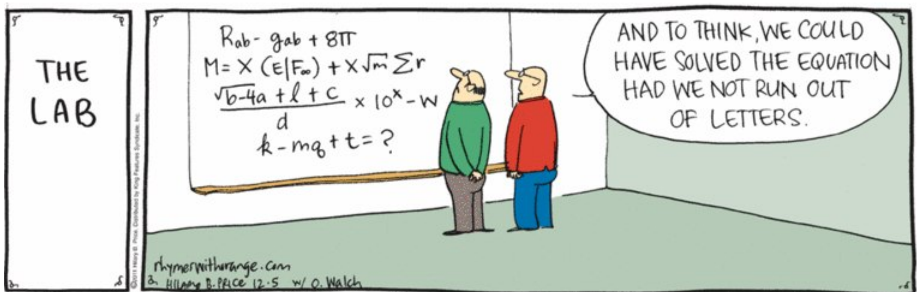
December 5, 2011 Physics 131 Prof. E. F. Redish

- **<u>Theme Music:</u> Might Could**
  - ***Adrift***
- **<u>Cartoon:</u> Hillary Putnam**
  - ***Rhymes with Orange***



12/5/11 Physics 131 **1**

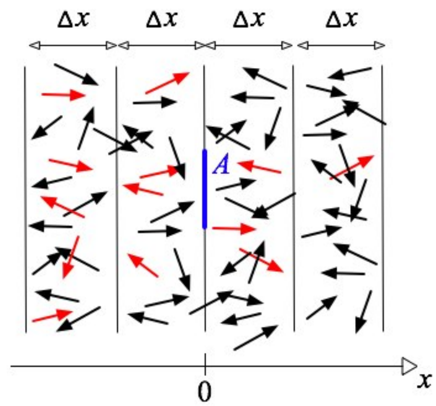
Outline

- Fick's law
  - The kinetic theory of diffusion
- Gradients
  - Gradient driven flow
  - Forces and potential energy
- Examples

12/5/11 Physics 131 **2**

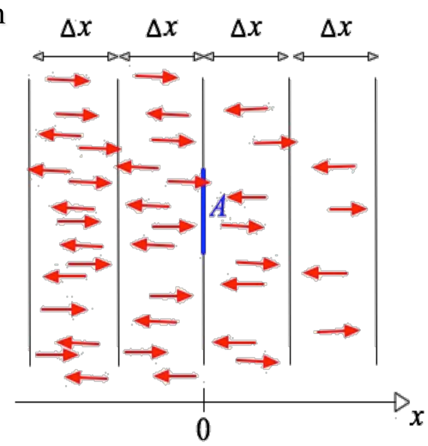
Diffusion: Fick's law (1D analysis)

- Uniform fluid containing (red) molecules with a varying concentration.
- Fluid molecules jiggle the (red) molecules around.



Fick's law: a simplified model of diffusion

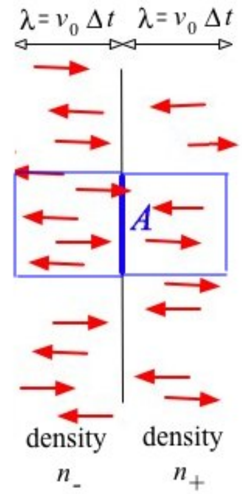
- The red molecules do a random walk (as a result of collisions with fluid molecules)
- Assume
  - Uniform density in each bin
  - Ignore up/down motions
  - Move with uniform (average) velocity
  - Choose bin width to be average distance red molecule travels before colliding.
  - Ask net amount going through a surface of area A in a time



How many cross A in a time Δt?

- Number hitting A from left= $\frac{1}{2} n_{-} \left(A v_0 \Delta t \right)$
- Number hitting A from right= $\frac{1}{2} n_{+} \left(A v_0 \Delta t \right)$
- Net = $\frac{1}{2} \left(n_{-} - n_{+} \right) \left(A v_0 \Delta t \right)$
- Rate (per unit area per unit time) =

$$J = \tfrac{1}{2} \left(-\frac{dn}{dx} \lambda \right) v_0 = -\left(\frac{\lambda v_0}{2} \right) \frac{dn}{dx}$$



12/5/11 Physics 131 **5**

Fick's law

- 1D result

$$J = -D \frac{dn}{dx} \quad D = \tfrac{1}{2} \lambda v_0$$

- For all directions (not just 1D) Fick's law becomes

$$\vec{J} = -D \vec{\nabla} n$$

12/5/11 Physics 131 **6**

The gradient

- If we want to take the derivative of a function of one variable, $y = df/dx$, it's straightforward.
- If we have a function of three variables – $f(x,y,z)$ – what do we do?
- The gradient is the **vector derivative.** To get it at a point (x,y,z)
  - Find the direction in which f is changing the fastest.
  - Take the derivative by looking at the rate of change in that direction.
  - Put a vector in that direction with its magnitude equal to the maximum rate of change.
  - The result is the vector called $\vec{\nabla} f$

12/5/11 Physics 131 **7**

What's a gradient good for?

- Flow is often driven by a change of a scalar quantity:
  - Diffusion – *Fick's law* (concentration gradient)
  - Fluid flow – *HP law* (pressure gradient)
  - Heat flow – *Fourier's law* (temperature gradient)
  - Electric current flow – *Ohm's law* (voltage gradient)
- Force is the gradient of potential energy

$$\vec{F}_{type} = -\vec{\nabla} U_{type}$$

12/5/11 Physics 131 **8**