

SOUND WAVES-BEATS

What would you hear (detect) if two sound waves of slightly* different frequencies arrived at your ear at the same time.

Let us put detector at $x = 0$ and have two waves

$$S_1 = S_m \sin(kx + \omega t)$$

$$\frac{\omega}{k} = V_s = \frac{\omega'}{k'}$$

$$S_2 = S_m \sin(k'x + \omega' t)$$

$x = 0$



Ear

Superpose S_1 and S_2 at $x = 0$

$$\begin{aligned} S &= S_m (\sin \omega t) + S_m \sin(\omega' t) \\ &= 2S_m \cos\left(\frac{\omega - \omega'}{2}t\right) \cos\left(\frac{\omega + \omega'}{2}t\right) \end{aligned}$$

* $(\omega - \omega') \ll \omega$ $[(f' - f) \ll f]$ so $\frac{\omega + \omega'}{2} \approx \omega$

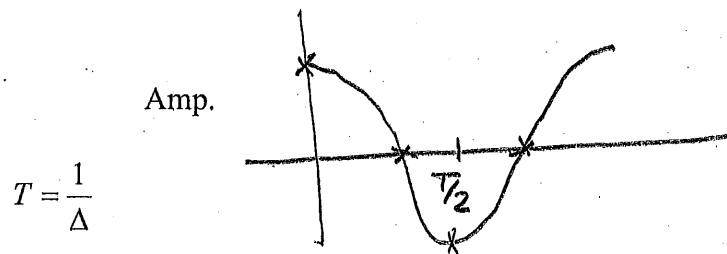
$$S = 2 \sin \cos \left[2\pi \left(\frac{f - f'}{2} \right) t \right] \sin \omega t$$

So you perceive a wave of frequency f whose amplitude varies as

$$\cos \frac{2\pi(f - f')}{2} t$$

or at a frequency $\Delta = \left(\frac{f - f'}{2} \right)$

That is if you plot amplitude as a function of t you get:



Note that in one period there are two maxima and two zeros so you will detect

$$f_B = |f - f'|$$

as the number of Beats when both sounds arrive together, you hear one frequency, but its amplitude has $|f - f'|$ maxima and $|f - f'|$ zeros every second.

The two bars in the Demo had frequencies $f = 440\text{Hz}$ and $f' = 445\text{Hz}$ you heard 440Hz and five zeros/maxima per second.