March 29, 2017
Physics 132 Prof. E. F. Redish
■Theme Music: R.E.M.
What's the Frequency, Kenneth?
■ Cartoon: Abstruse Goose


## Outline

- Go over Quiz 7

Discuss makeup exam

- Recap: The math of oscillations
- The hanging mass
- The pendulum
$■$ Oscillations and circular motion




## The Equation of the Day

## The Harmonic Oscillator

$\frac{d^{2} x}{d t^{2}}=-\omega_{0}^{2} x$
$x(t)=A \cos \left(\omega_{0} t+\phi\right)$


## Mathematical structure

■ Express $a=F^{\text {net }} / m$ in terms of derivatives.

$$
\frac{d^{2} x}{d t^{2}}=-\omega_{0}^{2} x
$$

$\square$ Except for the constant, this is like having a functions that is its own second derivative.

$$
\frac{d^{2} f}{d t^{2}}=-f
$$

$\square$ In calculus, we learn that $\sin (\mathrm{t})$ and $\cos (\mathrm{t})$ work like this. How about: $x=\cos t$ ?

## Interpreting the Result

■ We can easily take the derivatives to show
 solution $x(t)=A \cos \left(\omega_{0} t\right)$ satisfies the N2 equations
$\square$ What do the various terms mean?

- $A$ is the maximum displacement - the amplitude.
- What is $\omega_{0}$ ? If $T$ is the period (how long it takes to go through a full oscillation) then

$$
\begin{aligned}
& \omega_{0} t: 0 \rightarrow 2 \pi \\
& t \quad: 0 \rightarrow T \\
& \omega_{0} T=2 \pi \quad \Rightarrow \quad \omega_{0}=\frac{2 \pi}{T}
\end{aligned}
$$

$-1 / T$ is called the frequency and the unit $1 /$ second $=$ Hertz.

$$
f=\frac{1}{T}
$$

$$
\omega_{0}=2 \pi f
$$

## Graphs: $\sin (\theta)$ vs $\cos (\theta)$

■ Which is which? How can you tell?
$\square$ The two functions sin and cos are derivatives of each other (slopes), but one has a minus sign. Which one?
How can you tell?




## PhET sim: Trig Tour

Values
$(x, y)=\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
angle $=\frac{\pi}{3}$
$\cos \theta=\frac{x}{1}=\frac{1}{2}$

| degrees |
| :--- |
| radians |



3/27/17 https://phet.colorado.edu/en/simulation/trig-tour 9

## Graphs: $\sin (\theta)$ vs $\sin \left(\omega_{0} \mathrm{t}\right)$

$\square$ For angles, $\theta=0$ and $\theta=2 \pi$ are the same so you only get one cycle.
$\square$ For time, $t$ can go on forever so the cycles repeat.

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What does changing $\omega_{0}$ do to this graph?


## Interpreting the Result

■ What about the starting point?
Using cos means you always start at a peak when $t=0$. That might not always be true.


$$
\begin{aligned}
x(t) & =A \cos \left(\omega_{0}\left(t-t_{0}\right)\right) \\
& =A \cos \left(\omega_{0} t-\omega_{0} t_{0}\right)=A \cos \left(\omega_{0} t-\phi\right)
\end{aligned}
$$

