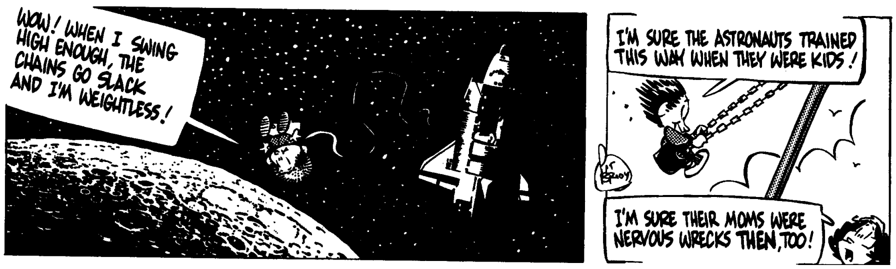
March 27, 2017 Physics 132 Prof. E. F. Redish

- **Theme Music: Fleetwood Mac**
  Silver Springs

  Cartoon: Pat Brady
  Rose is Rose



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Outline

- Quiz 7
- Harmonic oscillation
- Mass on a spring
- Oscillatory math: sines and cosines
- Equation of motion
  – Physical interpreation

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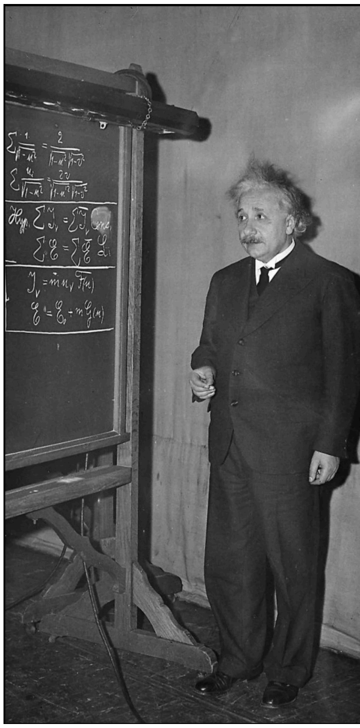

The Equation of the Day

The Harmonic Oscillator

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$x(t) = A\cos(\omega_0 t + \phi)$$

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Oscillation and waves: I − physics

- Broadly, Physics has two ways of building understanding of matter:
  - "Particles" – bits of matter and their rules of behavior (interactions, forces, Newton's laws)
  - "Waves" – motion of vibrating patterns (oscillating matter and fields, Huygens' principle, Maxwell's equations)
- Interestingly, at the sub-atomic level these two approaches are both required and blend into something new and different from either.

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Oscillation and waves: II − biology

- The physics of oscillations and waves have important implications for biology.
  - Many things in biology oscillate (carry out a repeating varying pattern)
  - Biological systems use oscillating waves to get information about their environment: sound, light
  - Waves carry rich information about their sources. Biology researchers (and physicists and astronomers) use the complex structure of waves to probe and gain information about biological systems.

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Oscillation and waves: III − pedagogy

- Waves are complicated mathematical concepts. Just as the concept of "field" was a step up in complexity from "particle" or "object", "wave" is a step up in complexity from "field".
  - We'll now consider not just a field distributed in space but we'll study how it can change in both space and time.
- We will have to consider oscillations in both space and time − functions of two variables.
  - We'll build the math required slowly, starting with the oscillation of one object: the harmonic oscillator.

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Foothold ideas: Harmonic oscillation

- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves, the force is in the direction of pushing it back to its equilibrium position.
- When it gets back to its equilibrium, it's still moving so it overshoots.

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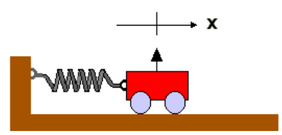
Toy Model system: Mass on a Spring



- Consider a cart of mass *m* attached to a light (mass of spring << *m*) spring.
- Choose the coordinate system so that when the cart is at 0 the spring it at its rest length
- Recall the properties of an ideal spring.
  - When it is pulled or pushed on both ends it changes its length.

$$T = k\Delta l$$

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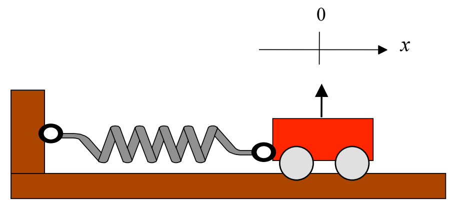
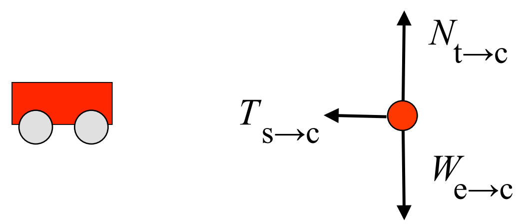
Analyzing the forces: cart & spring

- FBD:
  What are the forces acting on the cart?





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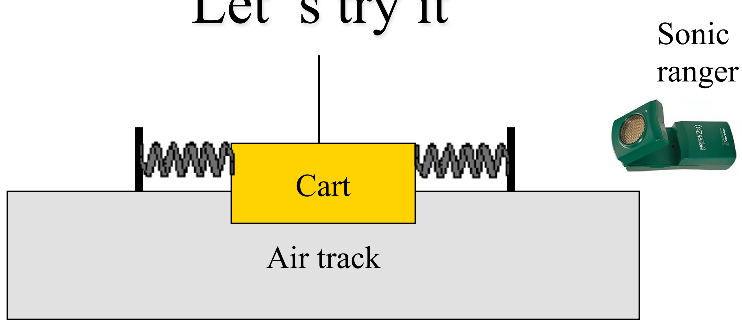
Let's try it



Why do we have two springs?

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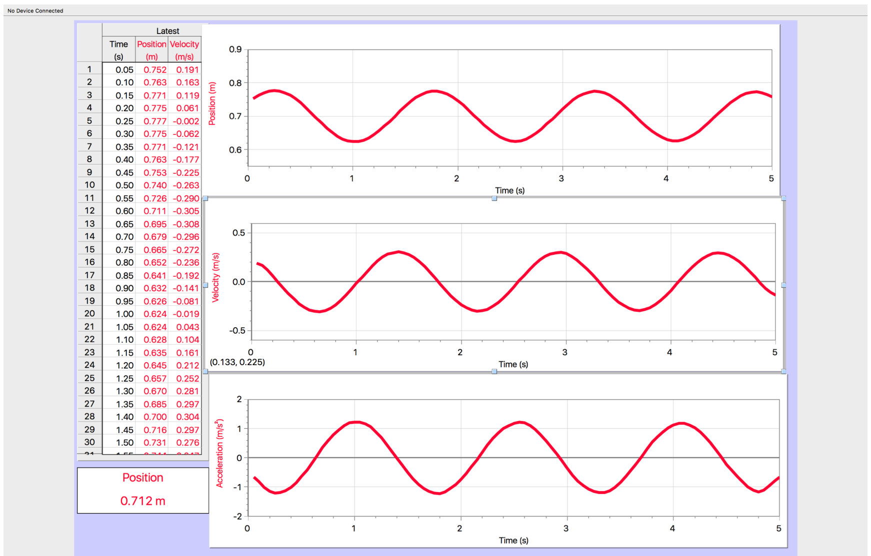
Result



	Time (s)	Position (m)	Velocity (m/s)
1	0.05	0.752	0.191
2	0.10	0.763	0.163
3	0.15	0.771	0.119
4	0.20	0.775	0.061
5	0.25	0.777	-0.002
6	0.30	0.775	-0.062
7	0.35	0.771	-0.121
8	0.40	0.763	-0.177
9	0.45	0.753	-0.225
10	0.50	0.740	-0.263
11	0.55	0.726	-0.290
12	0.60	0.711	-0.305
13	0.65	0.695	-0.308
14	0.70	0.679	-0.296
15	0.75	0.665	-0.272
16	0.80	0.652	-0.236
17	0.85	0.641	-0.192
18	0.90	0.632	-0.141
19	0.95	0.626	-0.081
20	1.00	0.624	-0.019
21	1.05	0.624	0.043
22	1.10	0.628	0.104
23	1.15	0.635	0.161
24	1.20	0.645	0.212
25	1.25	0.657	0.252
26	1.30	0.670	0.281
27	1.35	0.685	0.297
28	1.40	0.700	0.304
29	1.45	0.716	0.297
30	1.50	0.731	0.276

Doing the Math: The Equation of Motion

- The N2 equation for the cart is

$$a = F_{net}/m = -kx/m = -\left(\frac{k}{m}\right)x$$

- What kind of a quantity is k/m?

$$\left[\frac{k}{m}\right] =$$

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Mathematical structure

- Express $a = F^{net}/m$ in terms of derivatives.

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

- Except for the constant, this is like having a functions that is its own second derivative.

$$\frac{d^2 f}{dt^2} = -f$$

- In calculus, we learn that sin(t) and cos(t) work like this. How about: $\boxed{x = \cos t}$?

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A better answer

- We have to have an *x* that has units of distance. Since cos is a ratio it is unitless.
- We can't take the cos of a dimensioned number. It has to be an angle − radian is a ratio of two distances so its dimensionless.
- This fixes it (if $[A]$ = L and $[\omega_0]$ = 1/T)

$$x(t) = A\cos\left(\omega_0 t\right)$$

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Interpreting the Result

- We'll leave it to our friends in math to show that these results actually satisfy the N2 equations.
- What do the various terms mean?
  - A is the maximum displacement — the *amplitude* of the oscillation.
  - What is ω_0? If T is the *period* (how long it takes to go through a full oscillation) then

$$\omega_0 t : 0 \rightarrow 2\pi$$
$$t \quad : 0 \rightarrow T$$
$$\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}$$

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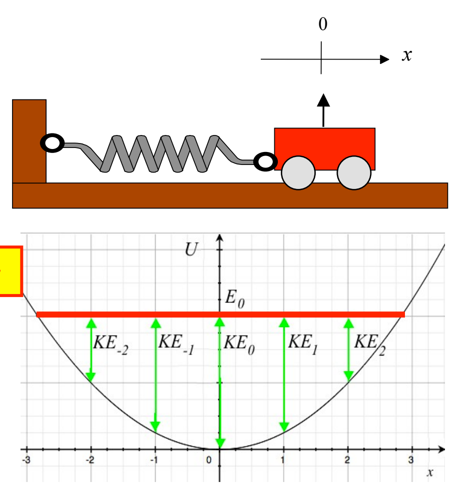
Analyzing the energy: cart & spring

- What are the energies in the cart-spring system?



PE of the spring

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$$

KE of the cart

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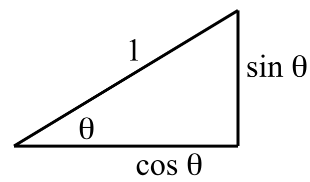
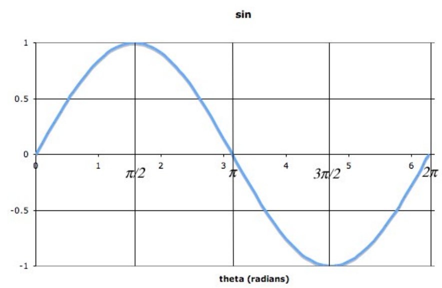
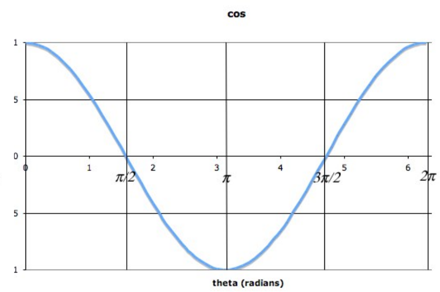
Graphs: sin(θ) vs cos(θ)

- Which is which? How can you tell?
- The two functions sin and cos are derivatives of each other (slopes), but one has a minus sign.
  Which one?
  How can you tell?





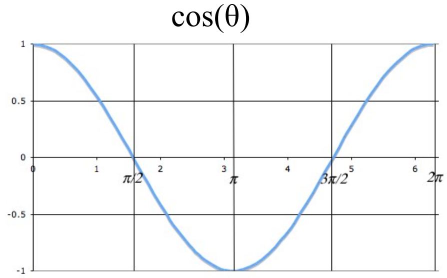


Graphs: sin(θ) vs sin(ω_0t)

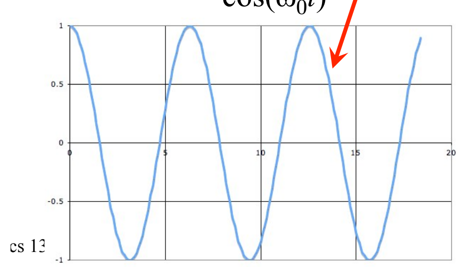
- For angles, θ = 0 and θ = 2π are the same so you only get one cycle.
- For time, *t* can go on forever so the cycles repeat.

What does changing ω_0 do to this graph?

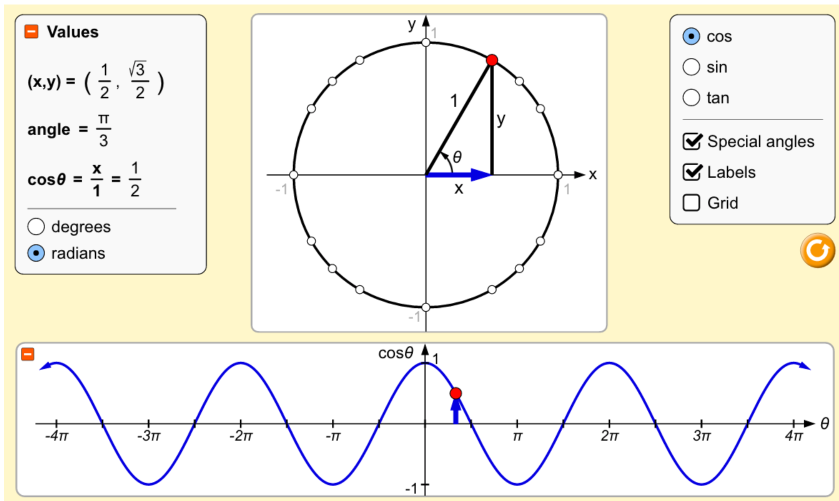
cos(θ)



cos($\omega_0 t$)



PhET sim: Trig Tour



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