January 30, 2017
Physics 132 Prof. E. F. Redish
$\square$ Theme Music: Blondie Atomic
■ Cartoon: Bill Amend
FoxTrot


## Building your toolbelt

Using math to make meaning in the physical world.


- Dimensional analysis
- Functional dependence / scaling
- Special cases / limiting cases
- Reading the physics in the representation (graphs)
- Reading the physics in the representation (equations)
- Changing physics equations to math (and back)
- The implications game


## The Dimensional Analysis Tool

■ Since we are mapping physical measurements into math, most of the quantities we use in physics are NOT NUMBERS. They are MEASUREMENTS.
■ This means they depend on an arbitrary scale we have chosen.

- In order that the equations we write keep their validity (the equation still holds) when we change our arbitrary scale dimensions must match on both sides of the $=$.
- Dimensions are arbitrary and depend on what choices we choose to think about changing. (e.g., moles, angles)


## Equations in physics (science) are NOT the same as equations in math!

We have four different kinds of measurements that we use so far:

- A measurement with a ruler (a length)
- A measurement with a clock (a time)
- A measurement with a scale (a mass)

- A measurement of electric strength (a charge)


## When we ask a symbol: "What measurements are you made of and how?" we will indicate it by using double square brackets:

 (making a Length measurement - L) $\llbracket \Delta t \rrbracket=\quad \begin{aligned} & \text { A time interval is found using a clock }\end{aligned}$
$\llbracket m \rrbracket=\quad \begin{aligned} & \text { A mass is found using a scale } \\ & \text { (making a mass measurement }-\mathrm{M})\end{aligned}$
$\llbracket \underbrace{Q}_{t / 2917} \rrbracket=\quad \begin{aligned} & \text { A charge is found using an ammeter } \\ & \left(\operatorname{mphaking}_{\text {Phics } \mathrm{H}_{2} \text { a current-time measurement }}-\mathrm{Q}\right)\end{aligned}$

## When we combine measurements we express it by showing how those measurements are combined

## $\llbracket \downarrow \rrbracket=$ (2) <br> A velocity is found by dividing a length measurement by a time measurement

$\llbracket a \rrbracket=(\omega) / \infty$
An acceleration is found by dividing a velocity measurement by a time measurement
$=$ (0)
Measurements, being a number with a unit, combine like algebraic symbols when combined by $\operatorname{mul}_{\text {Physcs }}$ t3iplying or dividing.

## When we have correct equations for symbols that we know it can tell us what measurements were combined to create that symbol.

$$
F=m a \quad \text { so } \quad \llbracket F \rrbracket=\llbracket m a \rrbracket
$$

So

$$
\llbracket F \rrbracket=\llbracket m \rrbracket \llbracket a \rrbracket=\overline{\operatorname{Ton}}(\mathrm{OQ}))
$$

# Since we don't want to be always drawing little scales, rulers, and clocks, we write them as "M", "L", and "T" but be careful not to confuse them with algebraic symbols that have values! 

## (Also, from laziness, we only write single instead of double brackets.)

## So read these as follows:

$$
\begin{array}{ll}
{[v]=\mathrm{L} / \mathrm{T}} & \begin{array}{l}
\text { a ruler measurement } \\
\text { by a clock measurement }
\end{array} \\
{[F]=\mathrm{ML} / \mathrm{T}^{2}} & \begin{array}{l}
\text { To get a force, multiply } \\
\text { a scale measurement by } \\
\text { a ruler measurement } \\
\text { and divide by two clock } \\
\text { measurements }
\end{array}
\end{array}
$$

# Keep separate your statement of what measurement tools you are using (dimensional analysis) from your actual values! 

■ These are not numbers!


## The Special Case / Extreme Case Tool

■ When we are working with symbolic equations, it often helps to think of specific cases (putting in numbers!) or considering extreme cases where we have strong intuitions as to what the result should be.

## The Estimation Tool

■ In the estimation game, you use whatever personal knowledge you have (and think you can trust) to build numbers in complex situations.

- This can help you
- Decide what you need to include and what to ignore when modeling
- Develop intuitions for large numbers (Use scientific notation!)
■ Be careful! Memorized (one-step) numbers often get crossed up. Find things you can trust and build crosslinks when possible.


## Every analysis we make in science is a model

■ This means we are ignoring some things and paying attention to others.
$\square$ We need to choose wisely which is which
$\square$ The art in science is in picking what really matters and what can safely be ignored - for the particular issue being considered at the time.

- That requires intuition about estimations


# Estimation is fundamental to all modeling 

A. A copper pot with a mass of 2 kg is sitting at room temperature $\left(20^{\circ} \mathrm{C}\right)$. If 200 g of boiling water $\left(100^{\circ} \mathrm{C}\right)$ are put in the pot, after a few minutes the water and the pot come to the same temperature. What temperature?
B. In the transformation that occurred in part A, how much thermal energy left the water? How much entered the copper?
C. If there were already 50 g of water in the pot (at room temperature) before the 200 g of hot water was added, what would the common temperature reached have been?

In solving the above problems, you almost certainly made a number of simplifying but unrealistic assumptions

## The Functional Dependence / Scaling Tool

■ This is one of your most important tools.
■ Different dependences show you that things may change in different ways when different things change, with some effects being much more important than others.

- A critical example in biology is Fick's Law.
- The fact that how long it takes something to diffuse a given distance is proportional to the square root of the time rather than the time is responsible for lots of structures in organismal anatomy.


## Foothold ideas: Conservation of Energy I

Mechanical energy

- The mechanical energy of a system of objects is conserved if all parts are taken into account.


$$
K E_{\text {initial }}+P E_{\text {initial }}=K E_{\text {final }}+P E_{\text {final }}
$$

- It is convenient to separate this into parts we might or might not want to consider the details of.
- Thermal energy: thekinetic energy of the random motion of molecules and the PE of their interactions is often grouped together as thermal energy.
- Chemical energy: The energy differences between separated atoms and molecules is due to the mechanical energy of electrons, often grouped as chemical energy.

$$
1 / 30 / 17 \quad \text { Physics } 132 \quad 22
$$

## Foothold ideas: <br> Conservation of Energy II

The thermal and chemical energies of an object are grouped together as the object's internal energy.

- Conservation of energy of a system of interacting objects with suppressed internal degrees of freedom now takes the form

$$
K E_{\text {initial }}+P E_{\text {initial }}+U_{\text {intital }}^{\mathrm{int}}=K E_{\text {final }}+P E_{\text {final }}+U_{\text {fnnal }}^{\mathrm{int}}
$$

Resistive forces transform energy from coherent mechanical energy of macroscopic objects into thermal and chemical reactions transform energy from chemical to other forms and back (usually thermal)

## Energy conservation with chemical reactions: 1

$\square$ Consider the collision of two molecules in isolation $\quad \mathrm{A}+\mathrm{B} \rightarrow \mathrm{A}+\mathrm{B}$

$$
K E_{A}+K E_{B}+P E_{A B}=\text { constant }
$$

$\square$ If the initial and final states both have the two molecules far apart, $U_{\mathrm{AB}} \sim 0$.

$$
K E_{A}+K E_{B}=\text { constant }
$$

## Energy conservation with chemical reactions: 2

$\square$ Consider the reaction of two molecules in isolation $\quad \mathrm{A}+\mathrm{B} \rightarrow \mathrm{C}+\mathrm{D}$
$\left(K_{A}+E_{A}\right)+\left(K_{B}+E_{B}\right)+P E_{A B}=\left(K_{C}+E_{C}\right)+\left(K_{D}+E_{D}\right)+P E_{C D}$
If the initial and final states both have the two molecules far apart, $P E_{\mathrm{AB}} \sim P E_{\mathrm{CD}} \sim 0$.

$$
\left(K_{A}+E_{A}\right)+\left(K_{B}+E_{B}\right)=\left(K_{C}+E_{C}\right)+\left(K_{D}+E_{D}\right)
$$

Note: The " $E$ "s here are molecular internal energies and are negative since the molecules are bound. The (positive)
bond energies from chemistry are given by $\varepsilon=-E>0$.

