WAVE OPTICS: INTERFERENCE AND DIFFRACTION

Radiation: Electromagnetic wave
Light: Transverse E.M. Wave

$\lambda_0 : 400nm < \lambda_0 < 800nm$ [in vacuum]

$f : 4\times10^{14} < f < 8\times10^{14} Hz$

Speed $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^{8} m/\sec$ in vacuum

$V = \dfrac{c}{n}$ in medium, $n > 1$

$\lambda_n = \dfrac{\lambda_0}{n}$

We can represent a light wave travelling along x as an $\underline{E}$-wave

$$E = E_m \sin(k\omega - \omega t + \Phi),\ \underline{E_m} \perp \hat{x}$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = \frac{2\pi}{f}$$

$$E_m = \text{amplitude},\ \Phi = \text{phase}$$

Superposition:
Recall that when more than one wave is present at the same point at the same time, the net effect is obtained by making an algebraic sum.

Let us consider two light waves

$$E_1 = E_m \sin(kx - \omega t + \Phi_1)$$
$$E_2 = E_m \sin(kx - \omega t + \Phi_2)$$

That is, they have the same wavelength and the same frequency but the phases are different.

As Discussed in class, emission of light involves an electron jumping from one energy level to another in its parent atom and each jump lets out a wave train about 3m long. Since there are "zillions" of atoms we have an enormous number of wave trains with arbitrary phases so a light wave from a typical source has a phase which varies randomly in time.

If you superpose E_1 and E_2 you will get

$$E = E_1 + E_2$$
$$= 2E_m \cos\frac{(\Phi_1 - \Phi_2)}{2}\sin\left(kx - \omega t + \frac{\Phi_1 + \Phi_2}{2}\right)$$

That is, a wave whose amplitude is

$$Amp = 2E_m \cos\left(\frac{\Phi_1 - \Phi_2}{2}\right)$$

Intensity $\quad I \;\alpha\; (Amp)^2$

So $\quad I \;\alpha\; 4E_m^{\;2}\cos^2\frac{(\Phi_1 - \Phi_2)}{2}$ [the factors $\frac{1}{2}\varepsilon_0 c$ are left out]

Two totally different situations arise

Case I The sources of E_1 and E_2 are INCOHERENT

That is, $(\Phi_1 - \Phi_2)$ is a random function of time. If so, I is also a random function of time. The observed value will be a time average:

$$<I> \;\alpha\; 4E_m^{\;2} < \cos^2\left(\frac{\Phi_1 - \Phi_2}{2}\right) >$$

But $\quad < \cos^2\left(\frac{\Phi_1 - \Phi_2}{2}\right) >= \frac{1}{2}$

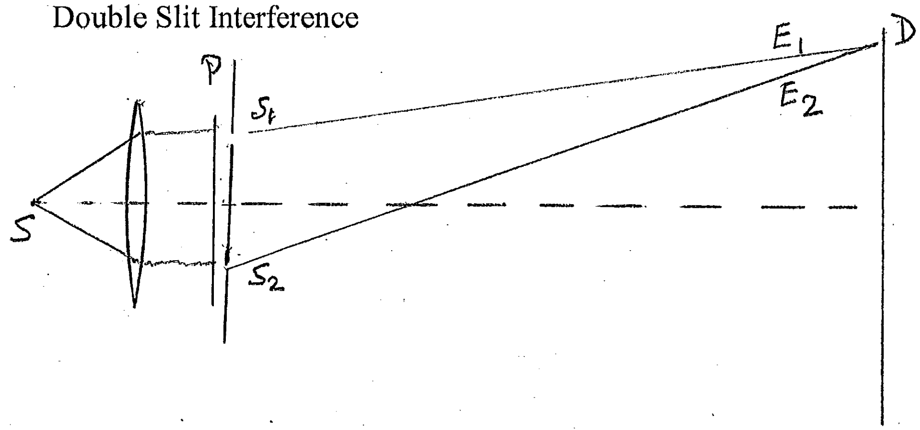
So $\quad <I> \;\alpha\; 2E_m^{\;2}$

Hence, for two incoherent sources the total intensity is just the sum of the two intensities. Two light bulbs just increase brightness.

Case II The sources of E_1 and E_2 are COHERENT

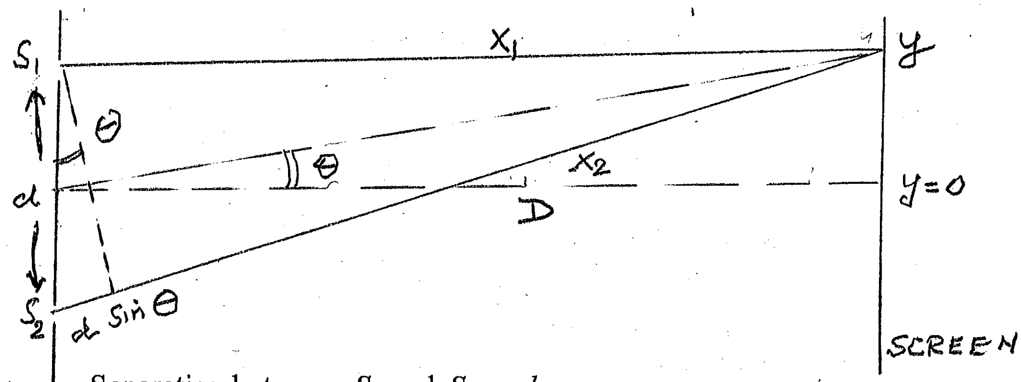
That is, the waves E_1 and E_2 are specially prepared in such a way that $(\Phi_1 - \Phi_2)$ is a fixed quantity (independent of time for our discussion) at any given location. [This is the case we discussed for sound waves a few weeks ago]. So how do we get two coherent light sources. We discuss two examples.

I Double Slit Interference



S is a point source of light located at the focal point of a convergent lens. After passing through the lens the light becomes a parallel beam. The corresponding wave front is a plane P travelling toward the right. Now, place a plate with two small holes each of width w separated by d. Assume $w << d$. The waves which emerge from S_1 and S_2 are both derived from the <u>SAME</u> wave front so at S_1 and S_2 they have same phase (say zero). By the time they arrive at the detector D their phases would have changed (see detail below) but $(\Phi_1 - \Phi_2)$ does NOT vary with time. We have two coherent sources producing E_1 and E_2 at D.

[In your experiment the source is a laser which produces a parallel beam. S_1 and S_2 are slits in a plate and you used a screen to view the interference pattern]



Separation between S_1 and S_2 $= d$
Distance to screen $= D$
Position of detector $= y$ [$y = 0$ at mid-point of sources]

x_1 = distance travelled by E_1
x_2 = distance travelled by E_2

At y: Phase of E_1, $\Phi_1 = \dfrac{2\pi}{\lambda} x_1$

Phase of E_2, $\Phi_2 = \dfrac{2\pi}{\lambda} x_2$

$$\left(\frac{\Phi_1 - \Phi_2}{2}\right) = \frac{2\pi}{\lambda}\left(\frac{x_1 - x_2}{2}\right)$$

If $(x_1 - x_2) = M\lambda$, $\quad M = 0, \pm 1, \pm 2 \ldots$

$$\frac{\Phi_1 - \Phi_2}{2} = M\pi$$

$$\cos^2\left(\frac{\Phi_1 - \Phi_2}{2}\right) = 1$$

So at such points I will be maximum

Condition for Maxima

$$(x_1 - x_2) = M\lambda, \qquad M = 0,\ \pm 1,\ \pm 2...$$

However, if $\quad (x_1 - x_2) = \left(m + \frac{1}{2}\right)\lambda, \qquad m = 0,\ \pm 1,\ \pm 2...$

$$\frac{\Phi_1 - \Phi_2}{2} = \left(m + \frac{1}{2}\right)\pi$$

$$\cos^2\left(m + \frac{1}{2}\right)\pi = 0 \qquad \underline{I = 0}$$

Condition for Minima

$$(x_1 - x_2) = \left(m + \frac{1}{2}\right)\lambda$$

From the picture you can see that

$$(x_1 - x_2) = d\sin\Theta$$

So

$$d\sin\Theta_M = M\lambda \qquad \text{[Maxima]}$$

$$d\sin\Theta_m = \left(m + \frac{1}{2}\right)\lambda \qquad \text{[Minima]}$$

And of course, all angles are small, $\frac{\lambda}{d} << 1$.

Consider the y-coordinate of the M^{th} maximum.

$$\frac{y_M}{D} = \tan\Theta_M = \sin\Theta_M$$

$$= \frac{M\lambda}{d}$$

Similarly, its next neighbor has

$$\frac{y_{M+1}}{D} = \frac{(M+1)\lambda}{d}$$

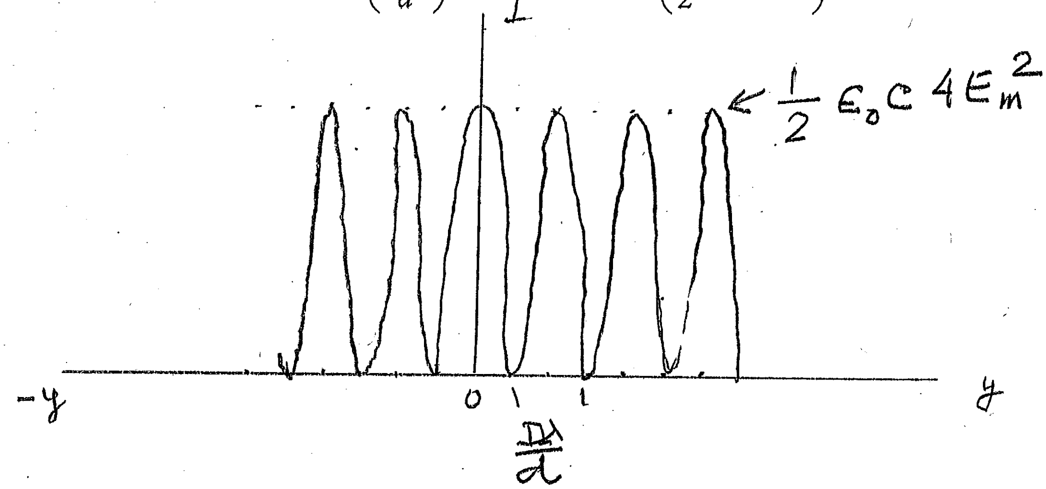
So

$$y_{M+1} - y_M = \frac{D\lambda}{d}$$

So for two slit interference

$$I = \frac{1}{2}\varepsilon_0 c 4 E_m{}^2 \cos^2\left(\frac{\Phi_1 - \Phi_2}{2}\right)$$

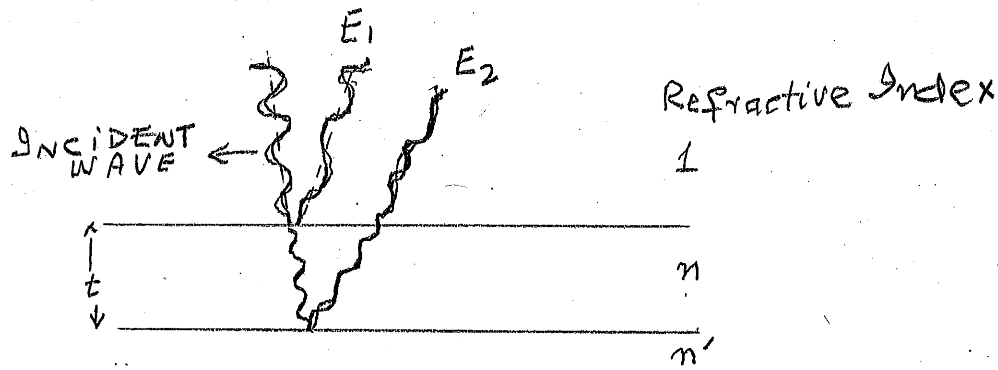
and consists of equally spaced $\left(\frac{D\lambda}{d}\right)$ equal intensity $\left(\frac{1}{2}\varepsilon_0 c 4E_m^{\ 2}\right)$ fringes.



Space Average intensity on screen

$$< I >= \frac{1}{2}\varepsilon_0 c 2E_m^{\ 2} \text{ !!} \qquad \left[< \cos^2 \Theta >= \frac{1}{2}\right]$$

II Thin Film Interference



A thin film of thickness t and refractive index n is deposited on a block of refractive index n'. A light wave is incident on the top surface at an angle of incidence of a fraction of a degree $(i \approx 0,\ r \approx 0,\ R \approx 0)$. On reflection from the top surface we get one wave (designated E_1), part of the light enters the film producing the second wave (E_2). Both E_1 and E_2 are derived from the same incident wave so their phase difference is a fixed quantity depending on the thickness of t. E_1 and E_2 are coherent.

The condition for maxima and minima are of course,

$$(x_1 - x_2) = M\lambda, \qquad M = 0,\ \pm 1,\ \pm 2...$$

Or

$$(x_1 - x_2) = \left(m + \frac{1}{2}\right)\lambda, \qquad m = 0,\ \pm 1,\ \pm 2...$$

However, now we must also consider what happens to the phase when a wave undergoes reflection.

Recall what we learnt while studying reflection of waves on stretched strings except now we cast it in terms of E-wave.

Incident wave
$E_i = E_{mi} \sin(kx - wt)$
Reflected Wave
$E_r = E_{mr} \sin(kx + wt)$

and $\dfrac{E_{mr}}{E_{mi}} = \dfrac{V_1 - V_2}{V_1 + V_2}$

Let us compare waves at $x = 0$ where the reflection occurs.

$$E_i = E_{mi} \sin(-wt) = E_{mi} \sin(wt + \pi)$$
$$E_r = E_{mr} \sin wt$$

Two Cases Arise

i) $V_1 < V_2$ $\quad [n_1 > n_2]$

$\dfrac{E_{mr}}{E_{mi}}$ is negative

$$E_i = E_{mi} \sin(wt + \pi)$$
$$E_r = E_{mr} \sin(wt + \pi)$$

No Phase Change.

ii) $V_1 > V_2$ $\quad [n_1 < n_2]$

$\dfrac{E_{mr}}{E_{mi}}$ is positive

$$E_i = E_{mi} \sin(wt + \pi)$$
$$E_r = E_{mr} \sin wt$$

phase change of π on reflection.

Now let us consider interference between E_1 and E_2.

First, extra distance travelled by E_2 is $2t$ but refractive index is n so wavelength in medium is $\frac{\lambda_0}{n}$ where λ_0 is wavelength in air.

Next, if $n' > n$ $[v' < v]$ there is π phase change for both E_1 and E_2 so condition for maximum is

$$2nt = M\lambda_0 \qquad M = 0,\ \pm 1,\ \pm 2 \ldots$$

However if $n' < n$ $[v' > v]$. Only E_1 has a phase change while E_2 has none so condition for maximum becomes

$$\left(2nt - \frac{\lambda_0}{2}\right) = M\lambda_0$$

Notice a phase change of π is like a path difference of $\frac{\lambda_0}{2}$.

Colors of thin films of oil on water, or the surface of soap bubbles, arise because of thin film interference. Non-reflecting glass is produced by depositing a thin layer of transparent material and ensuring destructive interference for $\lambda_0 \sim 600nm$ [Green Light].

MULTIPLE SOURCE INTERFERENCE – ALL SOURCES COHERENT

# of sources   Amplitudes

2

3

4