conductor.
- The distance, $d' = d - l$, used to calculate the ΔV, is only the place where there is an E field, so putting the conductor in reduces the ΔV for a given charge.

$$C = \frac{1}{4\pi k_C}\frac{A}{d'}$$



Consider what happens with an insulator

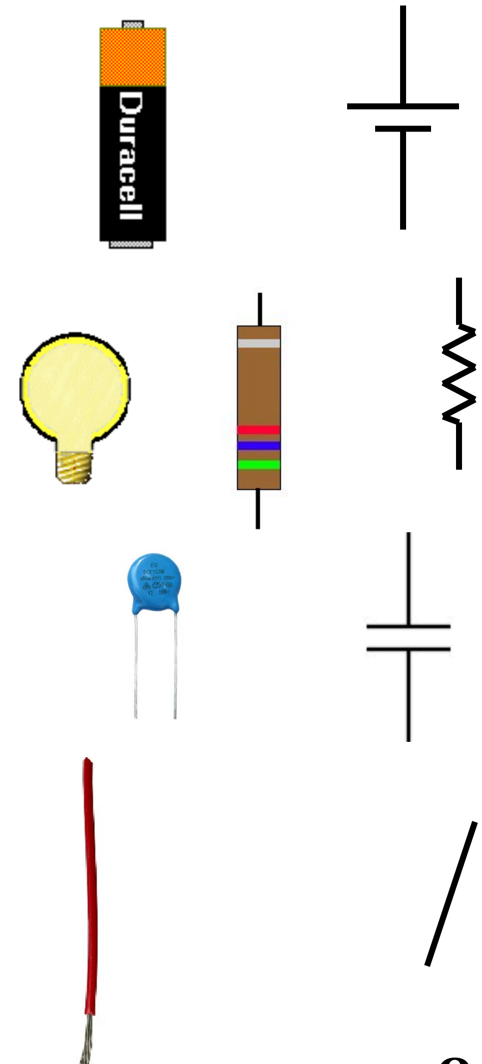
- We know that charges separate even with an insulator.
- This still reduces the field inside the material, just not to 0.
- The field reduction factor is defined to be κ (*the dielectric constant*).



$$E_{\text{inside material}} = \frac{1}{\kappa} E_{\text{if no material were there}}$$

Electric circuit elements

- *Batteries* —devices that maintain a constant electrical pressure difference across their terminals (like a water pump that raises water to a certain height).
- *Resistances* —devices that have significant drag and oppose current. Pressure will drop across them.
- *Capacitors* – devices that can maintain a separation of charge if there is a potential difference maintained across the,
- *Wires* — have very little resistance. We can ignore the drag in them (mostly – as long as there are other resistances present).



Foothold Idea: Local Neutrality

- Most matter is made of of an equal balance of two kinds of charges: positive and negative.
- Since the electric force is <u>very</u> strong, mostly the + and - charges overlap closely and cancel each other. (Large energy in BF!)
- Small imbalances in the cancellation leads to:
  - polarization forces
  - potential drop across a resistance
  - observed electric forces.

Foothold ideas: Electric charges in fluids

- ***Electroneutrality*** – Opposite charges in materials attract each other strongly. Pulling them apart to create a charge unbalance costs energy. This tends to make small volumes of fluid electrically neutral.
- ***Energy-Entropy balances*** – When there are situations of non-uniformity, electrical forces (energy) can balance or be balanced by random thermal motion (entropy). Two important cases are:
  - **Debye shielding** – introduced unbalanced charge
  - **Nernst potential** – non-uniform concentrations of ions

Foothold ideas:

- ***Debye length***– A charge imbedded in an ionic solution is shielded by the ions pulling up towards the charge. The amount of imbalance is determined by a balance of the thermal fluctuation energy against the repulsive electrostatic energy arising from the imbalance.

$$\lambda_D = \sqrt{\frac{\kappa k_B T}{k_C q^2 c_0}}$$

$$V(r) = \frac{k_C Q}{\kappa r} e^{-r/\lambda_D}$$

- ***Nernst potential*** – When a membrane permits only one kind of ion to pass, diffusion from the side with a greater concentration of that kind of ion will build up a potential difference due to ions moving to the side with the lower concentration.

$$\Delta V = \frac{k_B T}{q} \ln\left(\frac{c_1}{c_2}\right)$$

Resistivity and Conductance

- The resistance factor in Ohm's Law separates into a geometrical part (L/A) times a part independent of the size and shape but dependent on the material.
- This coefficient is called the *resistivity* of the material (ρ). Its reciprocal (g) is called *conductivity*. The reciprocal of the resistance is called the *conductance* (G).

$$R = \left(\frac{bL}{q^2 nA} \right) = \rho \frac{L}{A} = \frac{1}{g}\frac{L}{A} = \frac{1}{G}$$

Foothold ideas: Currents

- Charge is moving: How much?

$$I = \frac{\Delta q}{\Delta t}$$

- How does this relate to the individual charges?

$$I = q\, n\, A\, v$$

- Constant flow means pushing force balances the drag force

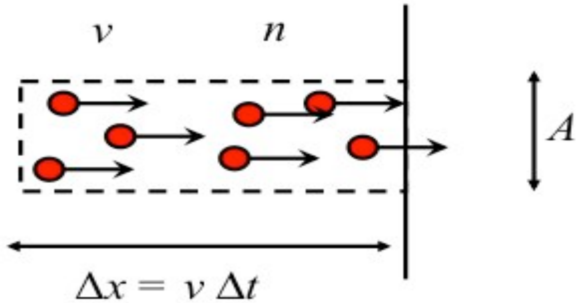
$$ma = F_e - bv$$

$$a = 0 \quad \Rightarrow \quad v = F_e / b$$

- What pushes the charges through resistance? Electric force implies a drop in *V*!

$$F_e = qE$$

$$\Delta V = -\frac{E}{L}$$



Ohm's Law

- Current proportional to velocity
- Due to resistance, Electric force proportional to velocity.
- Force proportional to "electric pressure drop" = "electric PE"
- Therefore, current proportional to "electric PE"

$$\Delta V = IR$$

$$I = qnAv \quad \Rightarrow \quad v = \frac{I}{qnA}$$

$$qE = bv$$

$$\Delta V = EL \quad \Rightarrow \quad E = \frac{\Delta V}{L}$$

$$\Rightarrow \quad \frac{q\Delta V}{L} = \frac{bI}{qnA}$$

$$\Delta V = I\left(\frac{bL}{q^2 nA}\right) \equiv IR$$

Foothold ideas: Kirchhoff's principles

1. ***Flow rule***: The total amount of current flowing into any volume in an electrical network equals the amount flowing out.
2. ***Ohm's law***: in a resistor, $\Delta V = IR$
3. ***Loop rule***: Following around any loop in an electrical network the potential has to come back to the same value (sum of drops = sum of rises).

Very useful heuristic

- The Constant Potential Corollary (CPC)
  – Along any part of a circuit with 0 resistance, then $\Delta V = 0$, i.e., the voltage is constant since in any circuit element

$$\Delta V = IR$$

$$R = 0 \Rightarrow \Delta V = 0$$

$$(\text{even if } I \neq 0)$$

Electric Power

- The rate at which electric energy is depleted from a battery or dissipated (into heat or light) in a resistor is

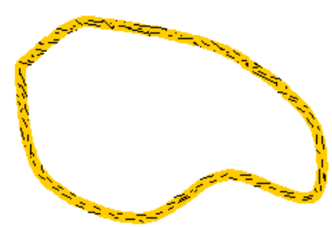
$$Power = \frac{dW}{dt} = \frac{d}{dt}\left(q\Delta V\right) = \frac{dq}{dt}\Delta V = I\Delta V$$

Units

■ Current (*I*)	**Ampere** = Coulomb/sec
■ Voltage (*V*)	**Volt** = Joule/Coulomb
■ E-Field (*E*)	Newton/Coulomb = Volt/meter
■ Resistance (*R*)	**Ohm** = Volt/Ampere
■ Capacitance (*C*)	**Farad** = Volt/Coulomb
■ Power (*P*)	**Watt** = Joule/sec

Analogy 1: The rope model

- Since like charges repel strongly, there can’t be a buildup of charge anywhere in the circuit (unless we make a special arrangement -- capacitance).
- Moving charges push other movable charges in front of them. The electrons move like links in a chain or rope.



Analogy 2 (Drude model): Ping-pong balls and nail board

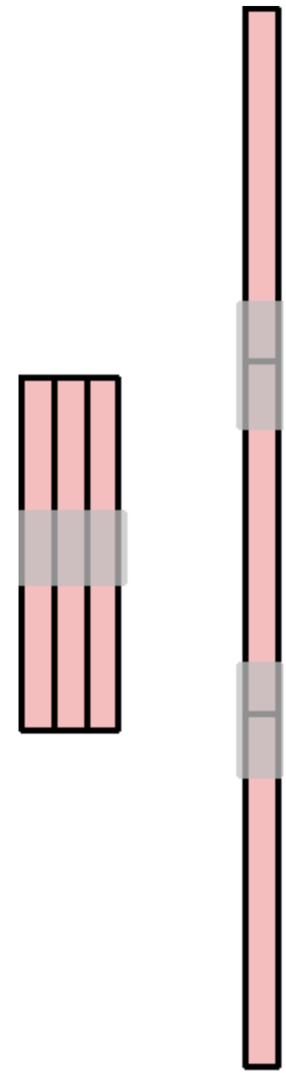
- In this analogy, we treat the electrons as small particles that can move freely through the conductor. (ping-pong balls)
- The ions that form the fixed body of the conductor are treated as fixed. (nails)
- The electron move freely between the ions until they hit them. Then they scatter in a random direction.

Analogy 3: Water flow

- The rope analogy fails because electrons can go either way at a junction. A current can split in a way a rope cannot.
- Water flow is a useful analogy because water
  - can divide
  - is conserved and cannot be compressed.

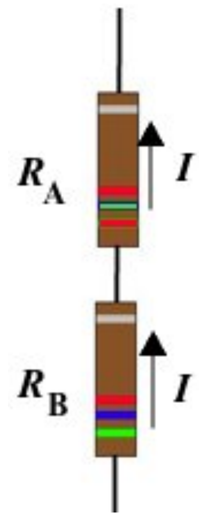
Analogy 4:
Air flow

- Pressure is analogous to electric potential.
- Pressure drop produces flow.
- Amount of flow depends on what is connected across a pressure drop.



Series and parallel

- Series
  - Same current flows through both devices

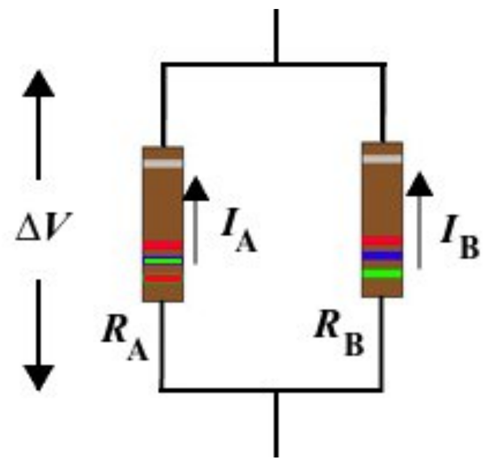


$$I = \frac{\Delta V_A}{R_A} = \frac{\Delta V_B}{R_B}$$

$$\frac{\Delta V_A}{\Delta V_B} = \frac{R_A}{R_B}$$

$$\Delta V = \Delta V_A + \Delta V_B$$
$$= I(R_A + R_B)$$

- Parallel
  - Same voltage drop across both devices



$$\Delta V = I_A R_A = I_B R_B$$

$$\frac{I_A}{I_B} = \frac{R_B}{R_A}$$

$$I = I_A + I_B$$
$$= \Delta V\left(\frac{1}{R_A} + \frac{1}{R_B}\right)$$

Oscillation and waves: I − physics

- Broadly, Physics has two ways of building understanding of matter:
  - "Particles" – bits of matter and their rules of behavior (interactions, forces, Newton's laws)
  - "Waves" – motion of vibrating patterns (oscillating matter and fields, Huygens' principle, Maxwell's equations)
- Interestingly, at the sub-atomic level these two approaches are both required and blend into something new and different from either.

Oscillation and waves: II − biology

- The physics of oscillations and waves have important implications for biology.
  - Many things in biology oscillate (carry out a repeating varying pattern)
  - Biological systems use oscillating waves to get information about their environment: sound, light
  - Waves carry rich information about their sources. Biology researchers (and physicists and astronomers) use the complex structure of waves to probe and gain information about biological systems.

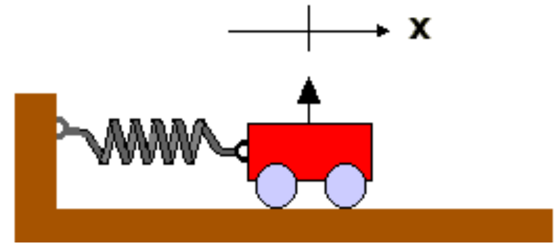
Oscillation and waves: III − pedagogy

- Waves are complicated mathematical concepts. Just as the concept of "field" was a step up in complexity from "particle" or "object", "wave" is a step up in complexity from "field".
  - We'll now consider not just a field distributed in space but we'll study how it can change in both space and time.
- We will have to consider oscillations in both space and time − functions of two variables.
  - We'll build the math required slowly, starting with the oscillation of one object: the harmonic oscillator.

Foothold ideas: Harmonic oscillation

- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves,
  the force is in the direction of
  pushing it back to its equilibrium position.
- When it gets back to its equilibrium,
  it's still moving so it overshoots.

Toy Model system: Mass on a Spring

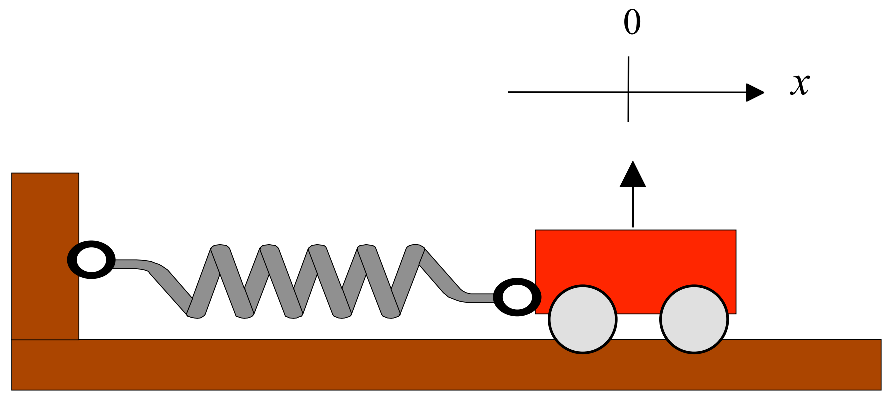


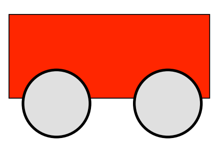
- Consider a cart of mass *m* attached to a light (mass of spring << *m*) spring.
- Choose the coordinate system so that when the cart is at 0 the spring it at its rest length
- Recall the properties of an ideal spring.
  - When it is pulled or pushed on both ends it changes its length.

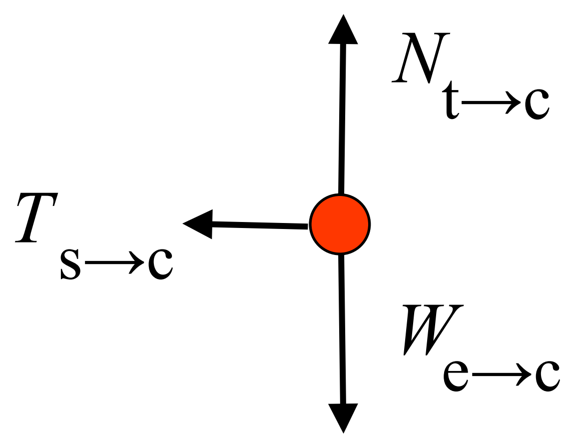
$$T = k\Delta l$$

Analyzing the forces: cart & spring

- FBD:
  What are
  the forces
  acting on the cart?

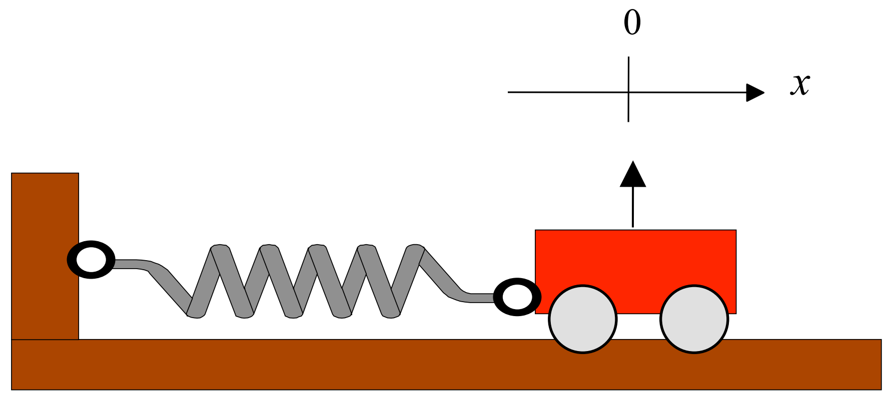


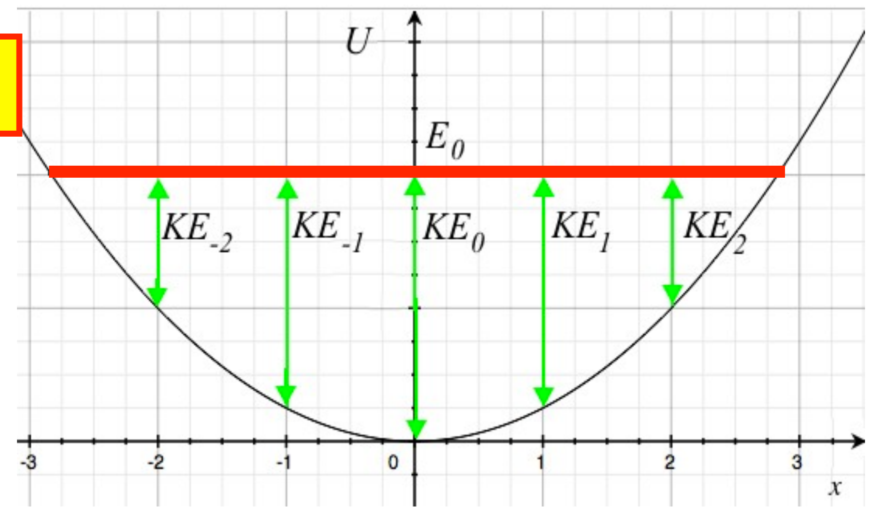




Analyzing the energy: cart & spring

- What are the energies in the cart-spring system?



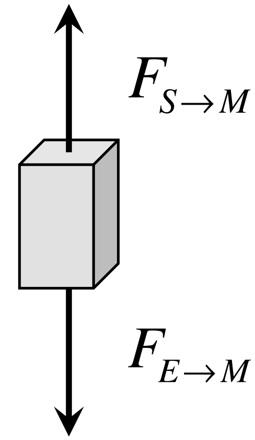


PE of the spring

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

KE of the cart

Summary with Equations: Mass on a spring



$$a = \frac{1}{m} F^{net} \qquad F^{net} = -kx$$

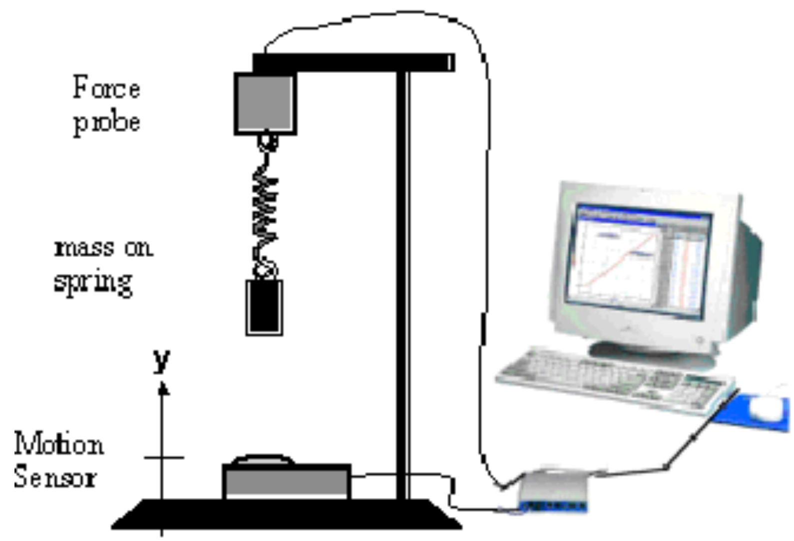
Measured from where?

$$a = -\omega_0^2 x \qquad \omega_0^2 = \frac{k}{m}$$

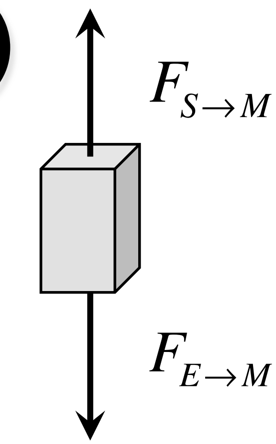
$$x(t) = A\cos(\omega_0 t + \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$

Interpret!



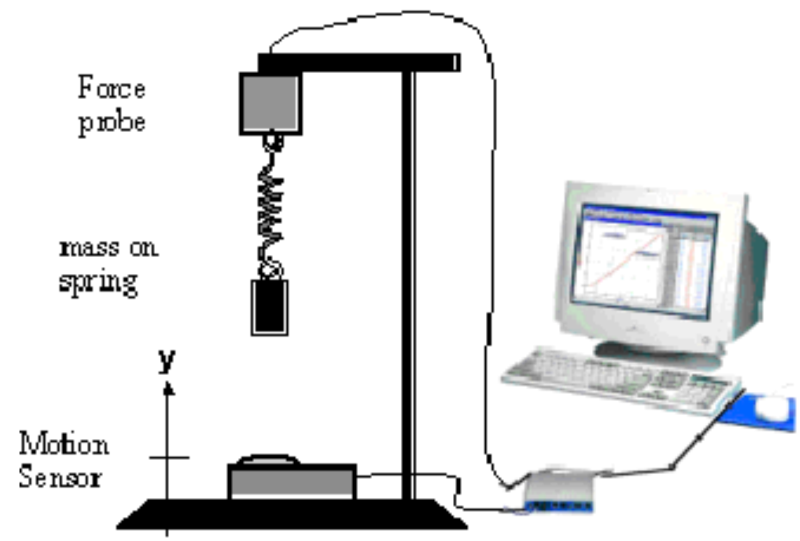
Summary with Equations: Mass on a spring (Energy)



Measured from where?

$$E = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}k\left(\Delta l\right)^2$$

$$E_i = E_f$$



The small angle approximation

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + ...$$

$$\cos\theta = 1 - \theta^2 + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + ...$$

$$\tan\theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + ...$$

This is how these are calculated! (Didn't you ever wonder how they did that?)

But these are often good enough.

$\sin\theta \approx \theta$ — *Good to 1% for θ < 1/4 rad (15°)*

$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ — *Good to 1% for θ < 1/3 rad (20°)*

$\tan\theta \approx \theta$ — *Good to 1% for θ < 1/4 rad (15°)*

Pendulum motion energy

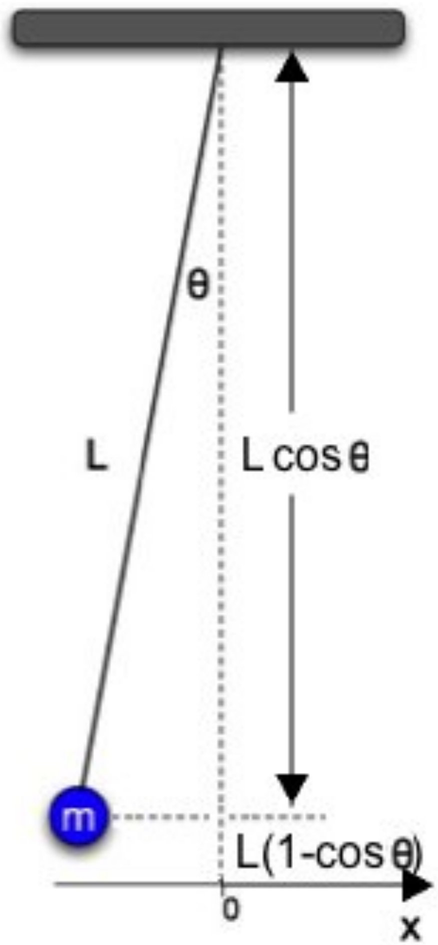
$$E_0 = \tfrac{1}{2}mv^2 + mgh = \tfrac{1}{2}mv^2 + mgL(1-\cos\theta)$$

$$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}[mgL]\theta^2$$

$$\theta \approx \sin\theta = \frac{x}{L}$$

$$E_0 \approx \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad k = \frac{mg}{L}$$



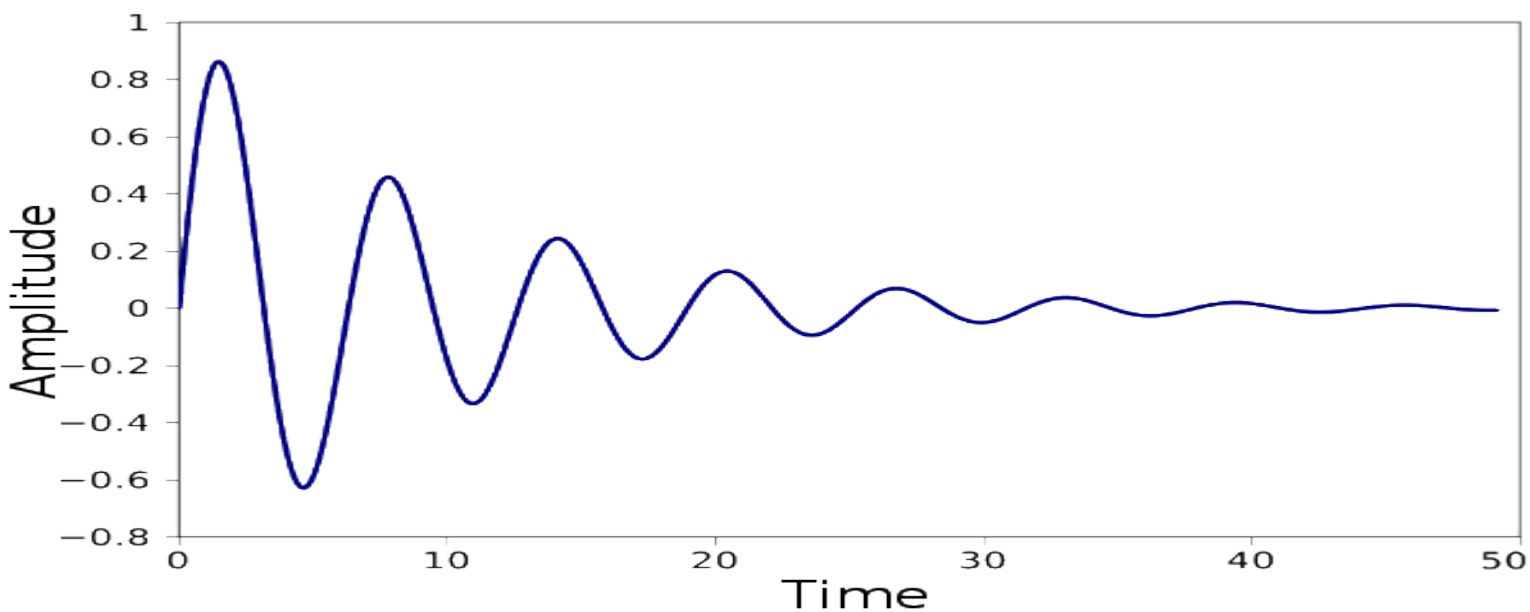
Same as mass on a spring!

Just with a different $\omega_0^2 = k/m = g/L$

What's the period? Why doesn't it depend on m?

Foothold ideas: Damped oscillator

- Our toy model of an oscillator gave the result $x(t) = A\cos(\omega_0 t)$.
- As we watch, it doesn't do that. What are we missing?



Foothold ideas: Damped oscillator 1

- Amplitude of an oscillator tends to decrease. Simplest model is viscous drag.

$$ma = -kx - bv$$

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = 0 \qquad \gamma = \frac{b}{m} \quad \omega_0 = \sqrt{\frac{k}{m}}$$

- Solution:

$$x(t) = A_0 e^{-\gamma t/2} \cos(\omega_1 t + \phi)$$

$$\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

Foothold ideas: Damped oscillator 2

- Competing time constants:

$$\frac{\gamma}{2} = \frac{1}{\tau} \qquad \frac{\omega_0}{2\pi} = \frac{1}{T} \qquad Q = \frac{\omega_0}{\gamma} = \pi\frac{\tau}{T}$$

Decay time (τ)

Period (T)

Tells which force dominates: restoring or damping. (Q)

- If:

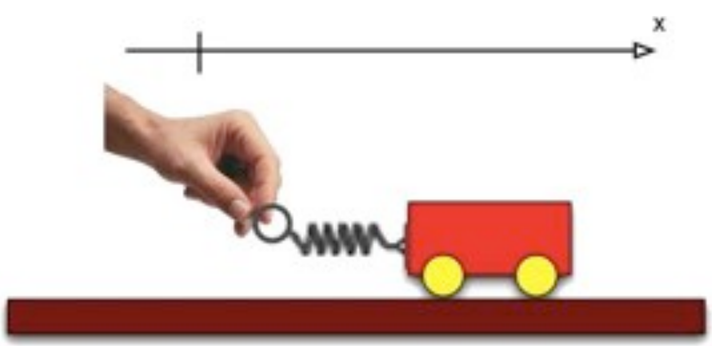
$\omega_0 > \gamma/2$ underdamped: oscillates

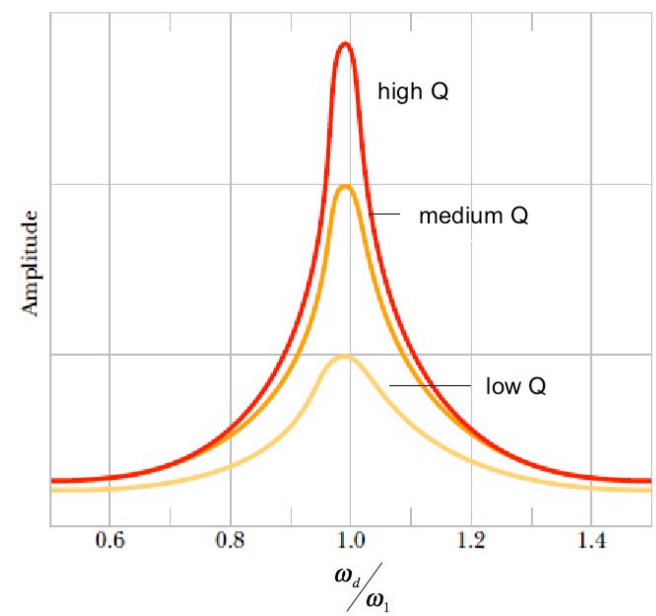
$\omega_0 = \gamma/2$ critically damped: no oscillation, fastest decay

$\omega_0 < \gamma/2$ over damped: no oscillation, slower decay

Foothold ideas: Driven oscillator

- Adding an oscillating force.
- When the extra oscillating force (driver) matches the natural frequency of the oscillator you get a big displacement (**resonance**). Otherwise, not much.





Foothold principles: Mechanical waves

- *Key concept*: We have to distinguish the motion of the bits of matter and the motion of the pattern.
- *Pattern speed*: a disturbance moves into a medium with a speed that depends on the properties of the medium (but not on the shape of the disturbance)
- *Matter speed*: the speed of the bits of matter depend on both the size and shape of the pulse and on the pattern speed.
- *Mechanism*: the pulse propagates by each bit of string pulling on the next.

Foothold principles: Waves on a stretched string

- A stretched string can propagate both transverse and longitudinal waves. In both cases the pattern and the matter motions have to be distinguished..
- *Pattern speed*: a disturbance moves on the string with the speed where τ is the tension and μ is the mass density (M/L).

$$v_0 = \sqrt{\frac{\tau}{\mu}}$$

- *Matter speed*: the matter in a transverse wave moves with a velocity that depends on the slope of the wave at that point (dy/dx) times v_0.

Foothold principles: Mechanical waves 2

- *Superposition*: when one or more disturbances overlap, the result is that each point displaces by the sum of the displacements it would have from the individual pulses. (signs matter)
- *Beats*: When sinusoidal waves of <u>different frequencies</u> travel <u>in the same direction</u>, you get variations in amplitude (when you fix either space or time) that happen at a rate that depends on the difference of the frequencies.
- *Standing waves*: When sinusoidal waves of the <u>same frequency</u> travel in <u>opposite directions</u>, you get a stationary oscillating pattern with fixed nodes.

Beats

- Adding two sinusoidal oscillations with nearby frequencies leads to alternate enhancement and cancellation producing *pulses*.
(When we do this with a space oscillations with nearby wavelengths we call the result *wave packets*.)
- This comes from the trig identity

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

which gives

$$A\sin(\omega_1 t) + A\sin(\omega_2 t) = 2A\sin(\bar{\omega} t)\cos\left(\frac{\Delta\omega}{2} t\right)$$

$$\bar{\omega} = \frac{\omega_1 + \omega_2}{2} \qquad \Delta\omega = \omega_1 - \omega_2$$

Standing Waves

- Some points in this pattern (values of x for which $kx = n\pi$) are always 0. (NODES)
- We can tie the string down at these points and still let it wiggle in this shape. (Why???)
- To wiggle like this (all parts oscillating together) we need to have (Why???)

$$L = n\frac{\lambda}{2}$$

- We still have $v_0 = \omega/k$ that is $v_0 = \lambda f$