SOLUTIONS – 2 ONE DIMENSIONAL MOTION

FORMULAE

DISPLACEMENT: CHANGE OF POSITION

VECTOR

$$\Delta \underset{\rightarrow}{x} = (x_2 - x_1)\hat{x}$$

AVERAGE VELOCITY

$$\langle \underset{\rightarrow}{v} \rangle = \frac{x(t_2) - x(t_1)}{(t_2 - t_1)} \hat{x}$$

INSTANTANEOUS VELOCITY

$$\underset{\rightarrow}{v} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right) \hat{x}$$

Measures rate of change of position $\underset{\rightarrow}{x}$ with time.

INSTANTANEOUS ACCELERATION

$$\underset{\rightarrow}{a} = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right) \hat{x}$$

measures rate of change of $\underset{\rightarrow}{v}$ with time

MOTION WITH CONSTANT ACCELERATION

$$\underset{\rightarrow}{a} = a\hat{x}$$

$$\underset{\rightarrow}{v} = (v_i + at)\hat{x}$$

(change of v is area under a vs t graph.

$$\underset{\rightarrow}{x} = (x_i + v_i t + \tfrac{1}{2} a t^2)\hat{x}$$

or

$$\underset{\rightarrow}{x} = \left(\frac{v_i + v_f}{2}\right) t \hat{x} \qquad (v_f = v_i + at)$$

$$v^2 = v_i^2 + 2a(x - x_i)$$

FREEFALL All unsupported objects near surface of Earth have **CONSTANT** acceleration

$$\vec{a} = -9.8 m/s^2 \hat{y}$$

Hence

$$\vec{v} = (v_i - 9.8t)\hat{y}$$

$$\vec{y} = (y_i + v_i t - 4.9t^2)\hat{y}$$

$$v^2 = v_i^2 - 19.6(y - y_i)$$

UNIFORM MOTION $\vec{a} = 0$

$$\vec{v} = v\hat{x}$$

$$\vec{x}(t) = (x_i + vt)\hat{x}$$

change of x is area under v vs t graph.

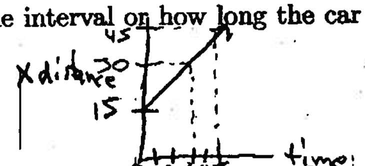
Chapter 2

5) Velocity is the relationship between an object's displacement and time. Note that the word dis-place-ment simply means shift in location. What we physicists mean when we use displacement is how much an object shifted in location and in what direction.

The displacement of our car is $(x_f - x_i)\hat{x} = (30-15)\hat{x} = 15\hat{x}$. Where I'm assuming the book is being straight with me and gave me 30m and 15m along the x axis (for example) as opposed to 30m north and 15m east or some such (you can see how that would mess things up?

The time interval that our car spent traveling that distance is 3 seconds.

Now given this information I can construct a ratio that instantly tells me how far the car has been displaced given a time interval on how long the car has traveled given a displacement.



$velocity = \vec{v} = \frac{15m}{3s}\hat{x} = 5\frac{m}{s}\hat{x}$

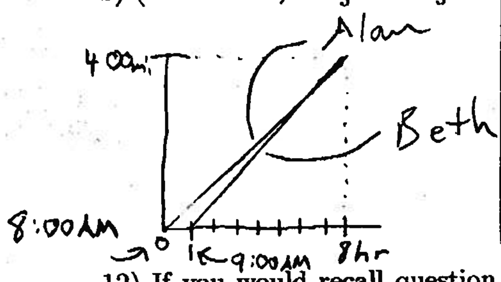
a) $(5\frac{m}{s} \times 1.5s)\hat{x} = 7.5m\hat{x}$. Are we done? Remember that the car started at 15m at t=0. So the final answer is $7.5m + 15m\hat{x} = 22.5m\hat{x} \approx 23m\hat{x}$

b) $(5\frac{m}{s} \times 5.0s + 15m)\hat{x} = 40m\hat{x}$

8) We can solve this problem in many ways but since both parts a and b require knowing the time elapsed while traveling why don't we calculate this immediately.

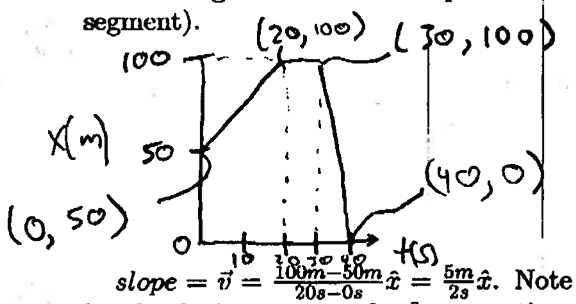
a) Alan: $t = 400mi \times \frac{1}{50\frac{mi}{hr}} = 8hrs$ Beth: $t = 400mi \times \frac{1}{60\frac{mi}{hr}} = 6\frac{2}{3}hrs$ Even though Alan has a one hour handicap it looks like Beth wins (barely)

b) $(8hrs - 1hr) - 6\frac{2}{3}hrs = \frac{1}{3}hrs.$



12) If you would recall question 5 the velocity is just a ratio (and a direction) where displacement is in the numerator and the time interval is in the denominator. Now as it happens the slope of a graph is the rise over run or change in the y axis over the change in the x axis. In this question the y axis is x (confusing I know) and the x axis is t. This just so happens to be the displacement over the time interval! To calculate the slope of

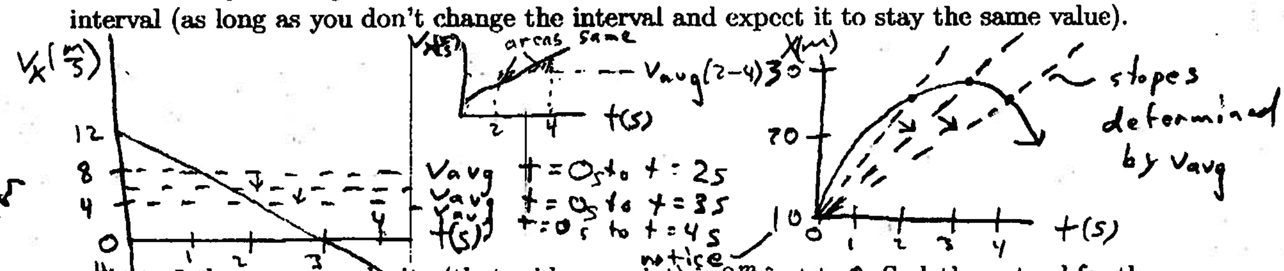
the first segment I subtract a point at t=20 from a point at t=0 (the endpoints of the line segment).



$slope = \vec{v} = \frac{100m-50m}{20s-0s}\hat{x} = \frac{5m}{2s}\hat{x}$. Note that t=10s is in that time interval so this calculated velocity answers the first question.

$slope = \vec{v} = \frac{0m-100m}{40s-30s}\hat{x} = -10\frac{m}{s}\hat{x}$. t=35s is in this time interval and velocity is constant so we have the velocity for the second part of the question.

14) There is a neat way to think about these problems. Find a point in a time interval in which the velocity is higher about half the time and lower the other half (try the midpoint : P). If the velocity is constantly changing then the distance you gain spending time above this point is exactly balanced out by the distance you lose spending time below this point. That means you can pretend that the velocity was at this middling value over the whole interval (as long as you don't change the interval and expect it to stay the same value).



Car reverses direction at t=3

At t=2 the average velocity (that midway point) is $8\frac{m}{s}\hat{x}$ ~~at t=2~~. So let's pretend for the purposes of calculating distance traveled at t=2 that v is constant at $8\frac{m}{s}$. $8\frac{m}{s} \times 2s = 16m$.

16m + 10m $\hat{x}$ = 26m $\hat{x}$

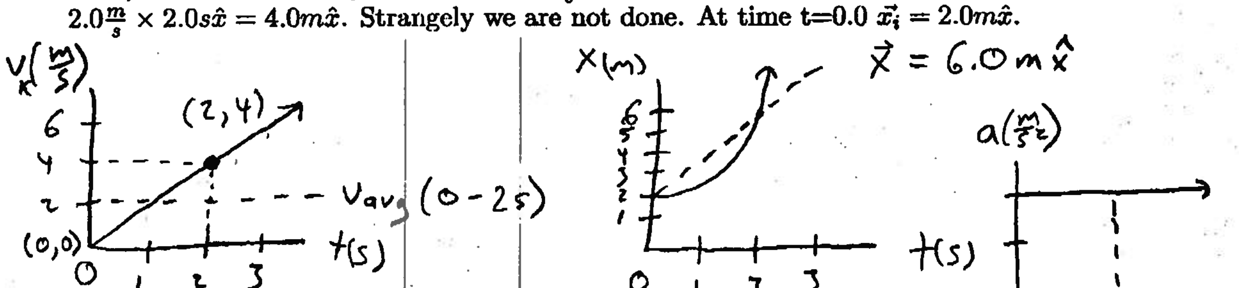
By the way you can also use the formula $v_{\vec{average}} = \left(\frac{v_f+v_i}{2}\right)\hat{x} = \left(\frac{4\frac{m}{s}+12\frac{m}{s}}{2}\right)\hat{x} = 8\frac{m}{s}\hat{x}$. Multiplying this by 2s is equivalent to finding the area under the curve so don't worry that we're straying from the text or the class.

At 3s the average velocity is $6\frac{m}{s}\hat{x}$. The total distance is $6\frac{m}{s} \times 3s\hat{x} = 18m\hat{x}$. (18m+10m)$\hat{x}$ = 28m$\hat{x}$

At 4s $v_{\vec{average}} = \frac{v_f+v_i}{2}\hat{x} = \frac{-4\frac{m}{s}+12\frac{m}{s}}{2}\hat{x} = 4\frac{m}{s}\hat{x}$. Distance is $4\frac{m}{s} \times 4s\hat{x} = 16m\hat{x}$

(16m+10m)$\hat{x}$ = 26m$\hat{x}$

17) At 2s the average velocity is $2.0\frac{m}{s}\hat{x}$. The total distance traveled in this time is $2.0\frac{m}{s} \times 2.0s\hat{x} = 4.0m\hat{x}$. Strangely we are not done. At time t=0.0 $\vec{x_i} = 2.0m\hat{x}$.



at t = 2.0s
$\vec{v}$ = 4.0m/s$\hat{x}$

acceleration is slope ~~of~~ of v vs t
$\vec{a}$ = 2m/s²$\hat{x}$

Pb24 FOR THIS PROBLEM !

use Eqn

$$v^2 = v_i^2 + 2a(x - x_i)$$

$\vec{v}_f = 300\,m/s\ \hat{x}$

$\vec{v}_i = 250\,m/s\ \hat{x}$

$(x - x_i) = 2000\,m\ \hat{x}$

$$a = \frac{v_f^2 - v_i^2}{2(x - x_i)} = \frac{(300)^2 - (250)^2}{2 \times 2000}$$

$$= 6.9\,m/s^2$$

$$\vec{a} = 6.9\,m/s^2\ \hat{x}$$

The answer is reasonable.

Pb26 IN FIRST 0.5sec motion is uniform

$\vec{v}_i = 20\,m/s\ \hat{x}$

reaction time $\Delta t = 0.5s$

$\vec{a} = -6\,m/s^2\ \hat{x}$

During 0.5s v_i is constant

$$\Delta \vec{x} = (20 \times 0.5)\,m\ \hat{x} = 10\,m\ \hat{x}$$

Once brake is on, use $v = 0$ in equation

$$v^2 = v_i^2 + 2a(x - x_i)$$

$$0 = (20)^2 - 2 \times 6\,(x - x_i)$$

and calculate $(x - x_i) = \frac{400}{12} = 33\,m$

Total $\Delta \vec{x} = (10 + 33)\,m\ \hat{x} = 43\,m\ \hat{x}$

Car will not hit obstacle

Pb 32 $\vec{v_i} = 19.6\,m/s\,\hat{y}$ $y_i = 0$

Free Fall

$\vec{v} = (v_i - 9.8t)\,\hat{y}$

$\vec{y} = (y_i + v_i t - 4.9t^2)\,\hat{y}$

$t = 1s$ $\vec{v} = (19.6 - 9.8)\,m/s\,\hat{y} = 9.8\,m/s\,\hat{y}$

$t = 2s$ $\vec{v} = (19.6 - 9.8\times2)\,m/s\,\hat{y} = 0\,\hat{y}$

$t = 3s$ $\vec{v} = (19.6 - 9.8\times3)\,m/s\,\hat{y} = -9.8\,m/s\,\hat{y}$

$t = 4s$ $\vec{v} = (19.6 - 9.8\times4)\,m/s\,\hat{y} = -19.6\,m/s\,\hat{y}$

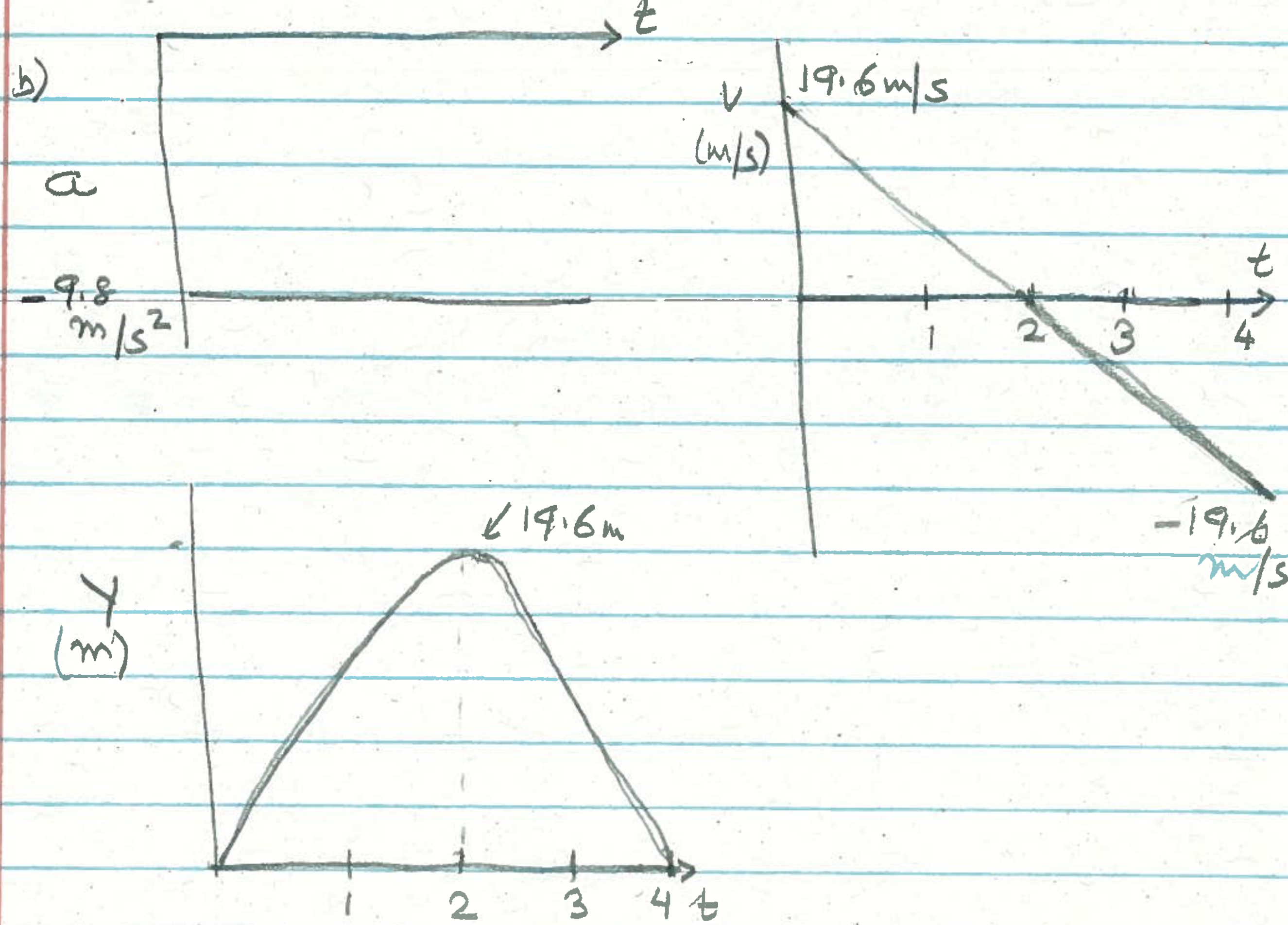
$t = 1s$ $y = (0 + 19.6 - 4.9)\,\hat{y} = 14.7\,m\,\hat{y}$

$t = 2s$ $y = (0 + 19.6\times2 - 4.9\times4)\,\hat{y} = 19.6\,m\,\hat{y}$

$t = 3s$ $y = (0 + 19.6\times3 - 4.9\times9)\,\hat{y} = 14.7\,m\,\hat{y}$

$t = 4s$ $y = (0 + 19.6\times4 - 4.9\times16)\,\hat{y} = 0$

Note: $\vec{a} = -9.8\,m/s^2\,\hat{y}$ whether the ball is travelling up or down b)



Pb 35 Two stones S_1 let go at $t=0$ with $\vec{v_i} = -2$ m/s, S_2 1 sec later, both arrive at $y=0$ at same time having started at $y_i = 50$m.

Free Fall $y = y_i + v_i t - 4.9t^2$

a) Consider S_1

$$0 = 50 - 2t - 4.9t^2$$

Solve quadratic in t

$$t = \frac{2 \pm \sqrt{4 + 4 \times 50 \times 4.9}}{-9.8} = 3 \text{ sec}$$

Keep −ive sign in numerator to get positive t

Both S_1, S_2 arrive together so S_2 hit 3 secs after S_1 was let go

For S_2 time of travel is $(3-1) = 2s$

so

$$y = 50 + v_i \times 2 - 4.9 \times 2^2$$

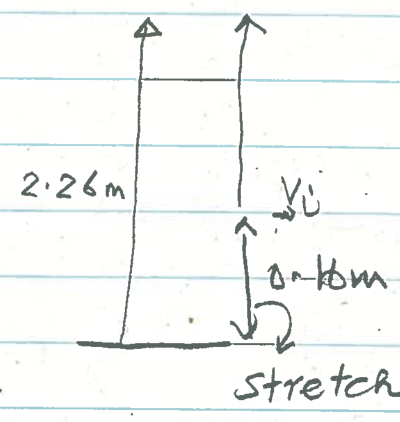
b) $$\vec{v_i} = -\frac{50 + 19.6}{2} \text{ m/s } \hat{y} = -15.2 \text{ m/s } \hat{y}$$

c) just before arrival at $y=0$

i) for S_1 $$\vec{v} = (-2 - 9.8 \times 3) \text{ m/s } \hat{y}$$
$$= -31.4 \text{ m/s } \hat{y}$$

for S_2 $$\vec{v_2} = (-15.2 - 9.8 \times 2) \text{ m/s } \hat{y}$$
$$= -34.8 \text{ m/s } \hat{y}$$

Pb 52 Once the leg stretch is over, bush baby is in free fall with $\vec{v_i}$ and $y_i = 0.16$ m



At 2.26 m, $\vec{v} = 0$.

$v^2 = v_i^2 - 19.6(y - y_i)$ $$0 = v_i^2 - 19.6(y - y_i)$$
$$v_i = 19.6 \times 2.1$$
$$\vec{v_i} = 6.42 \text{ m/s } \hat{y}$$

This velocity is acquired while travelling 0.16 m starting from rest

$$(6.42)^2 = 0 + 2a \times 0.16$$
$$\vec{a} = \frac{(6.42)^2}{0.32} \hat{y} = 128.6 \text{ m/s}^2 \hat{y}$$

which is ~13 times the acc. due to gravity.

Pb 56 $\quad (y = y_i + v_i t - 4.9t^2)\hat{y}$

$y_i = 2.0\,m$

$v_i = 15\,m/s.$

To return to ground

$$y = 0 = 2 + 15t - 4.9t^2$$

$$t = \frac{-15 \pm \sqrt{225 + 4\times2\times4.9}}{-9.8}$$

$$= \frac{-15 \pm \sqrt{264.2}}{9.8} = 3.2\,sec.$$

Here gain we kept -ive sign in numerator to get a positive answer for t.

Incidentally the negative value of t can be understood by looking at y vs t graph

noting that it would have to be launched from $y = 0$ with $\vec{v}_i = +16.36\,m/s\,\hat{y}$