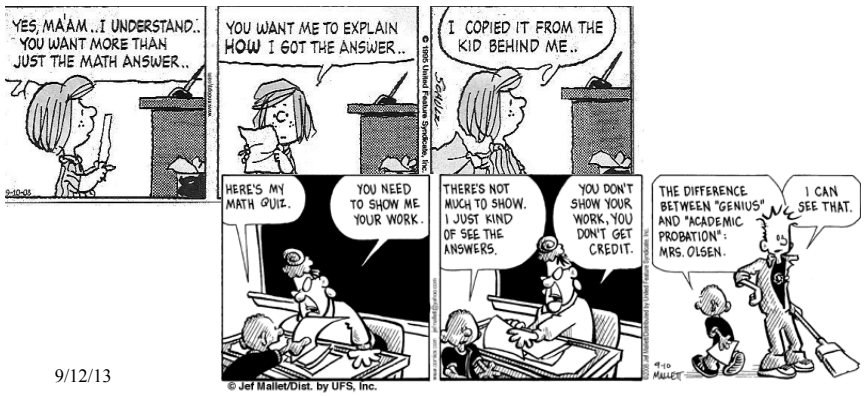
September 12, 2013 Physics 131 Prof. E. F. Redish

- **Theme Music:**
  ***Speed Racer Theme***
- **Cartoon: Charles Schultz / Jef Mallett**
  ***Peanuts / Frazz***

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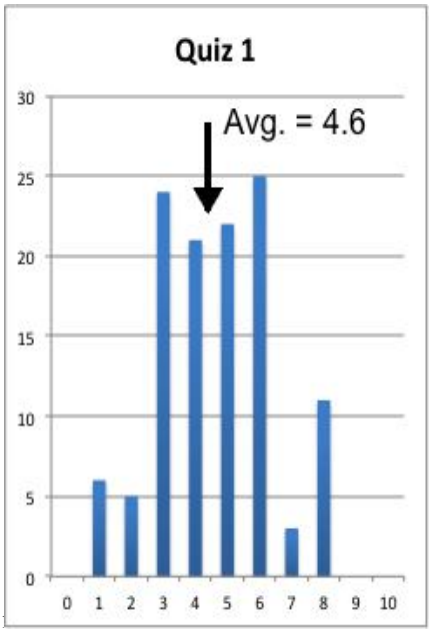
# Quiz 1

| | 1.2 | 1.3 | 2.1 | 2.2 | 2.3 |
|---|---|---|---|---|---|
| 1 | 71% | 28% | 3% | 15% | 26% |
| 2 | 7% | 15% | 10% | 62% | 18% |
| 3 | 22% | 56% | 57% | 14% | 56% |
| 4 | | | 2% | 9% | |
| 5 | | | 13% | | |

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# Example of a Diff Eq.

- Epidemiology: Number of people infected by a disease is proportional to the number of people.
- A simple model for the spread of infection

$$\frac{dI(t)}{dt} = AI(t) - BI(t)$$

$A =$ rate at which population gets infected

$B =$ rate at which infected people are cured (or die)

$$\frac{dI}{dt} = (A - B)I$$



# Use of Diff Eqs.

- In lots of cases – epidemiology, chemical reactions, and the motion of objects, the equations describing a system relate the values of something to its derivatives. Figuring out what happens next depends on being able to predict the future by knowing the derivative and "stepping".

Stepping rule: Suppose we know the value of something as a function of time at a given time, *f(t),* and we know its derivative, *df/dt* at that time. We can use that to predict the future!

$$\frac{df}{dt} = \frac{\Delta f}{\Delta t} = \frac{f_{end} - f_{beginning}}{\Delta t}$$

$$f_{end} - f_{beginning} = \left(\frac{df(t)}{dt}\right)\Delta t$$

$$f(t+\Delta t) - f(t) = \left(\frac{df(t)}{dt}\right)\Delta t$$

$$f(t+\Delta t) = f(t) + \left(\frac{df(t)}{dt}\right)\Delta t$$



# Foothold ideas: 1D Velocity

- Velocity is the rate of change of position
- Average velocity
  = (how far did you go?)/(how long did it take you?)

$$\langle v \rangle = \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity = same (but for short $\Delta t$)

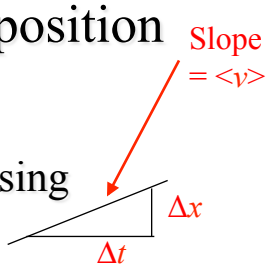
$$v = \frac{dx}{dt}$$

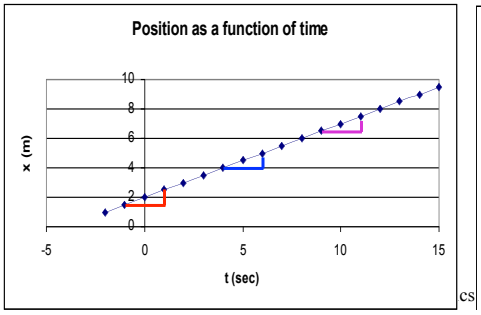
Can this velocity be negative as well as positive?

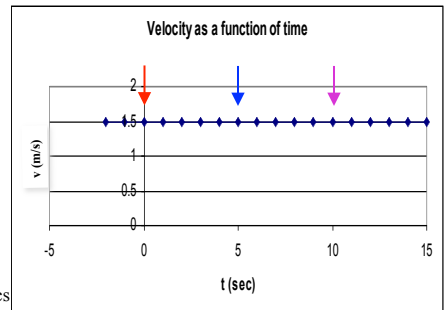
# Graphing velocity: Figuring it out from the position

- You can figure out the velocity graph from the position graph using

$$\langle v \rangle = \frac{\Delta x}{\Delta t} \qquad \Delta x = \langle v \rangle \Delta t$$

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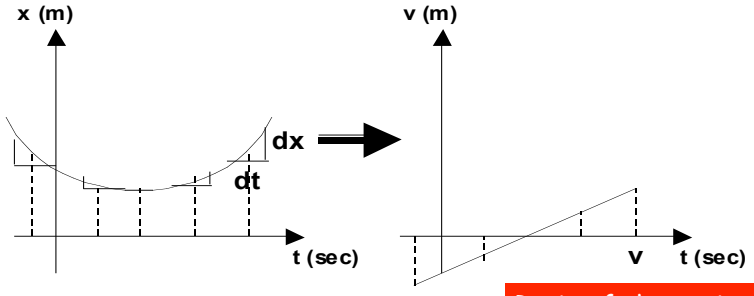
# Graphing Velocity: Figuring it out from the motion

- An object in uniform motion has constant velocity.
- This means the instantaneous velocity does not change with time. Its graph is a horizontal line.
- You can make sense of this by putting your mind in "velocity mode" and running a mental movie.

# Position to velocity




$$v(t) = \frac{dx}{dt}$$

Ratio of change in position that takes place to the (small) time interval

Difference of two positions at two (close) times

$$v(t) = \frac{x(t + \Delta t/2) - x(t - \Delta t/2)}{\Delta t}$$

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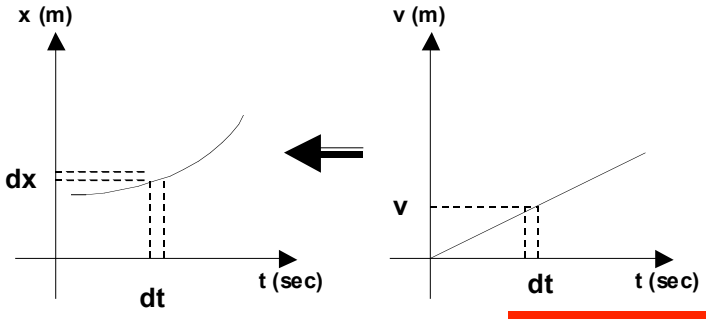
# Velocity to position




$$dx = v(t)\,dt$$

change in position that takes place in a small time interval

sum ("Σ") in the changes in position over many small time intervals

$$x = \sum dx = \int v(t)\,dt$$

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# What have we learned? Representations and consistency

- Visualizing where an object is at different times → a position graph
- Visualizing how fast an object is moving at different times → a velocity graph
- Position graph → velocity graph

$$\text{slopes } v = \frac{\Delta x}{\Delta t}$$

- Velocity graph → position graph

$$\text{areas } \Delta x = v\,\Delta t$$

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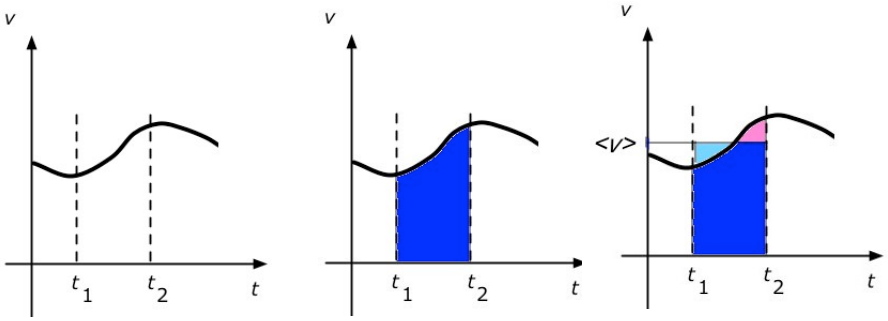
# Reading question: What the %*&$# does this mean?




$v$ is a function of $t$, $v(t)$

What is the average velocity between times $t_1$ and $t_2$?

The total displacement between those two times it the area under the curve. (Why?)

Adjust <$v$> (a constant) so $\Delta x = \langle v \rangle \Delta t$

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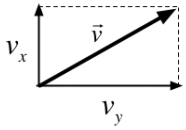
# Foothold ideas: Vector velocity and speed

- A displacement – a change in position – has a direction. This means
  *velocity = displacement/time interval*
  has one too. $\vec{v} = \frac{d\vec{r}}{dt}$

$$v_x\hat{i} + v_y\hat{j} = \frac{d}{dt}\left(x\hat{i} + y\hat{j}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j}$$

- We define speed as the magnitude of velocity. (No vector on this. Why?)

$$v = \sqrt{v_x^2 + v_y^2}$$






# The sonic ranger (motion detector)

- The sonic ranger measures distance to the nearest object by echolocation.
  - A speaker clicks 30 times a second. A microphone detects the sound bouncing back from the nearest object in front of it.
  - The computer calculates the time delay between and using the speed of sound (about 343 m/s at room temperature) it can calculate the distance to the object.

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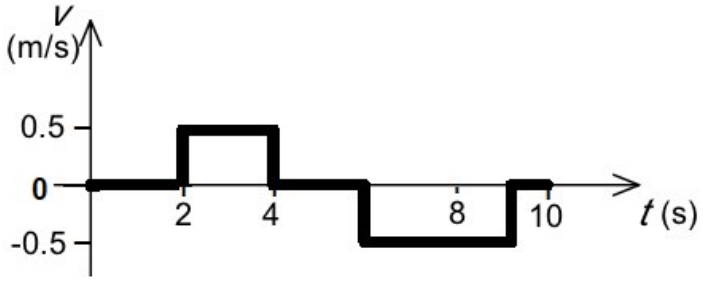
# Example

- How do you have to walk to make the sonic ranger produce the following velocity graph?




- Draw the position graph.