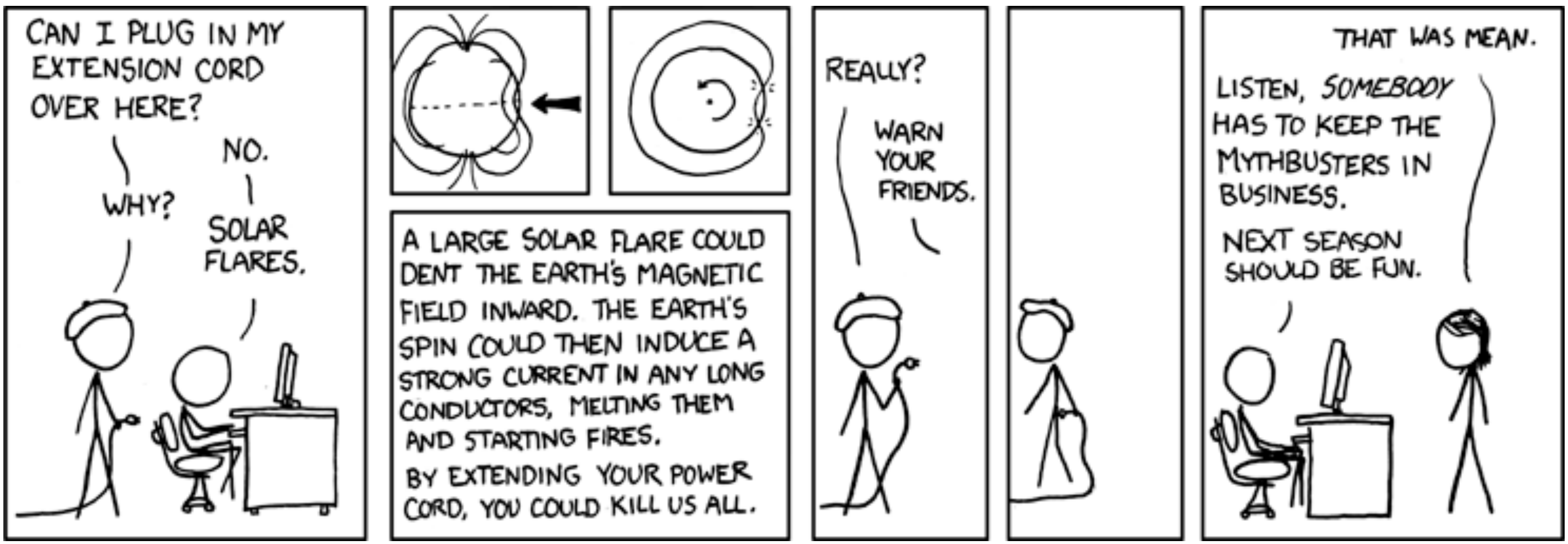
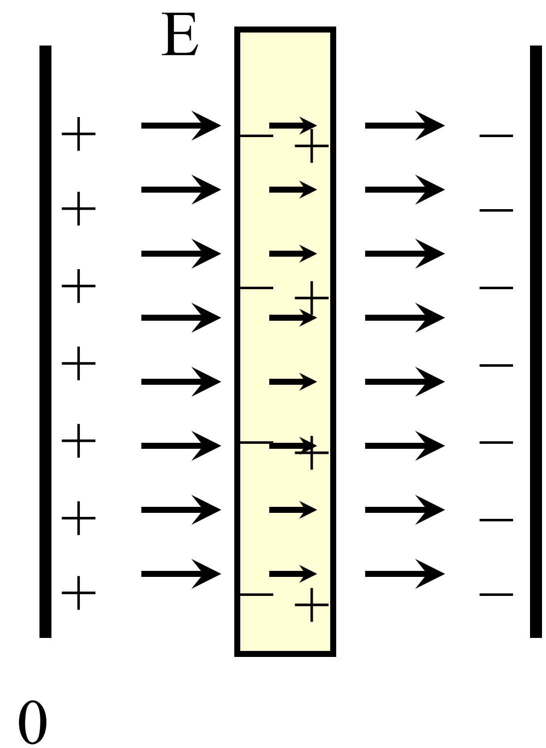
March 9, 2016 Physics 132 Prof. ~~E.F. Redish~~ B. Dreyfus

- **<u>Theme Music:</u> R.E.M., *Electrolite***
- **<u>Cartoon:</u> Randall Munroe, *xkcd***



Consider what happens with an insulator

- We know that charges separate even with an insulator.
- This still reduces the field inside the material, just not to 0.
- The field reduction factor is defined to be κ (*the dielectric constant*).



$$E_{\text{inside material}} = \frac{1}{\kappa} E_{\text{if no material were there}}$$

How a dielectric affects capacitance

- The dielectric **decreases** the electric field (for a given charge)
- This **decreases** the potential difference across the capacitor (for a given charge)
- This **increases** the capacitance of the capacitor

How a dielectric affects capacitance

- Capacitance without a dielectric:

$$C = \varepsilon_0 A/d$$

- Capacitance with a dielectric:

$$C = \kappa\varepsilon_0 A/d$$

Energy stored in a capacitor

- The potential difference across a capacitor is measured in volts (joules per coulomb)
- This represents the work it takes to move one unit of charge from one plate to the other
- $\Delta U = q\Delta V$

Energy stored in a capacitor

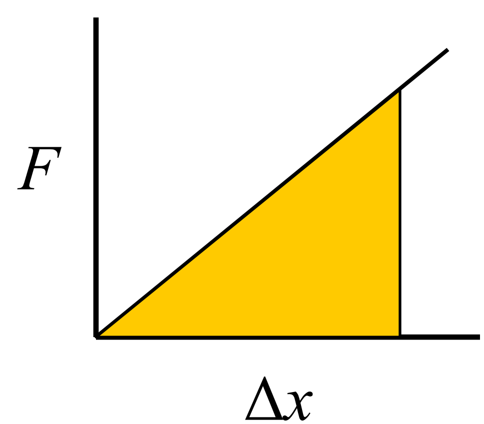
- The capacitance $C = Q/\Delta V$
- How much energy does it take to charge a capacitor from 0 to Q?
- If $\Delta U = q\Delta V$, *and* $\Delta V = Q/C$,
  can we just say that $\Delta U = Q^2/C$

Remember the potential energy stored in a spring…

- Hooke's Law: $F = k\Delta x$
- Work = $F\Delta x$
- **But** $U = ½\, kx^2$

Why the ½?

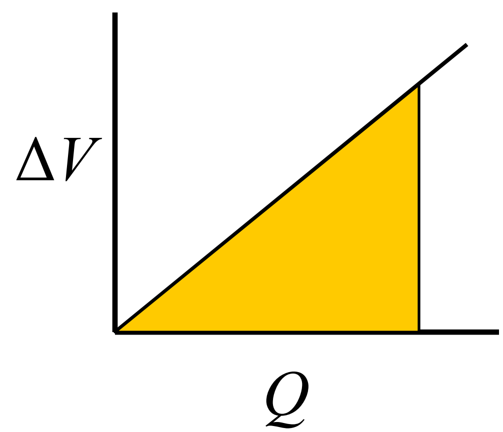
- Work = $F\Delta x$
- But in this case, F isn't constant!
- Integrate!



½ $(kx)\ x$

Similarly for a capacitor

– Work (for some charge q) = $q\Delta V$
– But in this case, ΔV isn't constant!
– Integrate!



$\frac{1}{2} Q\Delta V$

Energy stored in a capacitor

- $U = \frac{1}{2} Q\Delta V$
- And since $C = Q/\Delta V$
- $U = Q^2/2C$
- $U=1/2\ C(\Delta V)^2$

Mechanics→Stat mech

- We started with Newtonian mechanics
  - Looking at one object, or a small number of objects
- Then we went on to statistical mechanics
  - Looking at a **large** number of objects
  - Considering probability, random motion, entropy

Mechanics→Stat mech

- Electricity
  - So far we've also been looking at a small number of objects
- What happens when we combine electrical interactions with thermo / stat mech?
  - Debye length
  - Nernst potential

Electrostatics in a vacuum

- Coulomb's law: electric field (around a point charge) is proportional to $1/r^2$
- So it decreases as you get farther away, but never goes to zero

Electrostatics in a salt solution

- Ions can move around to “screen” charges, so the electric field is reduced

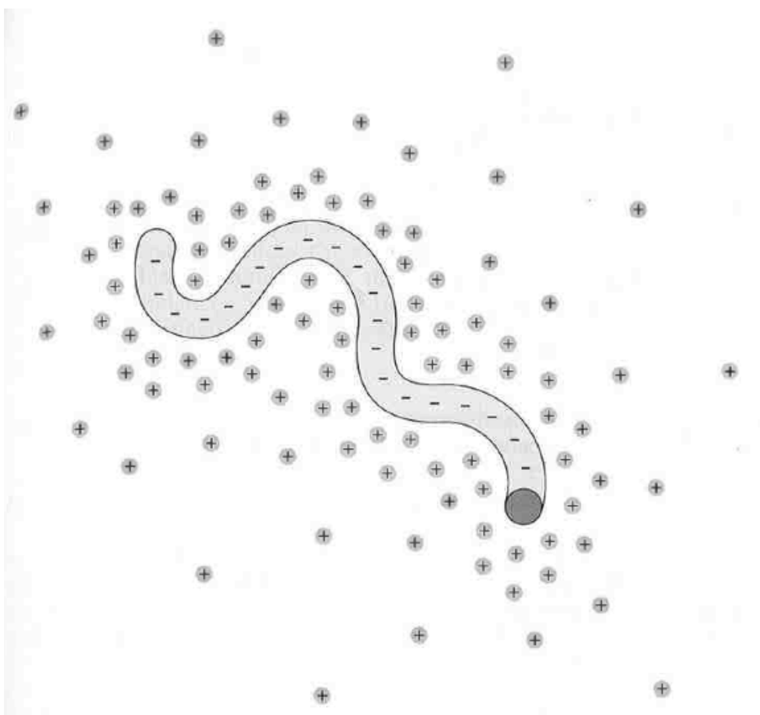


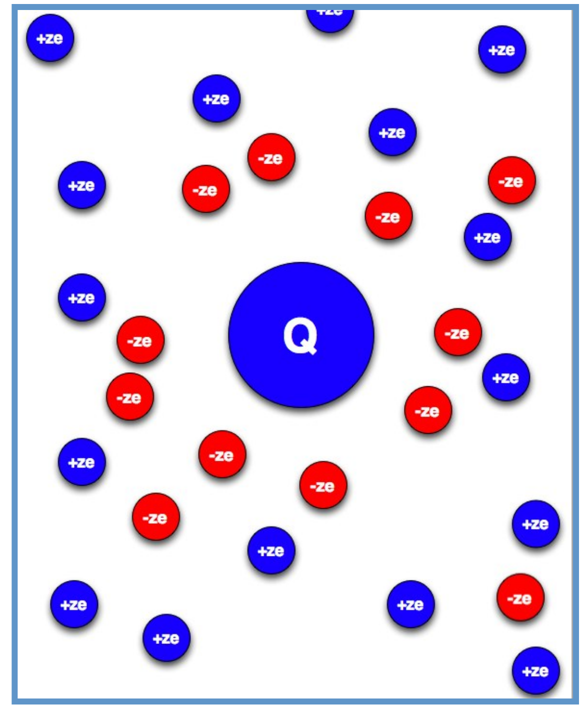
Figure 9.14 DNA in an ionic solution. The schematic shows the large negative charge density on the DNA molecule and the positive counterions in the surrounding solution.

Debye length

- Once again, it's energy vs. entropy!
- Effect of energy (forces):
  – electrical attraction
- Effect of entropy (random motion):
  – making everything spread out
- Debye length = how far out do we have to go before the electric field goes essentially to zero?

Debye length equations

- Charge embedded in an ionic solution.
  - Ion charge = ze
  - Concentration = c_0
  - Temperature = T
  - Dielectric constant = κ
- The ion cloud cuts off the potential



$$\lambda_D = \sqrt{\frac{k_B T}{8\pi\left(\frac{k_C z^2 e^2}{\kappa}\right)c_0}} = \sqrt{\frac{k_B T}{2\left(\frac{z^2 e^2}{\kappa\varepsilon_0}\right)c_0}}$$

$$V(r) = \frac{k_C Q}{\kappa r} e^{-r/\lambda_D}$$