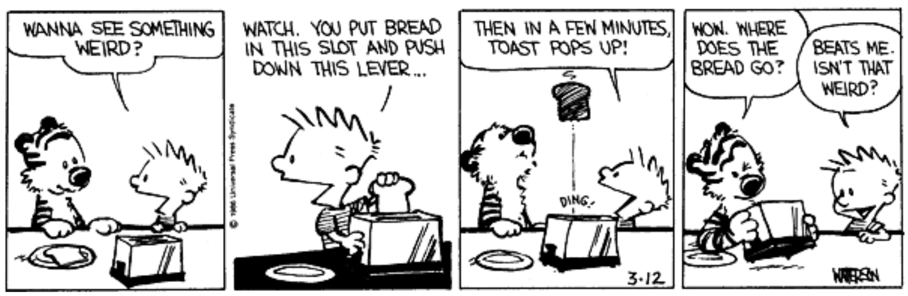
November 14, 2016 Physics 131 Prof. E. F. Redish

- **Theme Music:** **Bruce Springsteen**
  Working on a Dream
- **Cartoon:** **Bill Watterson**
  Calvin & Hobbes



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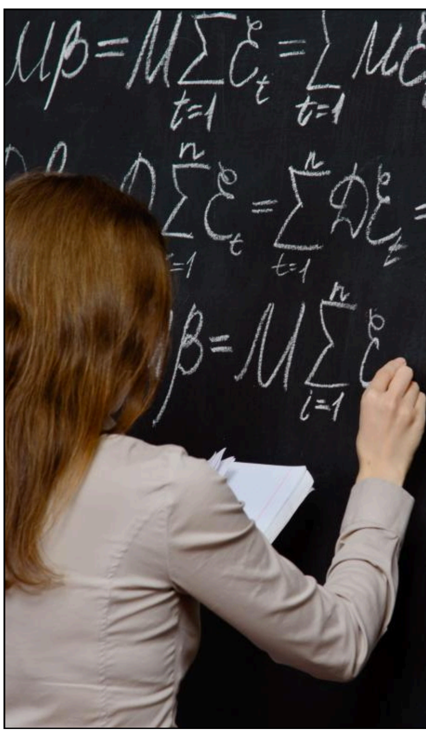

The Equation of the Day

The Work-Energy Theorem

$$\vec{F}^{net} \cdot \Delta\vec{r} = \Delta\left(\tfrac{1}{2}mv^2\right)$$

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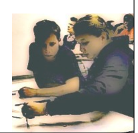
In many of your science classes you talk about "energy."

What is it?



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Energy

- N2 tells us that a force can change an object's velocity in one of two ways:
  - It can change the speed
  - It can change the direction
- Analyzing changes in speed leads us to study energy.
- Analyzing changes in direction leads us to study rotations.

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Kinetic Energy and Work

- Consider an object moving along a line feeling a constant net force, F^{net}. When it moves a distance Δx, how much does its speed change?

$$a = F^{net}/m$$

$$\frac{\Delta v}{\Delta t} = \frac{F^{net}}{m}$$

$$\frac{\Delta v}{\Delta t}\Delta x = \frac{F^{net}}{m}\Delta x$$

$$\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net}\Delta x}{m}$$

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$$\Delta v \frac{\Delta x}{\Delta t} = \frac{F^{net}\Delta x}{m}$$

$$\langle v \rangle \Delta v = \frac{F^{net}\Delta x}{m}$$

$$\frac{v_i + v_f}{2}(v_f - v_i) = \frac{F^{net}\Delta x}{m}$$

$$\tfrac{1}{2}(v_f^2 - v_i^2) = \frac{F^{net}\Delta x}{m}$$

$$\tfrac{1}{2}m(v_f^2 - v_i^2) = F^{net}\Delta x$$

Definitions:

Kinetic energy $= \tfrac{1}{2}mv^2$

Work done by a force F $= F\,\Delta x$

Result:

$$\Delta(\tfrac{1}{2}mv^2) = F^{net}\,\Delta x$$

The Work-Energy Theorem

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Foothold ideas: Kinetic Energy and Work

- Newton's laws tell us how velocity changes. The Work-Energy theorem tells us how **speed** changes (independent of direction).
- Kinetic energy = $\frac{1}{2}mv^2$
- Work done by a force = $F_x \Delta x$ or $F_{||} \Delta r$ ($F_{||}$ = the part of force || to displacement)
- Work-energy theorem: $\Delta(\frac{1}{2}mv^2) = F_{||}^{net} \Delta r$

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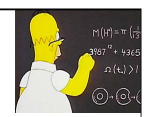
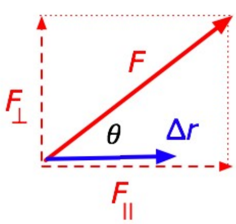
Work in another direction: The dot product



- Suppose we are moving along a line, but the force we are interested in in pointed in another direction? (How can this happen?)
- Only the part of the force in the direction of the motion counts to change the speed (energy).



$$\text{Work} = F_{||}\,\Delta r = F\cos\theta\,\Delta r \equiv \vec{F}\cdot\Delta\vec{r}$$

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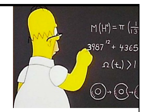


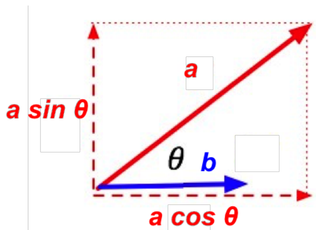
Dot products in general

$$F_{\parallel}\,\Delta r \equiv \vec{F}\cdot\Delta\vec{r} \qquad \vec{F}\cdot\Delta\vec{r} = F\cos\theta\,\Delta r$$

In general, for any two vectors that have an angle θ between them, the dot product is defined to be

$$\vec{a}\cdot\vec{b} = ab\cos\theta$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$$



The dot product is a scalar.
Its value does not depend on the coordinate system we select.

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