October 26, $2016 \quad$ Physics $131 \quad$ Prof. E. F. Redish
■ Theme Music: Robert Alda
Luck Be a Lady (from Guys \& Dolls)
■ Cartoon: Bill Amend
FoxTrot



## Foothold idea: <br> Coulomb's Law

- Point charges attract each other with a force whose magnitude is given by

$$
\vec{F}_{q \rightarrow Q}=-\vec{F}_{Q \rightarrow q}=\frac{k_{C} q Q}{r_{q Q}^{2}} \hat{r}_{q \rightarrow Q}
$$

$k_{\mathrm{C}}$ is put in to make the dimensions come out right.

$$
\left[k_{C}\right]=\left[\frac{F r^{2}}{q_{1} q_{2}}\right]=\frac{\mathrm{ML}^{2}}{\mathrm{~T}^{2}} \frac{\mathrm{~L}^{2}}{\mathrm{Q}^{2}}=\frac{\mathrm{ML}^{3}}{\mathrm{Q}^{2} \mathrm{~T}^{2}}
$$

## Adding forces for many charges!

$$
\begin{gathered}
\vec{F}_{q}=\vec{F}_{Q_{1} \rightarrow q}+\vec{F}_{Q_{2} \rightarrow q}+\vec{F}_{Q_{3} \rightarrow q}+\vec{F}_{Q_{4} \rightarrow q}+\ldots \\
\vec{F}_{q}=\frac{k_{c} q Q_{1}}{r_{1}^{2}} \widehat{r}_{1}+\frac{k_{c} q Q_{2}}{r_{2}^{2}} \vec{r}_{2}+\frac{k_{c} q Q_{3}}{r_{3}^{2}} \widehat{r}_{3}+\frac{k_{c} q Q_{4}}{r_{4}^{2}} \stackrel{r}{r}_{4}+\ldots
\end{gathered}
$$

where

$$
\begin{array}{ll}
r_{1}=\text { distance from } Q_{1} \text { to } q & \hat{r}_{1}=\text { direction from } Q_{1} \text { to } q \text { (mag. 1, no units!) } \\
r_{2}=\text { distance from } Q_{2} \text { to } q & \hat{r}_{2}=\text { direction from } Q_{2} \text { to } q \text { (mag. 1, no units!) }
\end{array}
$$

## E field for many charges!

$$
\vec{E}=\frac{\vec{F}_{q}}{q}
$$

$$
\vec{F}_{q}=\frac{k_{C} q Q_{1}}{r_{1}^{2}} \widehat{r}_{1}+\frac{k_{C} q Q_{2}}{r_{2}^{2}} \widehat{r}_{2}+\frac{k_{C} q Q_{3}}{r_{3}^{2}} \widehat{r}_{3}+\frac{k_{C} q Q_{4}}{r_{4}^{2}} \widehat{r}_{4}+\ldots
$$

where
$\begin{array}{ll}r_{1}=\text { distance from } Q_{1} \text { to } q & \widehat{r}_{1}=\text { direction from } Q_{1} \text { to } q \text { (mag. 1, no units!) } \\ r_{2}=\text { distance from } Q_{2} \text { to } q & \widehat{r}_{2}=\text { direction from } Q_{2} \text { to } q \text { (mag. 1, no units!) }\end{array}$

## Foothold principles: Fick's first Law

■ If a set of molecules is not distributed uniformly in 1 D (there is a concentration gradient) there will be an effective flow of those molecules according to
(or in 3D) $\vec{J}=-D \vec{\nabla}_{n} \quad J=-D \frac{d n}{d x}$
■ In a gas, the diffusion constant $D$ is given by $\frac{1}{2 \sqrt{3}} \lambda \bar{v}$
$\square$ In a liquid, the diffusion constant is given by $D=\frac{k_{B} T}{6 \pi \mu R}$

## Foothold principles:

## Fick's second Law

The average square displacement of a random walking molecule in a thermal bath after a time $t$ is given in $3 \mathrm{D}^{-1}$ by Fick's second law:

$$
\left\langle\Delta r^{2}\right\rangle=\left\langle\Delta x^{2}\right\rangle+\left\langle\Delta y^{2}\right\rangle+\left\langle\Delta z^{2}\right\rangle=6 D \Delta t
$$

- The radius of a small blob of chemical in a liquid will grow at this rate.
- The displacement, $\Delta r=\sqrt{\left\langle\Delta r^{2}\right\rangle}$, only grows like $\sqrt{\Delta t}$ For larger organisms, this is too slow and is the reason transport systems for air and blood have evolved.


## 2D Simulations: Multiple representations



## 2D Simulations: Multiple representations

1. Watch all the particles.
2. Look at the density of the particles

- What do the colors represent?

3. Look at a plot of the density along a slice through the middle.

- What it will look like and what it will do.

4. Look at the motion of individual particles.
