November 28, 2011 Physics 131 Prof. E. F. Redish

## ■ Theme Music: Mason Williams

## Classical Gas

■ Cartoon: Jef Mallet
Frazz


11/28/11


Physics 131


1

## Outline

■ Modeling Matter:
The Kinetic Theory of Gases

- Maxwell's Theoretical Model
- Bouncing off the wall

Relating to the Ideal Gas Law

## Phenomenology: <br> The Ideal Gas Law

$\square$ The result is written

$$
p V=n_{\text {moles }} R T
$$

$\square$ where $R$ is a constant independent of the kind of gas you have.
■ $R=8.31 \mathrm{~J} / \mathrm{mol}^{-}{ }^{\circ} \mathrm{K}$
■ This result holds for any dilute gas.
(It has corrections if the gas gets too dense.)

## Question

■ What happens if you have a box of gas at STP and you pump in a bunch of molecules of a different kind of gas and wait until things settle down?
■ Will you have the same $p$ ? $T$ ?


## Foothold ideas:

## Kinetic Theory

■ Newton' s laws tell us that motion continues forever
 unless something unbalanced tries to stop it, yet we observe motion always dies away.

- Our model of matter as lots of little particles in continual motion lets us "hide" the energy of motion that has "died away" at the macro level in the internal incoherent motion.
- The model unifies the idea of heat and temperature with our ideas of motion of macroscopic objects.


## Quantifying the model

■ Each molecule that bounces off the wall exerts a force on the wall.
$\square$ The force on the wall will be the change in momentum of all the molecules that bounce off the wall in a time $\Delta t$ divided by $\Delta t$. (We've used N3!)

$$
F=\frac{d p}{d t}
$$

How much does the momentum of one molecule change when it hits a wall?
$F_{\text {wall } \rightarrow \text { molecule }}=m \frac{\Delta v_{x}}{\Delta t}=-F_{\text {molecule } \rightarrow \text { wall }}$
$=\frac{2 m v_{x}}{\Delta t}$


## How many molecules hit an area A of the wall in a time $\Delta t$ ?

- The up-down motion doesn't matter. As many will come in as will leave.

- So we can model it like this:



## How many molecules hit an area A of the wall in a time $\Delta t$ ?

$\square$ There are $n \mathrm{~V}$ in the box where $\mathrm{V}=$ volume $=A v_{\mathrm{x}} \Delta t$.

- Half are going left, half right. Only the half going right count.
- Therefore the number in the box that hit in a time $\Delta t$ is:

$$
n A v_{\mathrm{x}} \Delta t
$$



So the force is...

$$
F=\left(\frac{2 m v_{x}}{\Delta t}\right)\left(\frac{1}{2} n A v_{x} \Delta t\right)=n m v_{x}^{2} A
$$

So the pressure is...

$$
p=\frac{F}{A}=n m v_{x}^{2}=\frac{N}{V} m v_{x}^{2}
$$

Use the full velocity instead of $x . .$.

$$
\begin{array}{llll}
v^{2}=v_{x}^{2}+v_{y}^{2}+v_{z}^{2} & \text { Each direction is the same so } \\
v^{2}=3 v_{x}^{2} & \text { or } & v_{x}^{2}=\frac{1}{3} v^{2} & \text { so } \\
p V=\frac{1}{3} N m v^{2} & & & \\
& & & \\
\end{array}
$$

## Interpreting the Ideal Gas Law

$\square$ To relate our model to the IGL, note that since the number of molecules in one mole is the same (Avogadro's number) $N=n_{\text {moles }} N_{A}$ where $N_{\mathrm{A}}=6.02 \times 10^{23}$. So
$p V=\frac{1}{3} N m v^{2}=\frac{1}{3} n_{m o l} N_{A} m v^{2}=n_{m o l} R T$
so
$R T=\frac{1}{3} N_{A} m v^{2}$
$T=\frac{2}{3}\left(\frac{N_{A}}{R}\right)\left(\frac{1}{2} m v^{2}\right)$

## Put the equations together

$$
p V=N \frac{2}{3}\left(\frac{1}{2} m v^{2}\right) \quad p V=n R T
$$

Make the $N$ parts look alike.

$$
n=N / N_{A}
$$

$$
p V=N\left(\frac{R}{N_{A}}\right) T
$$

Define: $k_{B}=\left(\frac{R}{N_{A}}\right)$ so $p V=N k_{B} T$

## Interpreting



■ The "physicist's form" of the ideal gas law lets us interpret where the $p$ comes from and what $T$ means.
$\square p$ arises from molecules hitting the wall and transferring momentum to it;

- $T$ corresponds to the KE of one molecule (up to a constant factor).

$$
p=N m v_{x}^{2} \quad k_{B} T=\frac{2}{3}\left(\frac{1}{2} m v^{2}\right)
$$

## The Ideal Gas Law




